Edge Trimagic Total Labeling for Disconnected Graphs

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Abstract - An edge trimagic total labeling of a graph G(V, E) with p vertices and q edges is a bijection f from the set of vertices and edges to 1, 2, ..., p+q such that for every edge uv in E, f(u)+f(uv)+f(v) is either λ1 or λ2 or λ3. An edge trimagic total graph is called a super edge trimagic total if f(V) = {1, 2, ..., p}. An edge trimagic total graph is called a superior edge trimagic total if f(E) = {1, 2, ..., q}. In this paper we prove the disconnected graphs nP, K1, nC and nC6 admis edge trimagic total labeling.

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I. Introduction

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. All graphs considered here are finite, simple and undirected. The useful survey on graph labeling by J.A. Gallian(2012) can be found in [4].

A walk of a graph G is an alternating sequence of vertices and edges beginning and ending with vertices, in which each edge is incident with the two vertices immediately preceding and following it. A walk which begins and ends at the same vertex is called a closed walk. If the terminal vertices are distinct, the walk is known as an open walk. An open walk in which no vertex appears more than once is called a path. A graph is said to be connected if there is at least one path between every pair of vertices otherwise, it is disconnected. The union of two graphs G1 = (V1, E1) and G2 = (V2, E2) is another graph G3 = G1∪G2 whose vertex set V3 = V1∪V2 and the edge set E3 = E1∪E2. A simple graph in which there exists an edge between every pair of vertices is called a complete graph; the complete graph with n vertices is denoted by Kn. A bigraph G is a graph whose vertex set V can be partitioned into two subsets V1 and V2 such that every edge of G joins a point of V1 to a point of V2. If G contains every edge joining V1 and V2, then G is a complete bigraph. If V1 and V2 have m and n vertices, we write G = Km,n.

A star is a complete bigraph K1,n. A sun is a cycle Cn with an edge terminating in a vertex of degree one attached to each vertex, it is denoted by Sn. The t copies of the sun graph St is denoted by tSt.[8]

In 1970, Kotzing and Rosa introduced edge magic total labeling [7]. In 2004, J. Basker Bubujee introduced edge bimagic labeling of graphs [1]. In 2013, C. Jayasekaran, M. Regees and C. Davidraj introduced edge trimagic total labeling of graphs[5]. We proved that some families of graphs are edge trimagic total in [5, 6, 9, 10, 11]. In this paper, we prove that the disconnected graphs K1, p∪K1, q∪K1, r (p ≤ q ≤ r), nP∪K1, n+1, t copies of the sun graph tSn, nC4 and nC6 are edge trimagic total and nP3 is superior edge trimagic total.

II. Edge Trimagic Labeling for Disconnected Graphs

In this section we define the superior edge trimagic total labeling and prove that the disconnected graphs nP, K1, p∪K1, q∪K1, r (p ≤ q ≤ r), nP∪K1, n+1, t copies of the sun graph tSn, nC4 and nC6 are edge trimagic total.

1) Definition: An edge trimagic total labeling of a (p, q) graph G is a bijection f: V∪E→{1, 2, ..., p+q} such that for each edge...
The value of \( f(x) + f(xy) + f(y) \) is equal to any of the distinct constants \( k_1, k_2 \) or \( k_3 \). A graph \( G \) is said to be an edge trimagic total if it admits an edge trimagic total labeling. An edge trimagic total labeling of a graph is called a super edge trimagic labeling if \( f(V) = \{1, 2, \ldots, p\} \). An edge trimagic total labeling of a graph is called a superior edge trimagic total labeling if \( f(E) = \{1, 2, \ldots, q\} \).

2) Theorem: The graph \( nP_3 \) admits an edge trimagic total labeling for even \( n \).

Proof: Let \( V = \{u_i, v_i, w_i / 1 \leq i \leq n\} \) be the vertex set and \( E = \{u_iv_i, v_iw_i / 1 \leq i \leq n\} \) be the edge set of the disconnected graph \( nP_3 \). The graph \( nP_3 \) has 3n vertices and 2n edges.

Define a bijection \( f: V \to \{1, 2, \ldots, 3n\} \) such that
\[
f(u_i) = 3n+\frac{n+i+1}{2}, \quad f(w_i) = 3n+\frac{n+i+1}{2}, \quad f(v_i) = 3n+\frac{n+i+1}{2}
\]
for \( 1 \leq i \leq n, i \equiv 1(\mod 2) \); \( f(u_i) = 2n+\frac{n+i}{2}, \quad f(w_i) = 3n+\frac{n+i}{2}, \quad f(v_i) = 3n+\frac{n+i}{2}
\]
for \( 1 \leq i \leq n, i \equiv 0(\mod 2) \); \( f(u_i) = \frac{5n}{2}, \quad f(v_i) = \frac{5n}{2}, \quad f(w_i) = \frac{5n}{2} \)

For \( 1 \leq i \leq n \) and \( i \equiv 1(\mod 2) \),
\[
f(u_i)+f(u_iv_i)+f(v_i) = 3n+\frac{n+i+1}{2} + \frac{n+i+1}{2} + \frac{n+i+1}{2} + 5n-i+1
= 9n+2 = \lambda_1 \text{(say)}.
\]
For \( 1 \leq i \leq n \) and \( i \equiv 0(\mod 2) \),
\[
f(u_i)+f(u_iv_i)+f(v_i) = 2n+\frac{n+i}{2} + \frac{n+i}{2} + \frac{n+i}{2} + 5n-i+1
= \frac{17n+2}{2} = \lambda_2 \text{(say)}.
\]
Consider the edges \( v_iw_i \).
For \( 1 \leq i \leq n \) and \( i \equiv 1(\mod 2) \),
\[
f(v_i)+f(v_iw_i)+f(w_i) = 5n-i+1+\frac{i+1}{2} + 3n + \frac{i+1}{2} = 8n+2 = \lambda_3 \text{(say)}.
\]
For \( 1 \leq i \leq n \) and \( i \equiv 0(\mod 2) \),
\[
f(v_i)+f(v_iw_i)+f(w_i) = \frac{5n}{2} + \frac{i+1}{2} + 3n - \frac{n+1}{2} = \frac{17n+2}{2} = \lambda_2.
\]

Hence for each edge \( uv \in E \), \( f(u)+f(uv)+f(v) \) yields any one of the trimagic constants \( \lambda_1 = 9n+2, \quad \lambda_2 = \frac{17n+2}{2} \) and \( \lambda_3 = 8n+2 \).

Hence the graph \( nP_3 \) admits an edge trimagic total labeling for even \( n \).

3) Theorem: The graph \( nP_3 \) admits a superior edge trimagic total labeling for even \( n \).

Proof: We have proved that the graph \( nP_3 \) has an edge trimagic total labeling when \( n \) is even. The labeling given in the proof of the Theorem 2, the edges get labels \( f(u_i) = \frac{n+i+1}{2}, \quad f(v_i) = \frac{i+1}{2}, \quad f(w_i) = \frac{n+i+1}{2} \) for \( 1 \leq i \leq n \) and \( i \equiv 1(\mod 2) \); \( f(u_i) = \frac{n+i}{2}, \quad f(v_i) = \frac{n+i}{2}, \quad f(w_i) = \frac{n+i}{2} \) for \( 1 \leq i \leq n \) and \( i \equiv 0(\mod 2) \). Clearly the graph \( nP_3 \) has 2n edges and the 2n edges get labels 1, 2, \ldots, 2n. Hence the graph \( nP_3 \) is a superior edge trimagic total when \( n \) is even.

4) Example: A superior edge trimagic total labeling of the disconnected graph \( 4P_3 \) is given in fig. 1.

![Fig.1 Graph 4P3 with \( \lambda_1 = 38, \lambda_2 = 35 \) and \( \lambda_3 = 34 \).](http://www.ijmttjournal.org)
5) Theorem: The graph \( nP_3 \) admits an edge trimagic total labeling for odd \( n \).

**Proof:** Let \( V = \{ u_i, v_i, w_i / 1 \leq i \leq n \} \) be the vertex set and \( E = \{ u_iv_i, v_iw_i / 1 \leq i \leq n \} \) be the edge set of the disconnected graph \( nP_3 \). The graph \( nP_3 \) has \( 3n \) vertices and \( 2n \) edges.

Define a bijection \( f: V \cup E \rightarrow \{1, 2, \ldots, 5n\} \) such that

\[
\begin{align*}
\text{for } & 1 \leq i \leq n \quad \text{and } i \equiv 0(\text{mod } 2), \\
f(u_i) &= 3n + \frac{n+1}{2} + \frac{n+1}{2} + 5n - i + 1 \\
&= 9n + 2 = \lambda_1 \text{ (say)}, \\
\text{for } & 1 \leq i \leq n \quad \text{and } i \equiv 0(\text{mod } 2), \\
f(v_i) &= 5n - i + 1 + \frac{n+1}{2} + \frac{n+1}{2} - 1 \\
&= 8n + 1 = \lambda_2 \text{ (say)}, \\
\text{for } & 1 \leq i \leq n \quad \text{and } i \equiv 0(\text{mod } 2), \\
f(w_i) &= n + \frac{n+1}{2} + \frac{n+1}{2} + 1 + 5n - i + 1 \\
&= \frac{17n+2}{2} = \lambda_3 \text{ (say)}.
\end{align*}
\]

Consider the edges \( u_iv_i \): For \( 1 \leq i \leq n \) and \( i \equiv 1(\text{mod } 2), \\
f(u_i)f(u_i) + f(v_i) = 5n - i + 1 + \frac{n+1}{2} + \frac{n+1}{2} - 1 \\
= 8n + 1 = \lambda_2 \text{ (say)}. \\
\text{For } & 1 \leq i \leq n \quad \text{and } i \equiv 0(\text{mod } 2), \\
f(v_i)f(v_i) + f(w_i) = n + \frac{n+1}{2} + \frac{n+1}{2} + 1 + 5n - i + 1 \\
&= \frac{17n+2}{2} = \lambda_3 \text{ (say)}.
\]

Hence for each edge \( uv \in E, f(u) + f(uv) + f(v) \) yields any one of the trimagic constants \( \lambda_1 = 9n + 2, \lambda_2 = 8n + 1 \) and \( \lambda_3 = \frac{17n+2}{2} \).

Therefore, the graph \( nP_3 \) admits an edge trimagic total labeling when \( n \) is odd.

6) Theorem: The graph \( nP_3 \) admits superior edge trimagic total labeling when \( n \) is odd.

**Proof:** We have proved that the graph \( nP_3 \) has edge trimagic total labeling when \( n \) is odd. The labeling given in the proof of the Theorem 5, the edges get labels for \( i \equiv 1(\text{mod } 2), f(u_i) = \frac{n+1+1}{2} \) and \( f(v_iw_i) = \frac{i+1}{2} \) also for \( i \equiv 0(\text{mod } 2), f(u_i) = n + \frac{n+1}{2} + \frac{i+1}{2} + 1, \) \( f(v_i) = n + \frac{i}{2} + 1 \), \( 1 \leq j \leq n \) Clearly the graph \( nP_3 \) has \( 2n \) edges and the \( 2n \) edges get labels 1, 2, ..., 2n. Hence the graph \( nP_3 \) is a superior edge trimagic total.

7) Corollary: The disconnected graph \( 5P_3 \) admits an edge trimagic labeling for all \( n \).

8) Example: The disconnected graph \( 5P_3 \) given in fig. 2 is a superior edge trimagic total graph.

![Graph 5P3](http://www.ijmttjournal.org)

Fig. 2 Graph \( 5P_3 \) with \( \lambda_1 = 47, \lambda_2 = 44 \) and \( \lambda_3 = 41 \).

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9) **Theorem:** The graph $K_{1, p} \cup K_{1, q} \cup K_{1, r}$, $(p \leq q \leq r)$ admits an edge trimagic total labeling.

**Proof:** Let $V = \{u_v, v_j, w_k\}$, $1 \leq i \leq p+1; 1 \leq j \leq q+1; 1 \leq k \leq r+1$ be the vertex set and $E = \{u_iu_j, v_iw_j, w_kw_i\}$, $2 \leq i \leq p+1; 2 \leq j \leq q+1; 2 \leq k \leq r+1$ be the edge set of the graph $(K_{1, p} \cup K_{1, q} \cup K_{1, r})$.

Define a bijection $f: V \rightarrow \{1, 2, ..., 2p+2q+2r+3\}$ such that $f(u_i) = 1; f(v_j) = 2; f(w_k) = 3; f(u_i) = p+q+r+5-i, 2 \leq i \leq p+1; f(v_j) = q+r+5-j, 2 \leq j \leq q+1; f(w_k) = r+5-k, 2 \leq k \leq r+1$.

For the edges $u_iu_j$, $2 \leq i \leq p+1$;

- $f(u_i)+f(u_j)+f(u) = 1+p+q+r+2i+p+q+r+5–i = 2(p+q+r)+8 = \lambda_1$ (say).

For the edges $v_jv_k$, $2 \leq j \leq q+1$;

- $f(v_j)+f(v_k)+f(v) = 2+2p+q+r+2j+q+r+5–j = 2(p+q+r)+9 = \lambda_2$ (say).

For the edges $w_kw_i$, $2 \leq k \leq r+1$;

- $f(w_k)+f(w_i)+f(w) = 3+2p+2q+r+2k+r+5–k = 2(p+q+r)+10 = \lambda_3$ (say).

Hence for each edge $uv \in E$, $f(u)+f(uv)+f(v)$ yields any one of the magic constants $\lambda_1 = 2(p+q+r)+8, \lambda_2 = 2(p+q+r)+9$ and $\lambda_3 = 2(p+q+r)+10$.

Therefore, the disconnected graph $K_{1, p} \cup K_{1, q} \cup K_{1, r}$, $(p \leq q \leq r)$ admits an edge trimagic total labeling.

10) **Theorem:** The graph $K_{1, p} \cup K_{1, q} \cup K_{1, r}$, $(p \leq q \leq r)$ has a super edge trimagic total labeling.

**Proof:** We have proved that the graph $K_{1, p} \cup K_{1, q} \cup K_{1, r}$, $(p \leq q \leq r)$ has an edge trimagic total labeling. The labeling given in the proof of the Theorem 9, the vertices get labels $f(u_i) = 1; f(v_j) = 2; f(w_k) = 3; f(u_i) = p+q+r+5-i, 2 \leq i \leq p+1; f(v_j) = q+r+5–j, 2 \leq j \leq q+1; f(w_k) = r+5–k, 2 \leq k \leq r+1$.

The graph $K_{1, p} \cup K_{1, q} \cup K_{1, r}$, $(p \leq q \leq r)$ has $(p+q+r+3)$ vertices and the vertices get labels $1, 2, ..., (p+q+r+3)$.

Hence the graph $(K_{1, p} \cup K_{1, q} \cup K_{1, r})$ is a super edge trimagic total.

11) **Example:** The graph $(K_{1, s} \cup K_{1, t}, \emptyset \cup K_{1, 10})$ given in fig. 3 is a super edge trimagic total.

![Fig. 3 (K₁₆∪K₁₈∪K₁₁₀) with λ₁ = 56, λ₂ = 57 and λ₃ = 58.](http://www.ijmttjournal.org)

12) **Theorem:** The graph $nP_{n} \cup K_{1, n+1}$ has an edge trimagic total labeling when $n$ is odd.

**Proof:** Let $V = \{u_v, v_j, w_i\}$, $1 \leq i \leq n\}$ be the vertex set and $E = \{u_i, v_j, w_i\}$, $1 \leq i \leq n + 1\}$ be the edge set of the disconnected graph $nP_{n} \cup K_{1, n+1}$. The graph $nP_{n} \cup K_{1, n+1}$ has $3n+2$ vertices and $2n+1$ edges.

Define a bijection $f: V \cup E \rightarrow \{1, 2, ..., 5n+3\}$ such that

- $f(v_i) = n+\left\lfloor \frac{i+1}{2} \right\rfloor +1, f(u_i) = 2n+\left\lfloor \frac{i+1}{2} \right\rfloor +2$ for $1 \leq i \leq n, i \equiv 1(\mod 2)$,
- $f(v_i) = \frac{n+1}{2} +\left\lfloor \frac{i+1}{2} \right\rfloor, f(u_i) = 2n+\left\lfloor \frac{i+1}{2} \right\rfloor +2$ for $1 \leq i \leq n, i \equiv 0(\mod 2)$. 

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\[ f(v, u_i) = 4n+3 - i, \ 1 \leq i \leq n, \]
\[ f(w_j) = 2(n+1) - \frac{(i-1)}{2}, \ f(ww_i) = 4n+\frac{n+1}{2} + 2 \text{ for } 1 \leq j \leq n+1 \text{ and } j \equiv 1 \pmod{2}, \]
\[ f(w_j) = \frac{n+1}{2} - \frac{i}{2} + 1, \ f(ww_i) = 4n+\frac{n+1}{2} + \frac{i}{2} + 2 \text{ for } 1 \leq j \leq n+1 \]

and \( j \equiv 0 \pmod{2} \) and \( f(w) = n+1. \)

Now we prove that the graph \( nP_2 \cup K_{1, n+1} \) have three trimagic constants \( \lambda_1, \lambda_2 \) and \( \lambda_3. \)

Consider the edges \( v_iu_i. \)

For \( 1 \leq i \leq n \) and \( i \equiv 1 \pmod{2}, \)
\[ f(v_i)+f(v, u_i)+f(u_i) = n + \frac{n+1}{2} + 1 + 4n + 3 - i + 2n + \frac{i+1}{2} + 2 \]
\[ = 7n+7 = \lambda_1 \text{(say).} \]

For \( 1 \leq i \leq n \) and \( i \equiv 0 \pmod{2}, \)
\[ f(v_i)+f(v, u_i)+f(u_i) = \frac{n+1}{2} + \frac{i}{2} + 4n + 3 - i + 2n + \frac{n+1}{2} + \frac{i}{2} + 2 \]
\[ = 7n+6 = \lambda_2 \text{(say).} \]

Consider the edges \( w_j; \)

For \( 1 \leq j \leq n+1 \) and \( j \equiv 1 \pmod{2}, \)
\[ f(w)+f(w, w_j)+f(w_j) = n+1+4n+\frac{n+1}{2} + \frac{j}{2} + 2 + 2(n+1) - \left( \frac{i-1}{2} \right) \]
\[ = 7n+6 = \lambda_2 \text{(say).} \]

For \( 1 \leq j \leq n+1 \) and \( j \equiv 0 \pmod{2}, \)
\[ f(w)+f(w, w_j)+f(w_j) = n+1+4n+\frac{n+1}{2} + \frac{j}{2} + 2 + \frac{n+1}{2} - \frac{j}{2} + 1 \]
\[ = 6n+5 = \lambda_3 \text{(say).} \]

Hence for each edge \( uv \in E, \) \( f(u)+f(uv)+f(v) \) yields any one of the trimagic constants \( \lambda_1 = 7n+7, \lambda_2 = 7n+6 \) and \( \lambda_3 = 6n+5. \)

Thus the graph \( nP_2 \cup K_{1, n+1} \) has an edge trimagic total labeling when \( n \) is odd.

13) **Theorem:** The graph \( nP_2 \cup K_{1, n+1} \) admits a super edge trimagic total labeling for odd \( n. \)

**Proof:** We have proved that the graph \( nP_2 \cup K_{1, n+1} \) has edge trimagic total labeling when \( n \) is odd. The labeling given in the proof of the Theorem 12, the vertices get labels \( f(v_i) = n+\frac{i+1}{2}+1, f(u_i) = 2n+\frac{i+1}{2}+2 \) for \( 1 \leq i \leq n \) and \( i \equiv 1 \pmod{2}, \) and \( f(v_i) = \frac{n+1}{2} + \frac{1}{2}, f(u_i) = 2n+\frac{n+1}{2} + \frac{1}{2} + 2 \) for \( 1 \leq i \leq n \) and \( i \equiv 0 \pmod{2}. \) Also \( f(w) = n+1, f(w_j) = 2(n+1) - \left( \frac{i-1}{2} \right) \) for \( 1 \leq j \leq n+1 \) and \( j \equiv 1 \pmod{2}, \) and \( f(w_j) = \frac{n+1}{2} - \frac{j}{2} + 1 \) for \( 1 \leq j \leq n+1, j \equiv 0 \pmod{2}. \) Clearly the graph \( nP_2 \cup K_{1, n+1} \) has \( 3n+2 \) vertices and get labels \( 1, 2, \ldots, 3n+2. \)

Hence the graph \( nP_2 \cup K_{1, n+1} \) is a super edge trimagic total.

14) **Example:** The disconnected graph \( 5P_2 \cup K_{1, 6} \) given in fig.4 is a super edge trimagic total.

![Fig. 4 Graph 5P_2 \cup K_{1, 6} with \lambda_1 = 42, \lambda_2 = 41 \text{ and } \lambda_3 = 35.](image-url)
15) **Theorem:** The graph $nP_2 \cup K_{1, n+1}$ has an edge trimagic total labeling when $n$ is even.

**Proof:** Let $V = \{v_i, v_i/1 \leq i \leq n\} \cup \{w_j, w_j/1 \leq j \leq n + 1\}$ be the vertex set and $E = \{v_i u_i/1 \leq i \leq n\} \cup \{w_j w_j/1 \leq j \leq n + 1\}$ be the edge set of the disconnected graph $nP_2 \cup K_{1, n+1}$. The graph $nP_2 \cup K_{1, n+1}$ has $3n+2$ vertices and $2n+1$ edges.

Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 5n + 3\}$ such that
$$f(v_i) = n + \frac{i+1}{2} + 1, f(u_i) = 2n - \frac{n+1}{2} + 1 + \frac{i+1}{2} + 1, 1 \leq i \leq n, i = 1 \pmod{2}, f(v_i) = \frac{n+1}{2} + 1, f(u_i) = 2n + \frac{n+1}{2} + 1 + 1, 1 \leq i \leq n, i \equiv 0 \pmod{2}, f(v_i) = 4n + 3 - i, 1 \leq i \leq n;$$
$$f(w_j) = \frac{n}{2} + \frac{j}{2} + 1, f(ww_j) = 4n + \frac{n}{2} + 3, j = 1 \leq j \leq n+1, j \equiv 0 \pmod{2} \text{ and } f(w) = \frac{n}{2} + 1.$$

Now we have to prove the graph $nP_2 \cup K_{1, n+1}$ has three distinct trimagic constants $\lambda_1, \lambda_2$ and $\lambda_3$.

Consider the edges $v_i u_i$.
For $1 \leq i \leq n$ and $i \equiv 0 \pmod{2}$;
$$f(v_i)+f(v_i u_i) + f(u_i) = \frac{n+1}{2} + 1 + 4n + 3 - i + 2n + \frac{i+1}{2} - \frac{n}{2} + 1 = \frac{15n+12}{2} = \lambda_1 \text{(say)}.$$ 

For $1 \leq i \leq n$ and $i \equiv 0 \pmod{2}$;
$$f(v_i)+f(v_i u_i) + f(u_i) = \frac{n+1}{2} + 1 + 4n + 3 - i + 2n + \frac{n}{2} + \frac{i}{2} + 2 = 7n+6 = \lambda_2 \text{(say)}.$$ 

Consider the edges $ww_j$.
For $1 \leq j \leq n+1$ and $j \equiv 1 \pmod{2}$;
$$f(w_j)+f(ww_j) + f(w_j) = \frac{n+1}{2} + 4n + \frac{j+1}{2} + 2 + 2(n+1) + \frac{n}{2} - \frac{j}{2} = 7n+6 = \lambda_2 \text{ (say)}.$$ 

Hence for each edge uv $\in E$, $f(u)+f(uv)+f(v)$ yields any one of the trimagic constants $\lambda_1 = \frac{15n+12}{2}, \lambda_2 = 7n+6$ and $\lambda_3 = \frac{11n+10}{2}$. Thus the graph $nP_2 \cup K_{1, n+1}$ admits an edge trimagic total labeling when $n$ is even.

16) **Theorem:** The graph $nP_2 \cup K_{1, n+1}$ has a super edge trimagic total labeling for even $n$.

**Proof:** We have proved that the graph $nP_2 \cup K_{1, n+1}$ has an edge trimagic total labeling when $n$ is even. The labeling given in the proof of the theorem 15, the vertices get labels $f(v_i) = n + \frac{i+1}{2} + 1$, $f(u_i) = 2n - \frac{n+1}{2} + 1 + \frac{i+1}{2} + 1$ for $1 \leq i \leq n$ and $i = 1 \pmod{2}$; $f(v_i) = \frac{n+1}{2} + 1$, $f(u_i) = 2n + \frac{n+1}{2} + 1 + 1$ for $1 \leq i \leq n$ and $i \equiv 0 \pmod{2}$; $f(w_j) = 2n + \frac{n}{2} + \frac{j}{2} + 2$ for $1 \leq j \leq n+1$ and $j = 1 \pmod{2}$ and $f(w) = \frac{n}{2} + 1$. Clearly the graph $nP_2 \cup K_{1, n+1}$ has $3n+2$ vertices and get labels $1, 2, \ldots, 3n+2$. Hence the graph $nP_2 \cup K_{1, n+1}$ admits a super edge trimagic total labeling when $n$ is even.

17) **Corollary:** The disconnected graph $nP_2 \cup K_{1, n+1}$ admits a super edge trimagic labeling for all $n$.

18) **Example:** The graph $6P_2 \cup K_{1, 7}$ given in fig. 5 is a super edge trimagic total.
19) **Theorem:** For \( n \geq 3 \) and \( t \geq 1 \), the \( t \) copies of the sun graph \( tS_n \) admits an edge trimagic total labeling.

**Proof:**

Let \( tS_n \) be the \( t \) copies of the sun graph \( S_n \) with the vertex set \( V = \{v_1, u_1, \ldots, v_n, u_1, \ldots, u_n \} \) and the edge set \( E = \{\{v_i u_{i+1}, v_i u_j \mid 1 \leq i \leq n-1, 1 \leq j \leq t\} \cup \{v_1 v'_1, v'_{t+1} \} \} \).

Define a bijection \( f: V(tS_n) \cup E(tS_n) \to \{1, 2, \ldots, 4nt\} \) such that

\[
\begin{align*}
\text{for } v_i & : f(v_i) = n(j-1) + i, \\
\text{for } u_i & : f(u_i) = nt + n(j-1) + i, \\
\text{for } v'_i & : f(v'_i) = 4nt - 2nt(j-1) - 2(i-1), \\
\text{for } u'_i & : f(u'_i) = 4nt - 2nt(j-1) - 2(i-1) - 1,
\end{align*}
\]

where \( 1 \leq i \leq n-1, 1 \leq j \leq t \) and \( f(v'_1) = 4nt-2n \).

Now we have to prove that the graph \( tS_n \) has three distinct trimagic constants \( \lambda_1, \lambda_2 \) and \( \lambda_3 \).

For the edges \( v_i v_{i+1}, 1 \leq j \leq t; \)

\[
f(v_i) + f(v_{i+1}) + f(u_j) = n(j-1) + i + 4nt - 2nt(j-1) - 2(i-1) - 1 = \lambda_1 \text{(say)}.
\]

For the edges \( v_i u_j, 1 \leq i \leq n-1, 1 \leq j \leq t; \)

\[
f(v_i) + f(v'_{i+1}) + f(u'_j) = n(j-1) + i + 4nt - 2nt(j-1) - 2(i-1) + nt + n(j-1) + i = \lambda_2 \text{(say)}.
\]

Also for the edges \( u_i u'_j, 1 \leq i \leq n-1, 1 \leq j \leq t; \)

\[
f(v'_1) + f(v'_j) + f(u_j) = 4nt + 3 = \lambda_3 \text{(say)}.
\]

Hence for each edge \( uv \in E \), \( f(u) + f(uv) + f(v) \) yields any one of the trimagic constants \( \lambda_1 = 4nt-2n+3, \lambda_2 = 4nt+3 \) and \( \lambda_3 = 5nt+1 \).

Thus the \( t \) copies of a sun graph, \( tS_n \) admits an edge trimagic total labeling.

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Fig. 5 Graph \( 6P_{2 \Delta K_{1,3}} \) with \( \lambda_1 = 45, \lambda_2 = 48 \) and \( \lambda_3 = 38 \).
20) Example: A super edge trimagic total labeling of 3 copies of sun $S_5$ is given in fig. 6.

Fig. 6 Graph $3S_5$ with $\lambda_1 = 58$, $\lambda_2 = 63$ and $\lambda_3 = 76$.

21) Theorem: For $n \geq 3$ and $t \geq 1$, the $t$ copies the sun graph $tS_n$ is a super edge trimagic total.

Proof: We have proved that the graph $tS_n$ admits an edge trimagic total labeling. The labeling given in the proof of the Theorem 19, the vertices get labels $f(v^1_j) = n(j-1)+i$, $f(u^1_j) = nt+n(j-1)+i$, $1 \leq i \leq n-1$; $1 \leq j \leq t$. Clearly the graph $tS_n$ has $2nt$ vertices and get labels 1, 2, ..., $2nt$.

Hence the graph $tS_n$ is a super edge trimagic total graph

22) Theorem: The graph $nC_4$ admits an edge trimagic total labeling.

Proof: Let $V = \{u_i, v_i, w_i, x_i / 1 \leq i \leq n\}$ be the vertex set and $E = \{u_iv_i, v_iw_i, w_ix_i, x_iu_i / 1 \leq i \leq n\}$ be the edge set of the disconnected graph $nC_4$. The graph $nC_4$ has $4n$ vertices and $4n$ edges.

Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 8n\}$ such that $f(u_i) = i$, $f(v_i) = n+i$, $f(w_i) = 3n+i$, $f(x_i) = 2n+i$ for all $1 \leq i \leq n$; $f(u_iv_i) = 8n-2i+2$, $f(v_iw_i) = 6n-2i+1$, $f(w_ix_i) = 6n-2i+2$ and $f(x_iu_i) = 8n-2i+1$ for all $1 \leq i \leq n$.

Now we prove that the graph $nC_4$ admits an edge trimagic total labeling.

For the edges $u_iv_i$, $1 \leq i \leq n$;

$$f(u_i)+f(u_iv_i)+f(v_i) = i+8n-2i+2+n+i = 9n+2 = \lambda_1 \text{(say)}.$$  

For the edges $v_iw_i$, $1 \leq i \leq n$;

$$f(v_i)+f(v_iw_i)+f(w_i) = n+i+6n-2i+1+3n+i = 10n+1 = \lambda_2 \text{(say)}.$$  

For the edges $w_ix_i$, $1 \leq i \leq n$;

$$f(w_i)+f(w_ix_i)+f(x_i) = 3n+i+6n-2i+2+2n+i = 11n+2 = \lambda_3 \text{(say)}.$$
For the edges $x_iu_i$, $1 \leq i \leq n$;

$$f(x_i) + f(x_iu_i) + f(u_i) = 2n+i+8n-2i+1+i = 10n+1 = \lambda_2.$$  

Hence for each edge $uv \in E$, $f(u) + f(uv) + f(v)$ yields any one of the trimagic constant $\lambda_1 = 9n+2$, $\lambda_2 = 10n+1$ and $\lambda_3 = 11n+2$. Hence the disconnected graph $nC_4$ admits an edge trimagic total labeling.

23) **Theorem:** The graph $nC_4$ admits an edge trimagic total labeling.

**Proof:** We have proved that the graph $nC_4$ has an edge trimagic total labeling. The labeling given in the proof of the theorem 22, the vertices get labels $f(u_i) = i$, $f(v_i) = 3n+i$, $f(w_i) = 2n+i$ for all $1 \leq i \leq n$. Clearly the graph has $4n$ vertices and get labels $1, 2, \ldots, 4n$. Hence the graph $nC_4$ admits a super edge trimagic total labeling.

24) **Example:** A super edge trimagic total labeling of $4C_4$ is given in fig. 7

![Graph 4C 4](image)

Fig. 7 Graph $4C_4$ with $\lambda_1 = 38$, $\lambda_2 = 41$ and $\lambda_3 = 46$.

25) **Theorem:** The graph $nC_6$ admits an edge trimagic total labeling.

**Proof:** Let $V = \{u_i, v_i, w_i, x_i, y_i, z_i \mid 1 \leq i \leq n\}$ be the vertex set and $E = \{u_iw_i, w_ix_i, x_iy_i, y_iz_i, z_iu_i \mid 1 \leq i \leq n\}$ be the edge set of the disconnected graph $nC_6$. The graph $nC_6$ has $6n$ vertices and $6n$ edges.

Define a bijection $f: V \cup E \rightarrow \{1, 2, \ldots, 12n\}$ such that $f(u_i) = i$, $f(v_i) = 3n+i$, $f(w_i) = n+i$, $f(x_i) = 4n+i$, $f(y_i) = 2n+i$, $f(z_i) = 5n+i$ for all $1 \leq i \leq n$; $f(u_iw_i) = 12n-2i+2$, $f(v_iw_i) = 12n-2i+1$, $f(w_ix_i) = 8n-2i+1$, $f(x_iy_i) = 10n-2i+1$, $f(y_iz_i) = 8n-2i+2$ and $f(z_iu_i) = 10n-2i+2$ for all $1 \leq i \leq n$.

Now we prove that the graph $nC_6$ admits an edge trimagic total labeling.

For the edges $u_iv_i$, $1 \leq i \leq n$;

$$f(u_i) + f(u_iv_i) + f(v_i) = i+12n-2i+2+3n+i = 15n+2 = \lambda_3$$ (say).

For the edges $v_iw_i$, $1 \leq i \leq n$;

$$f(v_i) + f(v_iw_i) + f(w_i) = 3n+i+12n-2i+1+n+i = 16n+1 = \lambda_2$$ (say).

For the edges $w_ix_i$, $1 \leq i \leq n$;

$$f(w_i) + f(w_ix_i) + f(x_i) = n+i+8n-2i+1+4n+i = 13n+1 = \lambda_3$$ (say).

For the edges $x_iy_i$, $1 \leq i \leq n$;

$$f(x_i) + f(x_iy_i) + f(y_i) = 4n+i+10n-2i+1+2n+i = 16n+1 = \lambda_2$$.

For the edges $y_iz_i$, $1 \leq i \leq n$;

$$f(y_i) + f(y_iz_i) + f(z_i) = 2n+i+8n-2i+2+5n+i = 15n+2 = \lambda_3.$$
For the edges $z_iu_i$, $1 \leq i \leq n$;

$$f(z_i) + f(u_i) + f(v_i) = 5n + i + 10n - 2i + 2 + i = 15n + 2 = \lambda_i.$$  

Hence for each edge $uv \in E$, $f(u) + f(uv) + f(v)$ yields any one of the trimagic constant $\lambda_1 = 15n + 2$, $\lambda_2 = 16n + 1$ and $\lambda_3 = 13n + 1$. Hence the disconnected graph $nC_6$ admits an edge trimagic total labeling.

26) **Theorem:** The graph $nC_6$ admits a super edge trimagic total labeling.

**Proof:** We have proved that the graph $nC_6$ has an edge trimagic total labeling. The labeling given in the proof of the theorem 25, the vertices get labels $f(u_i) = i$, $f(v_i) = 3n + i$, $f(w_i) = n + i$, $f(x_i) = 4n + i$, $f(y_i) = 2n + i$, $f(z_i) = 5n + i$ for all $1 \leq i \leq n$. Clearly the graph $nC_6$ has $6n$ vertices and get labels $1, 2, \ldots, 6n$. Hence the graph $nC_6$ admits a super edge trimagic total labeling.

27) **Example:** A super edge trimagic total labeling of $3C_6$ is given in fig. 8.

![Graph 3C_6 with \lambda_1 = 47, \lambda_2 = 49 and \lambda_3 = 40.](image)

Fig. 8 Graph $3C_6$ with $\lambda_1 = 47, \lambda_2 = 49$ and $\lambda_3 = 40$.

### III. CONCLUSIONS

In this paper, we proved that the disconnected graphs $K_{1, p} \cup K_{1, q} \cup K_{1, r}$ ($p \leq q \leq r$), $nP_2 \cup K_{1, n+1}$, $t$ copies of the sun graph $T_n$, $nC_4$ and $nC_6$ are super edge trimagic total and $nP_3$ is superior edge trimagic total. There may be many interesting trimagic graphs can be constructed also in future.

### REFERENCES