On $\beta g^*$ Closed Sets in Topological Ordered Spaces

C.Dhanapakyam*1, K.Indirani*2

*1 Assistant Professor, Department of Mathematics
Rathnavel subramaniam College of Arts & Science, Coimbatore, India
*2 Associate Professor, Nirmala College for women Red fields, Coimbatore, India

Abstract
The aim of this paper is to introduce a new class of closed sets in topological ordered spaces called increasing $\beta g^*$-closed sets, decreasing $\beta g^*$-closed sets and balance $\beta g^*$-closed sets and obtain some of its characteristics.

Keywords - $\beta g^*$-closed set, $d\beta g^*$-closed set, $b\beta g^*$-closed set

I. INTRODUCTION
Leopoldo Nachbin [6] initiated the study of topological ordered spaces. A topological ordered spaces is a triple $(X, \tau, \leq)$ where $\tau$ is a topology on $X$ and $\leq$ is a partial order on $X$. For any $x \in X$, $\{y \in X | x \leq y\} = [x, \to]$ and $\{y \in X | y \leq x\} = [\to, x]$. A subset $A$ of a topological ordered space $(X, \tau, \leq)$ is said to be increasing if $A = i(A)$ and decreasing if $A = d(A)$ where $i(A) = \bigcup_{a \in A} [a, \to]$ and $d(A) = \bigcup_{a \in A} [\to, a]$. A subset of a topological ordered space $(X, \tau, \leq)$ is said to be balanced if it is both increasing and decreasing.

II. PRELIMINARIES
Throughout this paper $(X, \tau, \leq)$ represent topological ordered spaces on which no separation axioms are assumed unless otherwise mentioned. For any subset $A$ of a space $(X, \tau, \leq)$, $cl(A)$ and $int(A)$ denote the closure of $A$ and interior of $A$ respectively.

Definition 2.1: A subset $A$ of a space $(X, \tau)$ is called
1) a regular open set[3] if $A = int(cl(A))$ and regular-closed if $A = cl(int(A))$.
2) a $\beta$-open set [1] if $A \subset cl(int(cl(A)))$ and $\beta$-closed if $int(cl(int(A))) \subset A$.
3) a semi-open set[5] if $A \subset cl(int(A))$ and semi-closed if $int(cl(A)) \subset A$.

Definition 2.2: A subset $A$ of a topological space $(X, \tau)$ is called
1. Generalized closed (briefly g-closed) [4] if $cl(A) \subset U$ whenever $A \subset U$ and $U$ is open.

Definition 2.3.[7] A subset $A$ of a topological space $(X, \tau, \leq)$ is called
(i) an iclosed set if $A$ is an increasing set and closed set.
(ii) a dclosed set if $A$ is a decreasing set and closed set.
(iii) a bclosed set if $A$ is both an increasing and decreasing set and a closed set.

III. $x\beta g^*$-CLOSED SETS

Definition 3.1: A subset $A$ of $(X, \tau)$ is called $\beta g^*$-closed set if $gcl(A) \subset U$ whenever $A \subset U$ and $U$ is $\beta$ open.

Definition 3.2: A subset $A$ of $(X, \tau, \leq)$ is called $i\beta g^*$ closed set if it is both increasing and $\beta g^*$ closed set.

Remark 3.3: $A$ and $X$ are $i\beta g^*$ closed subset of $(X, \tau, \leq)$.

Theorem 3.4: Every iclosed set is an $i\beta g^*$ closed set but not conversely.

Proof: Every closed set is a $\beta g^*$ closed[3]. Then every iclosed set is an $i\beta g^*$ closed set.
Example 3.5: Let $X=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \leq = (a, a), (b, b), (c, c), (a, b), (b, c), (a, c)\}$ clearly $(X, \tau, \leq)$ is a topological ordered space. All closed sets are $\{c\}$, $\{b,c\}, X, \phi$. Then clearly $A=\{c\}$ is an $\beta g^*$ closed but not an ir closed set in $X$.

**Theorem 3.6:** Every increasing regular closed set is an $\beta g^*$ closed but not conversely.

**Proof:** Every regular closed set is an $\beta g^*$ closed set $[3]$. Then every $i \beta g^*$ closed set.

Example 3.7: Let $X=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \leq = (a, a), (b, b), (c, c), (a, b), (b, c), (a, c)\}$ clearly $(X, \tau, \leq)$ is a topological ordered space. All closed sets are $\{c\}$, $\{b,c\}, X, \phi$ . Then clearly $A=\{c\}$ is an $\beta g^*$ closed but not an ir closed set in $X$.

**Definition 3.8:** A subset $A$ of $(X, \tau, \leq)$ is called an $d \beta g^*$ closed set if it both decreasing and $\beta g^*$ closed.

**Theorem 3.9:** Every $d$ closed set is an $d \beta g^*$ closed set and conversely.

**Proof:** Every closed d closed set is in a $d \beta g^*$ closed set $[3]$. Then every $d \beta g^*$ closed set is an $d \beta g^*$ closed set.

Example 3.10: Let $X=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \leq = (a, a), (b, b), (c, c), (a, b), (b, c), (a, c)\}$ clearly $(X, \tau, \leq)$ is a topological ordered space. All closed sets are $\{c\}$, $\{a,b\}, X, \phi$. Then clearly $A=\{a\}$ or $\{a,b\}$ is an $d \beta g^*$ closed set.

**Theorem 3.11:** Every decreasing regular closed set is a $d \beta g^*$ closed set but not conversely.

**Proof:** Every closed regular set is a $d \beta g^*$ closed set $[3]$. Then every $d \beta g^*$ closed set is an $d \beta g^*$ closed set.

Example 3.12: Let $X=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \leq = (a, a), (b, b), (c, c), (a, b), (b, c), (a, c)\}$ clearly $(X, \tau, \leq)$ is a topological ordered space. All $d \beta g^*$ closed sets are $\{c\}$, $\{a,b\}, X, \phi$. Then clearly $A=\{a\}$ or $\{a,b\}$ is an $d \beta g^*$ closed but not an $d \beta g^*$ closed set in $X$.

**Definition 3.13:** A subset $A$ of $(X, \tau, \leq)$ is called an $b \beta g^*$ closed set if it both increasing and decreasing $\beta g^*$ closed.

**Theorem 3.14:** Every $b$ closed set is an $b \beta g^*$ closed set but not conversely.

**Proof:** Every closed $b$ set is a $b \beta g^*$ closed set $[3]$. Then every $b \beta g^*$ closed set is an $b \beta g^*$ closed set.

Example 3.15: Let $X=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \leq = (a, a), (b, b), (c, c)\}$ clearly $(X, \tau, \leq)$ is a topological ordered space. All $b \beta g^*$ closed sets are $\{c\}$, $\{a,b\}$, $\{b,c\}, X, \phi$. Then clearly $A=\{c\}$ is an $b \beta g^*$ closed but not an $b \beta g^*$ closed set in $X$.

**Theorem 3.16:** Every $b$ regular closed set is $b \beta g^*$ closed but not conversely.

**Proof:** Every regular closed set is a $b \beta g^*$ closed set $[3]$. Then every $b \beta g^*$ closed set is an $b \beta g^*$ closed set.

Example 3.17: Let $X=\{a,b,c\}$, $\tau=\{X, \phi, \{a\}, \leq = (a, a), (b, b), (c, c)\}$ clearly $(X, \tau, \leq)$ is a topological ordered space. All $b \beta g^*$ closed sets are $\{a\}$, $\{b,c\}, \{a,b\}, \{b,c\}, X, \phi$. Then clearly $A=\{a\}$ is an $b \beta g^*$ closed but not a $b$ closed set in $X$.

IV. CONCLUSION

In this paper, we have introduced increasing $\beta g^*$ closed sets, decreasing $\beta g^*$ closed sets and balanced $\beta g^*$ closed sets and established their relationship with some of its characteristics in topological ordered spaces.

REFERENCES