Signed and Signed Product Cordial Labeling of Cylinder Graphs and Banana Tree

V.Jude Annie Cynthia[1] and E. Padmavathy[2,3]
[1] Department of Mathematics, Stella Maris College, Chennai,India.
[2] Research Scholar, Dept.of Mathematics, Stella Maris College, Chennai,India.
[3]Asst.Prof., Dept.of Mathematics, C.T.T.E College for women, Chennai, ,India.

Abstract

In this paper we investigate signed and signed product cordiality of cylinder graphs and banana tree.

Keywords - *Signed cordial, Signed product cordial, Cylinder Graphs, Banana tree.*

I. INTRODUCTION

We begin with finite, connected and undirected graph $G = (V(G), E(G))$without loops and multiple edges. Graph labelings of diverse types are currently the subject of much study. Most graph labeling methods trace their origin to one introduced by Rosa in 1967[1]. Labeled graph are becoming an increasingly useful family of mathematical models for a broad range of application. The state of the field is described in detail in Gallian's dynamic survey [4]. Results obtained so far, while numerous, are mainly piecemeal in nature and lack generality. Harary introduced *S*-Cordiality with the first letter of Signed Cordiality. In order to maintain compactness we will provide a brief summary of definitions.

Definition :1.1
A *graph labeling* is an assignment of integers to the vertices or edges, or both, subject the certain conditions.
Definition :1.2
A mapping $f: V(G) \to \{0,1\}$ is called *binary vertex labeling* of G under $f(v)$, called label of vertex v of G under f. The induced edge labeling $f^*: E(G) \to \{0,1\}$ defined by $f^*(uv) = |f(u) - f(v)|$, Let $v_f(i)$ and $e_f(j)$ are respectively the number of vertices labeled with i under f and the number of edges labeled with j under f^*. A binary vertex labeling of graph G is cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, A graph G is *cordial* if it admits cordial labeling.
Definition :1.3
A graph $G = (V, E)$ is called *signed cordial* if it is possible to label the edges with the number from the set $N = \{+1, -1\}$ in such a way that at each vertex v, the algebraic product of the labels on the edges incident with V is either $+1$ or -1 and the inequalities $|v_f(+1) - v_f(-1)| \leq 1$ and $|e_{f^*}(+1) - e_{f^*}(-1)| \leq 1$ are also satisfied, where $v_f(i), i \in \{+1, -1\}$ and $e_f(j), j \in \{+1, -1\}$ are respectively the number of vertices labeled with i and the number of edges labeled with j. A graph is called *signed-cordial* if it admits a signed-cordial labeling.
Definition :1.4
A vertex labeling of graph G $f: V(G) \to \{-1, +1\}$ with induced edge labeling $f^*: E(G) \to \{+1, -1\}$ defined by $f^*(uv) = f(u)f(v)$ is *signed product cordial labeling* if $|v_f(1) - v_f(-1)| \leq 1$ and $|e_f(1) - e_f(-1)| \leq 1$, where $v_f(i)$ and $e_f(j)$ are respectively the number of vertices labeled with i and the number of edges labeled with j. A graph G is *signed product cordial* if it admits signed product cordial labeling.
Definition :1.5
A *cylinder* $C_m \times P_n$, where $m, n \geq 3$ is a $P_m \times P_n$ grid with wraparound edge in each row. It is clear that the vertex set of $P_m \times P_n$ is $V = \{x_1x_2: 0 \leq x_i \leq d_i - 1, i = 1,2\}$ are two vertices $x = x_1x_2$ and $y = y_1y_2$ are linked by an edge, if $|x_1 - y_1| + |x_2 - y_2| = 1$.
Definition :1.6
A *banana tree*, $B_{n,k}$[18] is a graph obtained by connecting one leaf of each n copies of an $k -$star graph (S_k) to a new vertex. We denote the vertex as *root vertex*, denoted x. The vertices of distance 1 from the root vertex as the *intermediate vertices* denoted by $m_i, i = 1,2, \dots n$. The *center* of every S_k is denoted by l_i , $i = 1,2, \dots n$. We denoted the $j -$th leaf of the center l_{ij} $(j = 1,2, \dots k - 2)$.

II. LITERATURE SURVEY

The concept of cordial graph was introduced by Cahit [3]. Shee and Ho [16] proved that path union of cycles, Petersen graphs, trees, wheels, unicyclic graphs is cordial. Vaidya et al. [17] proved that graph obtained by joining two copies of cycles by a path of arbitrary length is cordial. Harary introduced S-Cordiality with the first letter of Signed Cordiality. Devaraj et al.[8] proved that Petersen graph, complete graph, book graph, Jahangir graph and flower graph are signed cordial.
The concept of signed product cordial labeling was introduced by Baskar Babujee [7]. P.Lawrence et al. [15] proved that arbitrary super subdivision of some graphs is signed product cordial. Santhi et al.[10],[11] proved that flower graph, Binary tree, k-square graphs, cycle related graphs, some star and bistar related graphs are signed product cordial. They have also proved that every signed product cordial labeling is a total signed product cordial labeling. Ulaganathan et al. [13] proved that duplicate graphs of Bistar, Double star and Triangular ladder graphs are signed product cordial. Lawrence et al. [14] proved that Face and Total face signed product cordial labeling of planar graphs. J.A. Cynthia et.al[5] proved that Signed product cordial labeling of Circulant network and splitting graph of circulant network and signed and signed product cordial labeling of grid graphs[6].

III. MAIN RESULTS

Theorem 1
The Cylinder $(C_m \times P_n)$,$m = n$, admits signed product cordial labeling.
Proof
Let G be a cylinder$(C_n \times P_n)$, $(n \geq 3)$. Let the vertices be v_{ij}, $0 \leq i \leq n-1,\ 0 \leq j \leq n-1$. The vertex labeling $f: V(G) \to \{+1, -1\}$ is given below
Case I: $n \equiv 0\ (mod\ 4)$
Subcase (i) i is even

$$v_{ij} = \begin{cases} +1 & j \text{ is even} \\ -1 & j \text{ is odd} \end{cases} \qquad 0 \leq i \leq n-1; 0 \leq j \leq n-1.$$

Subcase (ii) i is odd

$$v_{ij} = \begin{cases} +1 & i \equiv 1 mod 4 \\ -1 & i \equiv 3\ mod\ 4 \end{cases} \qquad 0 \leq i \leq n-1; 0 \leq j \leq n-1.$$

Case II: $n \equiv 2\ mod\ 4$

$$v_{ij} = \begin{cases} +1 & j \text{ is even} \\ -1 & j \text{ is odd} \end{cases} \qquad i = 0\ \&\ \frac{n}{2}; 0 \leq j \leq n-1.$$

$$v_{ij} = \begin{cases} +1 & i \text{ is odd} \\ -1 & i \text{ is even} \end{cases} \qquad i = 1,2, \ldots \frac{n}{2}-1, \frac{n}{2}+1, \ldots n-1; 0 \leq j \leq n-1.$$

Case III: n is odd

$$v_{ij} = \begin{cases} +1 & j \text{ is even} \\ -1 & j \text{ is odd} \end{cases} \quad 0 \leq i \leq n-1; 0 \leq j \leq n-2.$$

$$v_{i(n-1)} = \begin{cases} +1 & i \equiv 0,1\ mod(4) \\ -1 & i \equiv 2,3\ mod(4) \end{cases}; 0 \leq i \leq n-1.$$

The graph G satisfies the condition $|v_f(-1) - v_f(+1)| \leq 1$ and $|e_{f^*}(-1) - e_{f^*}(+1)| \leq 1$. Hence G is Signed Product cordial graph.
Illustration:

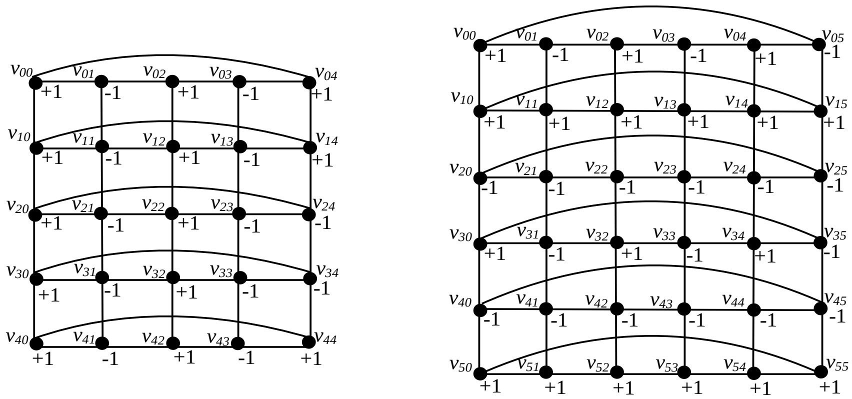

Fig.1 Signed Product Cordial labeling of Cylinders $(C_5 \times P_5)$ & $(C_6 \times P_6)$

Theorem 2

The Cylinder $(C_m \times P_n)$,$m = n$,admits signed cordial labeling.

Proof

Let G be a Cylinder $(C_n \times P_n), n \geq 3$. Let the horizontal edges of the cylinder be $e_{ij,i(j+1)}$, the vertical edges of the cylinder be $e_{ij,(i+1)j}$ and the wraparound edge be $e_{i0,i(n-1)}$.

Label the wraparound edges of the cylinder as follows:

$$e_{i0,i(n-1)} = \begin{cases} +1\,, i \text{ is even} \\ -1\,, i \text{ is odd} \end{cases} ; 0 \leq i \leq n-1$$

Label the horizontal edges of the cylinder as follows:

Case I: $n \equiv 0 \; mod \; 4$

Subcase(i) $i \equiv 0 \; mod \; 4$

$$e_{ij,i(j+1)} = \begin{cases} +1 \;\; , j \equiv 0{,}1 \; mod \; 4 \;\; ; \;\; 0 \leq i \leq n-4 \\ -1 \;\; , j \equiv 2{,}3 \; mod \; 4 \; ; \;\; 0 \leq j \leq n-2 \end{cases}$$

Subcase(ii) $i \equiv 2 \; mod \; 4$

$$e_{ij,i(j+1)} = \begin{cases} +1\,, j \equiv 2{,}3 \; mod \; 4\,; 2 \leq i \leq n-2 \\ -1\,, j \equiv 0{,}1 \; mod \; 4\,; 0 \leq j \leq n-2 \end{cases}$$

Subcase(iii) $i \equiv 1 \; mod \; 4$

$$e_{i0,i1} = -1\,; 1 \leq i \leq n-3$$

$$e_{ij,i(j+1)} = \begin{cases} +1, j \equiv 1{,}2 \; mod \; 4 \;\; ; \;\; 1 \leq i \leq n-3 \\ -1, j \equiv 0{,}3 \; mod \; 4 \;\; ; \;\; 1 \leq j \leq n-2 \end{cases}$$

Subcase(iv) $i \equiv 3 \; mod \; 4$

$$e_{i0,i1} = +1\,; 1 \leq i \leq n-1$$

$$e_{ij,i(j+1)} = \begin{cases} +1, j \equiv 0{,}3 \; mod \; 4 \;\; ; \;\; 3 \leq i \leq n-1 \\ -1, j \equiv 1{,}2 \; mod \; 4 \;\; ; \;\; 1 \leq j \leq n-2 \end{cases}$$

Case II: $n \equiv 2 \; mod \; 4$

$$e_{0j,0(j+1)} = \begin{cases} +1, j \text{ is even} \\ -1, j \text{ is odd} \end{cases} ; 0 \leq j \leq n-2$$

$$e_{(n-1)j,(n-1)(j+1)} = \begin{cases} +1, j \text{ is odd} \\ -1, j \text{ is even} \end{cases} ; 0 \leq j \leq n-2$$

Subcase(i) $i \equiv 1 \ mod\ 4$

$$e_{ij,i(j+1)} = \begin{cases} +1 \ , j \equiv 0,1 \ mod\ 4 \ ; \ 0 \leq i \leq n-5 \\ -1 \ , j \equiv 2,3 \ mod\ 4 \ ; \ 0 \leq j \leq n-2 \end{cases}$$

Subcase(ii) $i \equiv 3 \ mod\ 4$

$$e_{ij,i(j+1)} = \begin{cases} +1 \ , j \equiv 2,3 \ mod\ 4 \ ; 3 \leq i \leq n-2 \\ -1 \ , j \equiv 0,1 \ mod\ 4 \ ; 0 \leq j \leq n-2 \end{cases}$$

Subcase(iii) $i \equiv 2 \ mod\ 4$

$$e_{i0,i1} = +1 \ ; 2 \leq i \leq n-4$$

$$e_{ij,i(j+1)} = \begin{cases} +1, j \equiv 0,3 \ mod\ 4 \ ; \ 2 \leq i \leq n-4 \\ -1, j \equiv 1,2 \ mod\ 4 \ ; \ 1 \leq j \leq n-2 \end{cases}$$

Subcase(iv) $i \equiv 0 \ mod\ 4$

$$e_{i0,i1} = -1 \ ; 4 \leq i \leq n-2$$

$$e_{ij,i(j+1)} = \begin{cases} +1, j \equiv 1,2 \ mod\ 4 \ ; \ 4 \leq i \leq n-2 \\ -1, j \equiv 0,3 \ mod\ 4 \ ; \ 1 \leq j \leq n-2 \end{cases}$$

Case III: $n \equiv 1 \ mod\ 4$

Subcase(i) i is even

$$e_{ij,i(j+1)} = \begin{cases} +1 \ , j \equiv 0,1 \ mod(4) \ ; \ 0 \leq j \leq n-2 \\ -1 \ , j \equiv 2,3 \ mod(4) \ ; \ 1 \leq i \leq n-1 \end{cases}$$

Subcase(ii) i is odd

$$e_{ij,i(j+1)} = \begin{cases} +1 \ , j \equiv 2,3 \ mod(4) \ ; 0 \leq j \leq n-2 \\ -1 \ , j \equiv 0,1 \ mod(4) \ ; 1 \leq i \leq n-1 \end{cases}$$

Case IV: $n \equiv 3 mod\ 4$

$$\boldsymbol{e_{0j,0(j+1)} = -1 \ ; \ 0 \leq j \leq n-2}$$

Subcase(i) i is odd

$$e_{ij,i(j+1)} = \begin{cases} +1 \ , j \equiv 0,1 \ mod(4) \ ; \ 0 \leq j \leq n-2 \\ -1 \ , j \equiv 2,3 \ mod(4) \ ; \ 1 \leq i \leq n-2 \end{cases}$$

Subcase(ii) i is even

$$e_{ij,i(j+1)} = \begin{cases} +1 \ , j \equiv 2,3 \ mod(4) \ ; \ 0 \leq j \leq n-2 \\ -1 \ , j \equiv 0,1 \ mod(4) \ ; \ 2 \leq i \leq n-1 \end{cases}$$

Label the vertical edges of the cylinder as follows:

Case I : n is even

$$e_{ij,(i+1)j} = \begin{cases} +1 \ , j \ is\ even \ ; \ 0 \leq j \leq n-1 \\ -1 \ , j \ is\ odd \ ; \ 0 \leq i \leq n-2 \end{cases}$$

Case II : $n \equiv 1 \ mod\ 4$

Subcase(i) j is even

$$e_{ij,(i+1)j} = \begin{cases} +1 \ , i \equiv 0,1 \ mod(4) \ ; \ 0 \leq j \leq n-1 \\ -1 \ , i \equiv 2,3 \ mod(4) \ ; \ 0 \leq i \leq n-2 \end{cases}$$

Subcase(ii) j is odd

$$e_{ij,(i+1)j} = \begin{cases} +1\ , i \equiv 2,3\ mod(4)\ ;\ 0 \leq j \leq n-2 \\ -1\ , i \equiv 0,1\ mod(4)\ \ ;\ 0 \leq i \leq n-2 \end{cases}$$

Case III : $n \equiv 3\ mod\ 4$

$$e_{i0,(i+1)0} = +1; 0 \leq i \leq n-2$$

Subcase(i) j is even

$$e_{ij,(i+1)j} = \begin{cases} +1\ , i \equiv 2,3\ mod(4)\ ;\ 2 \leq j \leq n-1 \\ -1\ , i \equiv 0,1\ mod(4)\ \ ;\ 0 \leq i \leq n-2 \end{cases}$$

Subcase(ii) j is odd

$$e_{ij,(i+1)j} = \begin{cases} +1\ , i \equiv 0,1\ mod(4)\ ;\ 1 \leq j \leq n-2 \\ -1\ , i \equiv 2,3\ mod(4)\ \ ;\ 0 \leq i \leq n-2 \end{cases}$$

The graph G satisfies the condition $|v_f(-1) - v_f(+1)| \leq 1$ and $|e_{f^*}(-1) - e_{f^*}(+1)| \leq 1$. Hence G is Signed cordial graph.

Illustration:

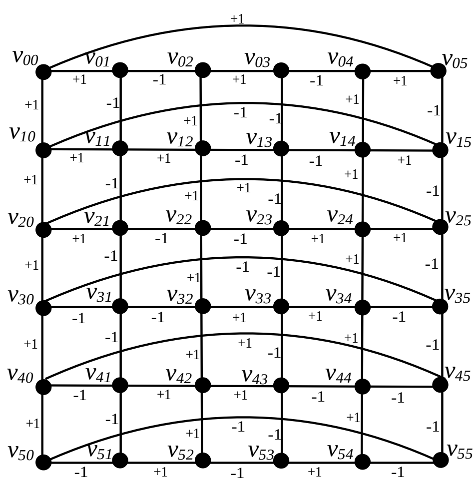

Fig.2 Signed Cordial labeling of Cylinder $(C_6 \times P_6)$

Theorem 3

The Banana Tree $B_{n,k}$, $n \geq 2, k \geq 4$, admits signed product cordial labeling.

Proof

Let G be $B_{n,k}$, $n \geq 2, k \geq 4$, label the vertex $x = -1, m_i = -1, l_i = +1, i = 1,2, \dots n$.

Case I: k is even

$$l_{ij} = \begin{cases} +1\ j\ is\ odd \\ -1\ j\ is\ even \end{cases} 1 \leq i \leq k-2; 1 \leq j \leq k-2$$

Case II: k is odd

Subcase(i) i is odd

$$l_{ij} = \begin{cases} +1\ j\ is\ odd \\ -1\ j\ is\ even \end{cases} 1 \leq i \leq k-2; 1 \leq j \leq k-2$$

Subcase(ii) i is even

$$l_{ij} = \begin{cases} +1\ j\ is\ even \\ -1\ j\ is\ odd \end{cases}\ 1 \le i \le k-2; 1 \le j \le k-2$$

The graph G satisfies the condition $|v_f(-1) - v_f(+1)| \le 1$ and $|e_{f^*}(-1) - e_{f^*}(+1)| \le 1$ for all n.
Hence the Banana tree $B_{n,k}, n \ge 2, k \ge 4$ is signed product cordial graph.

Illustration

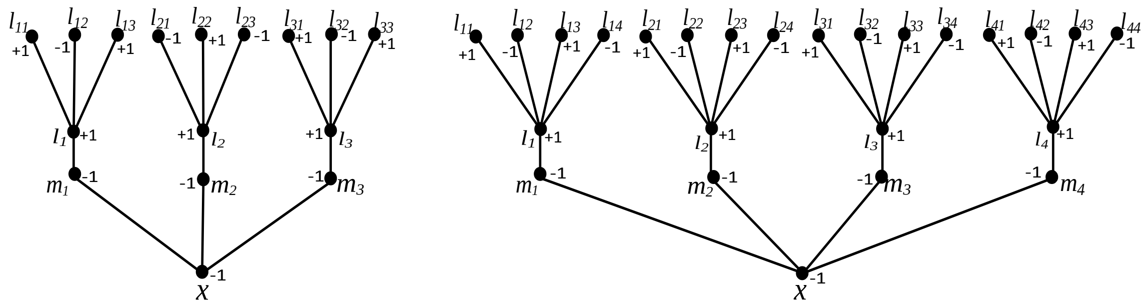

Fig.3 Signed product cordial labeling of Banana tree $B_{3,5}$ and $B_{4,6}$

Theorem 4
The Banana Tree $B_{n,k}$, $n \ge 2, k \ge 4$, admits signed cordial labeling for $n + k \not\equiv 2\ mod\ 4$ when n, k are both are odd.
Proof
Let G be $B_{n,k}$, $n \ge 2, k \ge 4$. Label the edges as follows

Case I: n is even

$$l_i l_{ij} = \begin{cases} +1\ i\ is\ odd \\ -1\ i\ is\ even \end{cases}; i = 1,2, \dots n; j = 1,2, \dots k-2.$$

Subcase(i) k is even

$$m_i l_i = +1\ \&\ x m_i = -1; i = 1,2, \dots n$$

Subcase(ii) k is odd

$$m_i l_i = \begin{cases} +1\ i\ is\ odd \\ -1\ i\ is\ even \end{cases}; i = 1,2, \dots n$$

$$x m_i = \begin{cases} +1\ i\ is\ even \\ -1\ i\ is\ odd \end{cases}; i = 1,2, \dots n$$

Case II: n is odd and k is even

$$m_i l_i = +1, x m_i = -1; i = 1,2, \dots n-1$$

$$m_n l_n = \begin{cases} +1 & k \equiv 2\ mod\ 4 \\ -1 & k \equiv 0\ mod\ 4 \end{cases}$$

$$x m_n = \begin{cases} -1 & k \equiv 2\ mod\ 4 \\ +1 & k \equiv 0\ mod\ 4 \end{cases}$$

$$l_i l_{ij} = \begin{cases} +1\ i\ is\ odd & i = 1,2, \dots n-1 \\ -1\ i\ is\ even & j = 1,2, \dots k-2 \end{cases}$$

$$l_n l_{nj} = \begin{cases} +1\ j\ is\ odd \\ -1\ j\ is\ even \end{cases} \quad j = 1,2, \dots k-2$$

Case III: n, k are odd ; $n + k \not\equiv 2\ mod\ 4$

$$m_i l_i = \begin{cases} +1\ i\ is\ odd \\ -1\ i\ is\ even \end{cases}; i = 1,2, \dots n$$

$$x m_i = \begin{cases} +1\ i\ is\ even \\ -1\ i\ is\ odd \end{cases}; i = 1,2, \dots n$$

$$l_i l_{ij} = \begin{cases} +1\ i\ is\ odd & i = 1,2, \dots n-1 \\ -1\ i\ is\ even & j = 1,2, \dots k-2 \end{cases}$$

Subcase(i) $k \equiv 1\ mod\ 4$

$$l_n l_{nj} = \begin{cases} +1\ j\ is\ even \\ -1\ j\ is\ odd \end{cases} \quad j = 1,2, \dots k-2$$

Subcase (ii) $k \equiv 3\ mod\ 4$

$$l_n l_{nj} = \begin{cases} +1\ j\ is\ odd \\ -1\ j\ is\ even \end{cases} \quad j = 1,2, \dots k-2$$

The graph G satisfies the condition $|v_f(-1) - v_f(+1)| \leq 1$ and $|e_{f^*}(-1) - e_{f^*}(+1)| \leq 1$. Hence the Banana Tree $B_{n,k}$, $n \geq 2, k \geq 4$, admits signed cordial labeling for $n + k \not\equiv 2\ mod\ 4$ when n, k are both odd.

Illustration:

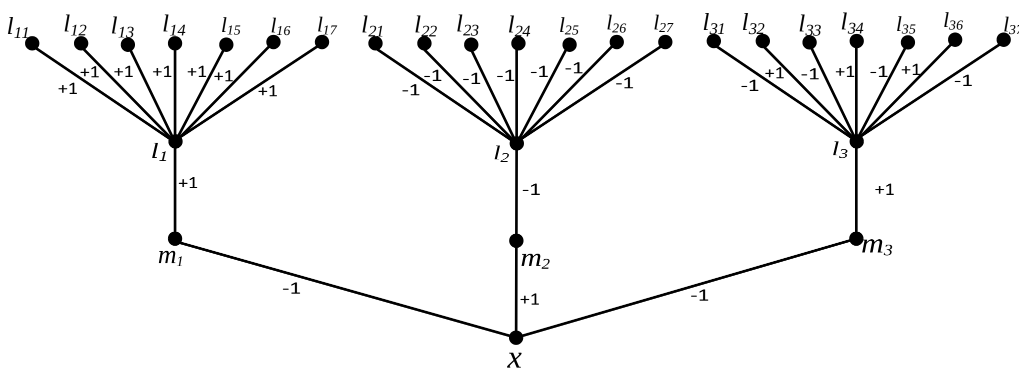

Fig.4 Signed cordial labeling of Banana tree $B_{3,9}$

IV. REMARKS

Santhi et al. [11](Theorem 3.2) proved that every signed product cordial labeling is a Total signed product cordial labeling. Thus the cylinder graphs and banana tree is also admits total signed product cordial labeling.

V. CONCLUSION

This paper presents the signed cordiality and signed product cordiality of cylinder graphs and banana tree. Further we intend to derive this labeling admits for some mesh derived architectures.

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