Multiple integrals involving the S generalized Gauss's hypergeometric function, class of polynomials, multivariable Aleph-function and multivariable I-function

F.Y. AYANT

1 Teacher in High School, France

ABSTRACT

In the present paper we evaluate a generalized multiple integrals involving the product of the S generalized Gauss hypergeometric function, multivariable Aleph-function, the multivariable I-function and general class of polynomials of several variables. The importance of the result established in this paper lies in the fact they involve the Aleph-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializing the parameters their in.

Keywords: Multivariable Aleph-function, general class of polynomials, multiple integral, S generalized hypergeometric function, multivariable I-function, multivariable H-function

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1. Introduction and preliminaries.

The function Aleph of several variables is an extension the multivariable I-function recently studied by C.K. Sharma and Ahmad [3], itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occurring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define:

\[ N(z_1, \ldots, z_r) = \mathcal{N}^{0,N}:M_1,N_1, \ldots, M_r,N_r \]

\[ P_1, Q_1, \tau_1, \cdots, P_r, Q_r, \tau_r : i, r, \tau_{i+1}: i, \cdots, P_{i(r)}, Q_{i(r)}, \tau_{i(r)} : i, r \]

\[ [(\alpha_{j(1)}, \alpha_{j(r)})_{1,N}], \left[ (\alpha_{j(1)}, \alpha_{j(r)})_{N+1}, P_1 \right], \ldots, \left[ (\alpha_{j(1)}, \alpha_{j(r)})_{M+1}, Q_1 \right] \]

\[ [(\delta_{j(1)}, \delta_{j(r)})_{1,N}], \left[ ([\gamma_{j(1)}], \gamma_{j(r)})_{N+1}, P_1 \right], \ldots, \left[ ([\gamma_{j(1)}], \gamma_{j(r)})_{M+1}, Q_1 \right] \]

\[ \psi(s_1, \ldots, s_r) \prod_{k=1}^{r} \theta_k(s_k) g_k^{s_k} \text{d}s_1 \cdots \text{d}s_r \quad (1.1) \]

with \( \omega = \sqrt{-1} \)

\[ \psi(s_1, \ldots, s_r) = \frac{\prod_{j=1}^{N} \Gamma(1 - a_j + \sum_{k=1}^{r} \alpha_{j(k)} s_k)}{\sum_{i=1}^{N} \prod_{j=N+1}^{r} \Gamma(a_{ji} - \sum_{k=1}^{r} \alpha_{ji(k)} s_k) \prod_{j=1}^{Q_i} \Gamma(1 - b_{ji} + \sum_{k=1}^{r} \beta_{ji(k)} s_k)} \quad (1.2) \]

and
Suppose, as usual, that the parameters $\theta_k(s_k) = \frac{\prod_{j=1}^{M_k} \Gamma(d_j^{(k)} - \delta_j^{(k)} s_k) \prod_{j=1}^{N_k} \Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)}{\sum_{i=1}^{R(k)} \prod_{j=M_k+1}^{d_j^{(k)}} \Gamma(1 - d_j^{(k)} - \delta_j^{(k)} s_k) \prod_{j=N_k+1}^{c_j^{(k)}} \Gamma(c_j^{(k)} - \gamma_j^{(k)} s_k)}}$ (1.3)

Suppose, as usual, that the parameters

$\alpha_j^{(k)}, j = n_k + 1, \ldots, P_j^{(k)}; \beta_j^{(k)}, j = 1, \ldots, N_k;$

$\epsilon_j^{(k)}, j = M_k + 1, \ldots, Q_j^{(k)}; \delta_j^{(k)}, j = 1, \ldots, M_k;$

with $k = 1 \cdots r, i = 1, \ldots, R, \epsilon_i^{(k)} = 1, \cdots, R(k)$

are complex numbers, and the $\alpha_j^{l}$s, $\beta_j^{l}$s, $\gamma_j^{l}$s and $\delta_j^{l}$s are assumed to be positive real numbers for standardization purpose such that

$U_i^{(k)} = \tau_i \sum_{j=N+1}^{P_i^{(k)}} \alpha_j^{(k)} - \tau_i \sum_{j=N_k+1}^{Q_i^{(k)}} \gamma_j^{(k)} + \tau_i \sum_{j=1}^{M_k} \beta_j^{(k)} - \tau_i \sum_{j=M_k+1}^{Q_i^{(k)}} \delta_j^{(k)} \leq 0$ (1.4)

The contour $L_k$ is in the $s_k$-$p$ lane and run from $\sigma - i\infty$ to $\sigma + i\infty$, where $\sigma$ is a real number with loop, if necessary, ensure that the poles of $\Gamma(a_j^{(k)} - \delta_j^{(k)} s_k)$ with $j = 1$ to $m_k$ are separated from those of $\Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k)$ with $j = 1$ to $N$ and $\Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)$ with $j = 1$ to $N_k$ to the left of the contour $L_k$. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as

$|\arg z_i| < \frac{1}{2} A_i^{(k)} \pi$, where

$A_i^{(k)} = \sum_{j=1}^{N} \alpha_j^{(k)} - \tau_i \sum_{j=N+1}^{P_i^{(k)}} \alpha_j^{(k)} - \tau_i \sum_{j=1}^{Q_i^{(k)}} \beta_j^{(k)} - \tau_i \sum_{j=N_k+1}^{Q_i^{(k)}} \gamma_j^{(k)} - \tau_i \sum_{j=M_k+1}^{Q_i^{(k)}} \delta_j^{(k)} > 0$, with $k = 1 \cdots r, i = 1, \cdots, R, \epsilon_i^{(k)} = 1, \cdots, R(k)$ (1.5)

The complex numbers $z_i$ are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the asymptotic expansion in the following convenient form:

$\mathcal{N}(y_1, \cdots, y_r) = 0(|y_1|^{\alpha_1}, \cdots, |y_r|^{\alpha_r}), \max(|y_1|, \cdots, |y_r|) \to 0$

$\mathcal{N}(y_1, \cdots, y_r) = 0(|y_1|^{\beta_1}, \cdots, |y_r|^{\beta_r}), \min(|y_1|, \cdots, |y_r|) \to \infty$

where, with $k = 1, \cdots, r$: $\alpha_k = \min[Re(d_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, M_k$ and

$\beta_k = \max[Re((\epsilon_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, N_k$
Serie representation of Aleph-function of several variables is given by

$$N(y_1, \ldots, y_r) = \sum_{G_1, \ldots, G_r} \sum_{g_1=0}^{\infty} \cdots \sum_{g_r=0}^{\infty} \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1! \cdots \delta_{g_r}G_r!} \psi(\eta_{G_1,g_1}, \ldots, \eta_{G_r,g_r})$$

$$\times \theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r}) y_1^{-\eta_{G_1,g_1}} \cdots y_r^{-\eta_{G_r,g_r}}$$

(1.6)

Where $\psi(\cdot, \ldots, \cdot)$, $\theta_i(\cdot)$, $i = 1, \ldots, r$ are given respectively in (1.2), (1.3) and

$$\eta_{G_1,g_1} = \frac{d^{(1)}_{g_1} + G_1}{\delta_{g_1}^{(1)}}, \ldots, \eta_{G_r,g_r} = \frac{d^{(r)}_{g_r} + G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions

$$\delta_{g_i}^{(i)}[d^{(i)}_{g_i} + P_i] \neq \delta_{j}^{(j)}[d^{(j)}_{g_j} + P_j]$$

(1.7)

for $j \neq M_i, M_i = 1, \ldots, n_{G_i,g_i}$; $P_i, N_i = 0, 1, 2, \ldots, y_i \neq 0, i = 1, \ldots, r$

(1.8)

In the document we will note:

$$G(\eta_{G_1,g_1}, \ldots, \eta_{G_r,g_r}) = \phi(\eta_{G_1,g_1}, \ldots, \eta_{G_r,g_r}) \theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r})$$

(1.9)

where $\phi(\eta_{G_1,g_1}, \ldots, \eta_{G_r,g_r}), \theta_1(\eta_{G_1,g_1}), \ldots, \theta_r(\eta_{G_r,g_r})$ are given respectively in (1.2) and (1.3)

We shall note the Aleph-function of $r$ variables

$$\kappa_{u,v}^{N_1,N_1} \left( \begin{array}{c} z_1 \\ \vdots \\ z_r \end{array} \right)$$

(1.10)

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral:

$$I(z_1, \ldots, z_s) = \int_{L_1} \cdots \int_{L_s} \xi(t_1, \ldots, t_s) \prod_{i=1}^{s} \phi_i(t_i) z_i^{t_i} dt_1 \cdots dt_s$$

(1.11)

The defined integral of the above function, the existence and convergence conditions, see Y.N Prasad [2]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of
the corresponding conditions for multivariable H-function given by as:

\[ |\arg z_k| < \frac{1}{2} \Omega^{(k)}_i \pi, \]

where

\[ \Omega^{(k)}_i = \sum_{k=1}^{n_i} \alpha_k^{(i)} - \sum_{k=n_i+1}^{p_i} \alpha_k^{(i)} + \sum_{k=1}^{m_i} \beta_k^{(i)} - \sum_{k=m_i+1}^{q_i} \beta_k^{(i)} + \left( \sum_{k=1}^{n_2} \alpha_k^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_k^{(i)} \right) + \cdots + \]

\[
\left( \sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)} \right) - \left( \sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)} \right) \]

\[ \quad \quad (1.13) \]

where \( i = 1, \ldots, s \)

The complex numbers \( z_i \) are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the asymptotic expansion in the following convenient form:

\[ \mathcal{N}(z_1, \ldots, z_s) = 0\left(|z_1|^{\alpha'_1}, \ldots, |z_s|^{\alpha'_s}\right), \max\left(|z_1|, \ldots, |z_s|\right) \rightarrow 0 \]

\[ \mathcal{N}(z_1, \ldots, z_s) = 0\left(|z_1|^{\beta'_1}, \ldots, |z_s|^{\beta'_s}\right), \min\left(|z_1|, \ldots, |z_s|\right) \rightarrow \infty \]

where, with \( k = 1, \ldots, z : \alpha'_k = \min\{\text{Re}(b_j^{(k)})/\alpha_j^{(k)}\}, j = 1, \ldots, m_k \) and \( \beta'_k = \max\{\text{Re}(\alpha_j^{(k)})/\beta_j^{(k)}\}, j = 1, \ldots, n_k \)

We will use these following notations in this paper:

\[ U = p_2, p_3, q_3, \ldots, p_{s-1}, q_{s-1}; V = 0, n_2, 0, n_3; \ldots; 0, n_{s-1} \]

\[ W = (p', q'); \ldots; (p^{(s)}, q^{(s)}); X = (m', n'); \ldots; (m^{(s)}, n^{(s)}) \]

\[ A = (a_{2k}, \alpha'_{2k})_{1, p_2}; \ldots; (a_{(s-1)k}, \alpha'_{(s-1)k})_{1, p_{s-1}} \]

\[ B = (b_{2k}, \beta'_{2k})_{1, q_2}; \ldots; (b_{(s-1)k}, \beta'_{(s-1)k})_{1, q_{s-1}} \]

\[ \mathfrak{A} = (a_{sk}, \alpha'_{sk})_{p_s, q_s}; \mathfrak{B} = (b_{sk}, \beta'_{sk})_{p_s, q_s} \]

\[ A' = (a_k^{(s)}, \alpha_k^{(s)})_{1, p'; \ldots; (a_k^{(s)}, \alpha_k^{(s)})_{1, p^{(s)}}}; B' = (b_k^{(s)}, \beta_k^{(s)})_{1, q'; \ldots; (b_k^{(s)}, \beta_k^{(s)})_{1, q^{(s)}}} \]

The multivariable I-function writes:

\[ I(z_1, \ldots, z_s) = I_{U; p_s, q_s; W} \begin{pmatrix} z_1 \vdots A; \mathfrak{A}; A' \vdots \cdot \vdots B; \mathfrak{B}; B' \end{pmatrix} \]

(1.20)
The generalized polynomials defined by Srivastava [4], is given in the following manner:

\[ S_{N_1', \ldots, N_t'}^{M_1', \ldots, M_t'}[y_1, \ldots, y_t] = \sum_{K_1=0}^{N_1'/M_1'} \cdots \sum_{K_t=0}^{N_t'/M_t'} \frac{(-N_1')_{M_1'K_1} \cdots (-N_t')_{M_t'K_t}}{K_1! \cdots K_t!} A[N_1', K_1; \ldots; N_t', K_t] y_1^{K_1} \cdots y_t^{K_t} \]  

(1.21)

where \( M_1', \ldots, M_t' \) are arbitrary positive integers and the coefficients \( A[N_1', K_1; \ldots; N_t', K_t] \) are arbitrary constants, real or complex. In the present paper, we use the following notation

\[ \alpha_1 = \frac{(-N_1')_{M_1'K_1}}{K_1!} \cdots \frac{(-N_t')_{M_t'K_t}}{K_t!} A[N_1', K_1; \ldots; N_t', K_t] \]  

(1.22)

2. S Generalized Gauss’s hypergeometric function

The S generalized Gauss’ hypergeometric function \( F_{p}^{(a,b;c)}(a, b, c; z) \) introduced and defined by Srivastava et al [5, page 350, Eq.(1.12)] is represented in the following manner:

\[ F_{p}^{(a,b;c)}(a, b, c; z) = \sum_{n=0}^{\infty} \frac{B_{p}^{(a,b;c)}(a, b, c; z)^{n}}{B_{p}^{(a,b;c)}(a, b, c; z)} \frac{z^{n}}{n!} \quad (|z| < 1) \]  

(2.1)

provided that \( \Re(p) \geq 0, \min \Re(a, b, c, z) > 0; \Re(c) > \Re(b) > 0 \)

where the S generalized Beta function \( B_{p}^{(a,b;c)}(x, y) \) was introduced and defined by Srivastava et al [5, page 350, Eq(1.13)]

\[ B_{p}^{(a,b;c)}(x, y) = \int_{1}^{x} t^{a-1}(1-t)^{b-1} F_{1} \left( \alpha; \beta; \frac{-p}{t^{	au}(1-t)^{\mu}} \right) dt \]  

(2.2)

provided that \( \Re(p) \geq 0, \min \Re(x, y, a, b, \alpha, \beta) > 0; \min \Re(\tau, \mu) > 0 \)

3. Required integral

We have the following integral, see (Marichev et [2], 3.3.3, Eq.5, page 589)

Lemma.

\[ \int_{x_1 \geq 0} \cdots \int_{x_n \geq 0} \frac{x_1^{v_1-1} \cdots x_n^{v_n-1}}{(1-x_1^{\alpha_1} - \cdots - x_n^{\alpha_n})^{\mu}} dx_1 \cdots dx_n = \Gamma \left[ \begin{array}{c} \frac{v_1}{\mu_1}, \ldots, \frac{v_n}{\mu_n}, 1 - \mu \\ 1 - \mu + \frac{v_1}{\mu_1} + \cdots + \frac{v_n}{\mu_n} \end{array} \right] \]  

(3.1)

where \( x_1^{\alpha_1} + \cdots + x_n^{\alpha_n} \leq 1, \alpha_i > 0, v_i \in \mathbb{R}^+, i = 1, \cdots, n \) and \( \Re(\mu) < 1 \)

4. Main integral

We note \( X_{v_1, \ldots, v_n, \mu} = \frac{x_1^{v_1} \cdots x_n^{v_n}}{(1-x_1^{\alpha_1} - \cdots - x_n^{\alpha_n})^{\mu}} \)
We have the following formula

**Theorem**

\[
\int_{x_{1} \geq 0} \cdots \int_{x_{n} \geq 0} \frac{x_{1}^{v_{1}-1} \cdots x_{n}^{v_{n}-1}}{1 - x_{1}^{\alpha_{1}} - \cdots - x_{n}^{\alpha_{n}}} \mu_{p} F_{\alpha, \beta, \gamma, \mu}(a, b, c, y; x_{1}, \cdots, x_{n}, \epsilon_{1}, \cdots, \epsilon_{R}) \prod_{j=1}^{r} N_{j} X_{\gamma_{j}}(\cdots) dy_{1} \cdots dy_{n} = \frac{1}{\alpha_{1} \cdots \alpha_{n}} \sum_{G_{1}, \cdots, G_{r}, \epsilon=0}^{\infty} \delta_{G_{1} G_{1}} \cdots \delta_{G_{r} G_{r}} \prod_{j=1}^{r} \left( \frac{B_{p}^{a, b, c} (b + n', c - b)}{B(b, c - b)} \right) \prod_{j=1}^{r} G(\eta_{G_{j}, G_{j}}) \prod_{j=1}^{r} \eta_{G_{j}, G_{j}}
\]

\[
\sum_{g_{1}=0}^{m_{1}} \cdots \sum_{g_{r}=0}^{m_{r}} \sum_{K_{1}=0}^{N_{1}/M_{1}^{*}} \cdots \sum_{K_{r}=0}^{N_{r}/M_{r}^{*}} \sum_{n'=0}^{\infty} \sum_{n'=0}^{\infty} (a)_{n'} \frac{B_{p}^{(a, b, c, \mu)} (b + n', c - b)}{B(b, c - b)} \prod_{j=1}^{r} \frac{G(\eta_{G_{j}, G_{j}})}{\prod_{j=1}^{r} \eta_{G_{j}, G_{j}}}
\]

\[
\frac{y_{1}^{n'}}{n'!} \frac{1}{z_{1}^{v_{1}}} \cdots \frac{1}{z_{r}^{v_{r}}} \prod_{j=1}^{r} K_{j} \prod_{j=1}^{r} \frac{y_{1}^{v_{1}}}{\alpha_{1}} \cdots \frac{y_{r}^{v_{r}}}{\alpha_{r}} \prod_{j=1}^{r} A^{1}_{1, n}
\]

\[
\begin{bmatrix}
K_{j} \eta_{1, j} + \sum_{j=1}^{r} \eta_{G_{j}, G_{j}} \alpha_{j} \frac{1}{\alpha_{j}} & \cdots & \frac{1}{\alpha_{r}} \alpha_{r}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\begin{pmatrix}
\mu' + n'b' + \sum_{j=1}^{r} K_{j} \mu_{j} + \sum_{j=1}^{r} \eta_{G_{j}, G_{j}} \beta_{j} - \eta_{1} \cdots - \eta_{R}
\end{pmatrix} A'
\end{bmatrix}
\]

Where

\[
B_{1} = \begin{bmatrix}
\mu' + n'b' + \sum_{j=1}^{r} K_{j} \mu_{j} + \sum_{j=1}^{r} \beta_{j} \eta_{G_{j}, G_{j}} \eta_{j} \sum_{i=1}^{n} \frac{v_{i} + n'a_{i} + \sum_{j=1}^{r} K_{j} \gamma_{i} + \sum_{j=1}^{r} \eta_{G_{j}, G_{j}} \alpha_{j}}{\alpha_{i}}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{\eta_{1}}{\alpha_{1}} \cdots \frac{\eta_{n}}{\alpha_{n}} - \epsilon_{1} \cdots \frac{\eta_{1}}{\alpha_{1}} \cdots \frac{\eta_{n}}{\alpha_{n}} - \epsilon_{R}
\end{bmatrix}
\]

and \( U, V, X, Y, A, B, \mathfrak{A}, \mathfrak{B}, A' \) and \( B' \) are defined respectively by (1.14), (1.15), (1.16), (1.17), (1.18) and (1.19)

Provided that

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a) \( \min \{a_i, b, \gamma_j, \mu_j, \alpha_k, \beta_k, \eta_l, \epsilon_i \} > 0, i = 1, \ldots, n; j = 1, \ldots, t; k = 1, \ldots, r; l = 1, \ldots, R \) and \( j = 1, \ldots, t \)

b) \( 1 > Re(\mu' + n'b') + \sum_{j=1}^{r} \beta_j \min_{1 \leq k \leq M_j} Re \left( \frac{d_k(j)}{\delta_k(j)} \right) + \sum_{j=1}^{R} \epsilon_j \min_{1 \leq k \leq m(j)} Re \left( \frac{b_k(j)}{\beta_k(j)} \right) > 0 \)

c) \( Re(v_i + n'a_i) + \sum_{j=1}^{r} \alpha_j \min_{1 \leq k \leq M_j} Re \left( \frac{d_k(j)}{\delta_k(j)} \right) + \sum_{j=1}^{R} \eta_j \min_{1 \leq k \leq m(j)} Re \left( \frac{b_k(j)}{\beta_k(j)} \right) > 0, i = 1, \ldots, n \)

d) \( |arg z_k| < \frac{1}{2} A_i^{(k)} \pi, \) where \( A_i^{(k)} \) is defined by (1.5) ; \( i = 1, \ldots, r \)

e) \( |arg Z_k| < \frac{1}{2} B_i^{(k)} \pi, \) where \( B_i^{(k)} \) is defined by (1.13) ; \( i = 1, \ldots, R \)

f) \( x_1^{\alpha_1} + \cdots + x_n^{\alpha_n} \leq 1, \alpha_i > 0, v_i \in \mathbb{R}, i = 1, \ldots, n \)

g\( (Re(p) > 0, \min Re(\alpha, \beta, \tau, \mu) > 0; Re(c) > Re(b) > 0) \)

**Proof**

First, expressing the generalized S generalized Gauss hypergeometric function \( F_p^{(\alpha, \beta; \tau; \mu)}(a, b; c, z) \) in series with the help of equation (2.1), the Aleph-function of \( r \) variables in series with the help of equation (1.6), the general class of polynomial of several variables \( S^{M_1, \ldots, M_r}_{N_1, \ldots, N_r} [y_1, \ldots, y_t] \) with the help of equation (1.21) and the I-function of \( s \) variables in defined by Prasad [2] in Mellin-Barnes contour integrals with the help of equation (1.12), changing the order of integrations ans summations (which is easily seen to be justified due to the absolute convergence of the integrals and the summations involved in the process) and then evaluating the resulting \( (x_1, \ldots, x_n) \)-integrals with the help of equation (3.1). Now interpreting the result obtained in the Mellin-barnes contour integral, we arrive at the desired result.

5. Particular case

If \( U = V = A = B = 0 \), the multivariable I-function defined by Prasad [2] reduces in multivariable H-function defined by Srivastava et al [6]. We have the following result.

**Corollary**

\[
\int_{x_1 > 0} \cdots \int_{x_n > 0} \frac{x_1^{v_1-1} \cdots x_n^{v_n-1}}{(1 - x_1^{\alpha_1} - \cdots - x_n^{\alpha_n})^{\mu}} F_p^{(\alpha, \beta; \tau; \mu)}(a, b; yX_{\alpha_1}, \ldots, yX_{\alpha_n}) S^{M_1, \ldots, M_r}_{N_1, \ldots, N_r} [y_1, \ldots, y_t] \left( \begin{array}{c} y_1 X_{\eta_1}^{\gamma_1}, \ldots, y_t X_{\eta_t}^{\gamma_t} \\ \mu_1, \ldots, \mu_t \end{array} \right) \\

\times \left( \begin{array}{c} z_1 X_{\alpha_1}^{\gamma_1}, \ldots, z_t X_{\alpha_t}^{\gamma_t} \\ \beta_1, \ldots, \beta_t \end{array} \right) \left( \begin{array}{c} Z_1 X_{\eta_1}^{\gamma_1}, \ldots, Z_t X_{\eta_t}^{\gamma_t} \\ \epsilon_1, \ldots, \epsilon_t \end{array} \right) \right) dx_1 \cdots dx_n = \frac{1}{\alpha_1 \cdots \alpha_n \Gamma_{G_1} \cdots \Gamma_{G_r}} \sum_{G_1 = 0}^{\infty} \cdots \sum_{G_r = 0}^{\infty} \left( \begin{array}{c} a_n \\ b_n \end{array} \right) \frac{B_p(b + n', c - b)}{B(b - c)} \delta_{G_1} \cdots \delta_{G_r} G(\eta_{G_1}, \eta_{G_2}, \ldots, \eta_{G_r})
\]

\[
\sum_{g_1 = 0}^{m_1} \cdots \sum_{g_r = 0}^{m_r} \sum_{K_1 = 0}^{[N_1/M_1]} \cdots \sum_{K_r = 0}^{[N_r/M_r]} \sum_{n' = 0}^{\infty} (a_n) \frac{B_p(a, \beta, \gamma, \delta; b + n', c - b)}{B(b - c)} \delta_{G_1} \cdots \delta_{G_r} G(\eta_{G_1}, \eta_{G_2}, \ldots, \eta_{G_r})
\]

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under the same notations and conditions that (4.1) with

\[
\alpha_1 \frac{y_1^{n'}}{n'!} z_1^{\eta_{1,\gamma_1}} \cdots z_r^{\eta_{r,\gamma_r}} y_1^{K_1} \cdots y_t^{K_t} H_{p_x+n+1,q_x+1}^{0,n_x+n+1,1;1} W \left( \begin{array}{c} Z_1 \\ \vdots \\ Z_R \end{array} \right)
\]

\[
\left[ 1 - \frac{v_1+n'a_1+\sum_{j=1}^{t} K_j \gamma_j+\sum_{j=1}^{r} \eta_{j,\gamma_j} \alpha_j}{\alpha_1} , \eta_1^{\alpha_1}, \cdots , \eta_t^{\alpha_t} \right]^1_{1,n}, \ldots , B_1 ,
\]

\[
(\mu' + n' b' + \sum_{j=1}^{t} K_j \mu_j + \sum_{j=1}^{r} \eta_{j,\gamma_j} \beta_j; -\eta_1, \cdots , -\eta_r), A : A'
\]

(5.1)

under the same notations and conditions that (4.1) with \( U = V = A = B = 0 \). \( B_1 \) are defined by (4.2).

6. Conclusion

In this paper we have evaluated a generalized multiple integrals involving the multivariable Aleph-function, a class of polynomials of several variables, the S generalized Gauss hypergeometric function and the multivariable I-function defined by Prasad [2]. The integral established in this paper is of very general nature as it contains multivariable Aleph-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

REFERENCES


