Mathematical Model For The Spread of Coronavirus Disease In Tamilnadu

Dr.T.Brindha¹, K.S.Haripriya²

¹Department of Mathematics PG, PSGR Krisnammal College for Women, Coimbatore, Tamil Nadu,
²II MSc Mathematics, PSGR Krisnammal College for Women, Coimbatore, Tamil Nadu.

Abstract - In this dissertation, we develop a mathematical model of the COVID-19 pandemic in the Tamil Nadu. We use real data from the Tamil Nadu government’s open-source website tn.data.gov.in against COVID-19 and propose a compartmental mathematical model for the spread of the COVID-19 disease. We use the public policies in the model to reduce the contact rate.

Keywords – COVID-19, Mathematical model, Basic reproduction number, Public Policies and Numerical Simulation.

I. INTRODUCTION

Of all pandemics that have appeared the Coronavirus disease (COVID-19) is the most dreadful disease which left everyone in the world to remain inside the house because of the spread. Despite the advanced technologies in medicine and science, it taught us that humanity remains powerless before nature. Since December 2019, the Coronavirus disease (COVID-19) has been spreading all over the world. During the outbreak in Wuhan, China, the infectious disease quickly spread into other countries. In India the first case was reported in Kerala on 27 January 2020 and finally reached Tamil Nadu on 07 March 2020; by a 45-year-old man who returned to India from Oman. The Tamil Nadu state has been under a lockdown since 25 March which was extended until 30 June with significant relaxations from 1 June. The state has enforced a stricter lockdown in four majority affected districts which include Chennai and its three neighboring districts of Chengalpattu, Thiruvallur, and Kancheepuram from 19 to 30 June. Currently, many mathematical models of the COVID-19 has been developed, mainly for a Coronavirus epidemic. We have developed a model describing the Coronavirus epidemic in Tamil Nadu, focusing on public policies, and numerical simulation.

II. MODEL PARAMETERS AND MODEL FORMULATION

We prepare a mathematical model on Coronavirus which is valid physically and hypothetical. Numerical illustration has been carried out for different parameters, functions, and self-data creation. The results have been represented graphically for infective, recovered, and mortality rate on physical conditions.

Considering the known characteristics of Coronavirus disease (COVID-19) pandemic from the obtained data, we assume that each person in the population (N) falls into one of the following compartments:

- **S (Susceptible)** – the number of persons who are not infected by the disease at time t but potentially able to get in contact in the future.
- **E (Exposed)** – the number of persons who are in incubation period after being infected by the disease. These persons don’t have any visible clinical signs. They can infect other people but with a lower probability than people in the infectious
stages.

- **I (Infected)** – the number of persons who start developing clinical signs and even become severe. These people will be either quarantined or admitted to the hospital for treatment.

- **R (Recovered)** – the number of persons who have survived the disease, is no longer infectious, and has developed natural immunity to the disease.

- **F (Fatality)** – the number of persons who are died because of the disease.

\[
\begin{align*}
\frac{dS}{dt} &= -\beta \frac{S(t)}{N} I(t) \\
\frac{dE}{dt} &= \beta \frac{S(t)}{N} I(t) - \alpha E(t) \\
\frac{dI}{dt} &= \alpha E(t) - (\gamma + \delta) I \\
\frac{dR}{dt} &= \gamma I \\
\frac{dF}{dt} &= \delta I
\end{align*}
\]

Here \( N = S + E + I + R + F \) is independent of time ‘t’ denote total population size. The parameters used are:

- \( \beta \) : Transmission coefficient from infected individuals
- \( \alpha \) : Rate at which exposed become infectious
- \( \gamma \) : Recovery rate of an infected person
- \( \delta \) : Disease induced death rate of an infected person

**A. DISEASE-FREE EQUILIBRIUM:**

The disease-free equilibrium is defined as the point at which no disease is present in the population. To determine the equilibrium point without disease, it is assumed on the one hand functions \( S, E, I, R, F \) are constant and on the other hand that we have no infection.

Thus, the disease-free equilibrium lies at the point

\[ Q_0 = (0, 0, \frac{\beta}{\delta}, 0, 0) \]

**B. BASIC REPRODUCTION NUMBER OF OUR MODEL:**

The basic reproduction number for our model is defined by

\[ R_0 = \frac{\beta}{{(\gamma + \delta)N}} \]

**Proof.** We use the method of the next-generation matrix to compute the Reproduction number \( R_0 \)

We get

\[
F = \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} \frac{\beta}{\gamma} & 0 \\ 0 & 0 \end{pmatrix} \quad V = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} -\alpha E \\ \alpha E - (\gamma + \delta)I \end{pmatrix}
\]

We have
\[ DF = \begin{pmatrix} \frac{\partial F_1}{\partial E} & \frac{\partial F_2}{\partial E} \\ \frac{\partial F_1}{\partial I} & \frac{\partial F_2}{\partial I} \end{pmatrix} = \begin{pmatrix} 0 & \beta S^0 \N \\ 0 & 0 \end{pmatrix} \]

And

\[ DV = \begin{pmatrix} \frac{\partial V_1}{\partial E} & \frac{\partial V_1}{\partial I} \\ \frac{\partial V_2}{\partial E} & \frac{\partial V_2}{\partial I} \end{pmatrix} = \begin{pmatrix} -\alpha & 0 \\ \alpha & -(\gamma + \delta) \end{pmatrix} \]

On \( E_0 = (S^0, 0) \), we get

\[ F = \begin{pmatrix} 0 & \beta S^0 \N \\ 0 & 0 \end{pmatrix} \]

And

\[ V = \begin{pmatrix} -\alpha & 0 \\ \alpha & -(\gamma + \delta) \end{pmatrix} \]

Thus we obtain

\[ -FV^{-1} = \begin{pmatrix} 0 & \beta S^0 \N \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\alpha} & 0 \\ \frac{1}{1(\gamma + \delta)} & \frac{1}{(\gamma + \delta)} \end{pmatrix} \]

\[ = \begin{pmatrix} \frac{\beta}{(\gamma + \delta) \N} & \frac{\beta S^0}{(\gamma + \delta) \N} \\ 0 & 0 \end{pmatrix} \]

\[ -FV^{-1} = \frac{\beta S^0}{(\gamma + \delta) \N} \]

The eigenvalue of the matrix are 0 and \( \frac{\beta S^0}{(\gamma + \delta) \N} \)

The basic reproduction number is defined as the dominant eigenvalue of the matrix \(-FV^{-1}\)

Therefore,

\[ R_0 = \frac{\beta S^0}{(\gamma + \delta) \N} \]
The basic reproduction number $R_0$ is defined as the number of cases that one infected person generates on average during his infectious period, in an uninfected population and without any special control measures. This number does not change during the spread of the disease.

The effective reproductive number $R_e(t)$ is defined as the number of cases that one infected person generates during his infectious period. This effective reproduction number depends on time, soon public policies (change during the spread of the disease). Furthermore $R_e(0) = R_0$ and the spread of the disease slows when $R_e(t) < 1$

Since

$$R_0 = \frac{\beta}{(\gamma + \delta)} \frac{S_0}{N}$$

Therefore

$$R_e(t) = \frac{\beta}{(\gamma + \delta)} \frac{S(t)}{N}$$

Here $R_e(t)$ known as $R_{effective}$ or Effective reproduction rate. This represents the number of people each infected individual goes on to infect which can change over time, depending on behavior, immunization, etc…The R number is not fixed, but can be affected by a range of factors, including not just how infectious a disease is but how it develops over time, how a population behaves, and any immunity already possessed.

III. PUBLIC POLICIES

The Tamil Nadu state government has taken several actions to contain the spread and impact of COVID-19. The following regulations detail the responsibilities of hospitals and individuals, and the powers of officials in relation to the diagnosis, treatment, and containment of COVID-19. These include:

- Creation of isolation wards in hospitals.
- Containment measures in an area once positive cases are detected.
- Mandatory 14-day home isolation for asymptomatic air travelers from COVID-19 affected countries.
- Shutting down of establishments, such as educational institutions, theatres, malls, etc.
- Banning interstate travel.
A. WITHOUT ANY PUBLIC POLICIES UNTIL JUNE 27, 2020:

Even though some districts were isolated the overall state lockdown was announced after June 27, 2020. The maximum number of people was tested positive in the middle of July. Though the lockdown was announced the people who are already in contact with the infected were tested positive so that the positive cases abruptly increased amid July therefore we consider this as without public policy stage since most of the positive cases are due to the contact with infected before the lockdown was announced. We take COVID-19 Database Chart for July-August month which is shown in Fig.1

![COVID-19 Data chart (Jul-Aug 2020)](chart1)

**Fig.1** COVID-19 Database Chart for July-August

B. WITH PUBLIC POLICIES SINCE JUNE 28, 2020:

From this date, the overall lockdown was announced. Since we cannot able to see the result of lockdown within few days we take the COVID-19 database for January 2021 which is shown in Fig.2

![COVID-19 Data chart (Jan 2021)](chart2)

**Fig.2** COVID-19 Database Chart for January month
C. RESULT OF THIS PUBLIC POLICIES:

From the obtained data we get to know that approximately the contact rate has dropped by 17% by the implementation of public policies. According to some surveys, it is said that the total number of COVID-19 cases would have been increased by 23% by now if the public policies are not announced in July and by January it would have been increased by 58% and above. The implementation of lockdown has forced many countries in flattening the epidemic curve and strengthening the health care system and improves environmental quality.

Though the Public policies do not give a quick decrease in the disease it eventually slows down the spread slowly. We can see that the positive cases were massively decreased after six months.

IV. NUMERICAL SIMULATION

Numerical Simulation for our compartment model which is a non-linear system is obtained by estimating it from the exponential equation.

Since our model is in exponential growth which is satisfied by

\[ I(t) = I_0 e^{mt} \]

Here m is considered to be not depending on the population size.

To find ‘m’, we use the method called Curve Fitting. Since it is a non-linear function with huge data it is unable to do manually. Curve Fitting helps us to find out the non-linear function of the given data accurately. Here we use Curve Fitting to fit the curve to the data of 16 July 2020 – 15 August 2020 and January 2021 respectively.

A. Estimating β, γ, δ for July-August:

At the onset of infection, almost the entire population is susceptible \( S \approx N \)
so \( I(t) \) first grows exponentially

\[ \frac{dI}{dt} \sim mI \]

Where, \( m = \beta - \gamma \)

\[ I(t) \sim I_0 e^{mt} \]

We can estimate \( m \) by looking at the data on an exponential plot

July-August COVID-19 data is shown in Fig.3
From the data the non-linear equation obtained is $y = 5432e^{0.0039x}$

Where ‘y’ is the number of positive cases and ‘x’, represents the time, from this we get the value of m as

$$m \approx 0.003$$

Estimating $\gamma$ directly from the data

$$\gamma = \frac{R(t+1) - R(t)}{I(t)}$$

The obtained value of $\gamma$ is $\gamma \approx 0.7974$

Then use $\beta = m + \gamma$, we get $\beta$ value as $\beta \approx 0.8274$

Estimating $\delta$ directly from the data

$$\delta = \frac{F(t+1) - F(t)}{I(t)}$$

The obtained value of $\delta$ is $\delta \approx 0.0127$

The total population of Tamil Nadu is $N = 77841267$ and $S^0 = 77793927$

Since the equation of $R_0$ is

$$R_0 = \frac{\beta}{(\gamma + \delta)} \frac{S^0}{N}$$

Substituting the obtained values in $R_0$ we get

$R_0 \approx 1.02064$  

here $R_0 > 1$
B. Estimating $\beta, \gamma, \delta$ for January:

January COVID-19 data is shown in Fig.4

![Curve Fitting graph for the COVID-19 of January 2021](image)

From the data the non-linear equation obtained is $y = 909e^{-0.021x}$

From this, we get the value of $m$ as $m \approx -0.021$

Estimating $\gamma$ directly from the data, the obtained value of $\gamma$ is $\gamma \approx 0.0238$

Then use $\beta = m + \gamma$, we get $\beta$ value as $\beta \approx 0.0038$

Estimating $\delta$ directly from the data, the obtained value of $\delta$ is $\delta \approx 0.0217$

The total population of Tamil Nadu is $N = 77841267$ and $S_0 = 77793927$

Substituting the obtained values in $R_0$ we get

$R_0 \approx 0.146161$  here $R_0 < 1$

CONCLUSION

In this paper, we have developed a mathematical model of COVID-19 for the Tamil Nadu. We have been able to estimate some parameters which have made it possible to fit the model to real data. It emerges from the model that the most important parameter here is the contact rate which is a time-dependent function (with respect to the public policies taken). A drastic reduction of the contact rate can lead to a considerable reduction in the number of infectious and of the duration of the epidemic.

From July 16th to August 15th of 2020 data we get the value for $R_0$ which greater than 1 which is considered as epidemic this is because the public policies were announced recently but when we take January 2021 data we get the value for $R_0$ is less than 1 which is not epidemic anymore. This shows that public policies play a big role in controlling the spread but it is not the solution to eradicate the disease because if the public policies were withdrawn there is still more chance to increase the spread like when the epidemic started. The only solution is to vaccinate the people before the public policies withdrawn.
REFERENCES


