Semipre Generalized Closed Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract—In this paper we introduce intuitionistic fuzzy semipre generalized closed mappings, intuitionistic fuzzy semipre generalized open mappings and intuitionistic fuzzy M-semipre generalized closed mappings and we study some of their properties. We provide the relation between intuitionistic fuzzy M-semipre generalized closed mappings and intuitionistic fuzzy semipre generalized closed mappings.

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I. INTRODUCTION

In 1965, Zadeh [12] introduced fuzzy sets and in 1968, Chang [3] introduced fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors were conducted on the generalization of this notion. The notion of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. 1997, Coker [4] introduced the concept of intuitionistic fuzzy topological spaces. In 2000, Seok Jong Lee and Eun Pyo Lee [8] investigated the properties of continuous, open and closed maps in the intuitionistic fuzzy topological spaces. In this direction we introduce the notions of intuitionistic fuzzy semipre generalized closed mappings, intuitionistic fuzzy semipre generalized open mappings and intuitionistic fuzzy M-semipre generalized closed mappings and study some of their properties.

II. PRELIMINARIES

Definition 2.1: [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form A = { (x, μA(x), νA(x)) / x ∈ X } where the functions μA: X → [0,1] and νA : X → [0,1] denote the degree of membership (namely μA(x)) and the degree of non-membership (namely νA(x)) of each element x ∈ X to the set A respectively, and 0 ≤ μA(x) + νA(x) ≤ 1 for each x ∈ X. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form A = { (x, μA(x), νA(x)) / x ∈ X } and B = { (x, µB(x), νB(x)) / x ∈ X }. Then
(i) A ⊆ B if and only if μA(x) ≤ µB(x) and νA(x) ≥ νB(x) for all x ∈ X,
(ii) A = B if and only if A ⊆ B and B ⊆ A,
(iii) A' = { (x, νA(x), μA(x)) / x ∈ X },
(iv) A ∩ B = { (x, μA(x) ∧ μB(x), νA(x) ∨ νB(x)) / x ∈ X },
(v) A ∪ B = { (x, μA(x) ∨ μB(x), νA(x) ∧ νB(x)) / x ∈ X }.
For the sake of simplicity, we shall use the notation A = (x, μA, νA) instead of A = { (x, μA(x), νA(x)) / x ∈ X }. Also for the sake of simplicity, we shall use the notation A = (x, μA, μB, (A/νA, B/νB)) instead of A = (x, (μA, B/νB), (A/νA, B/νB)). The intuitionistic fuzzy sets 0 = { (x, 0, 1) / x ∈ X } and 1 = { (x, 1, 0) / x ∈ X } are respectively the empty set and the whole set of X.

Definition 2.3: [4] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:
(i) 0, 1 ∈ τ,
(ii) G1 ∩ G2 ∈ τ, for any G1, G2 ∈ τ,
(iii) U Gi ∈ τ for any family {Gi / i ∈ J} ⊆ τ.
In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFT in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A' of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [4] Let (X, τ) be an IFTS and A = (x, μA, νA) be an IFS in X. Then
(i) int(A) = ∅ / G / G is an IFOS in X and G ⊆ A ,
(ii) cl(A) = ∅ / K / K is an IFCS in X and A ⊆ K ,
(iii) cl(A') = (int(A))°,
(iv) int(A') = (cl(A))°.

Definition 2.5: [5] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy semiclosed set (IFSCS in short) if int(cl(A)) ⊆ A,
(ii) intuitionistic fuzzy semiopen set (IFSOS in short) if A ⊆ cl(int(A)).

Definition 2.6: [5] An IFS A of an IFTS (X, τ) is an
(i) intuitionistic fuzzy preclosed set (IFPCS in short) if
\[ \text{cl}(\text{int}(A)) \subseteq A, \]
(ii) intuitionistic fuzzy preopen set (IFPOS in short) if \( A \subseteq \text{int}(\text{cl}(A)). \)

Note that every IFOS in \((X, \tau)\) is an IFPOS in \(X\).

**Definition 2.7:** [5] An IF A of an IFTS \((X, \tau)\) is an
(i) intuitionistic fuzzy \(\alpha\)-closed set (IF\(\alpha\)CS in short) if
\[ \text{cl}(\text{int}(A)) \subseteq A, \]
(ii) intuitionistic fuzzy \(\alpha\)-open set (IF\(\alpha\)OS in short) if \( A \subseteq \text{int}(\text{cl}(A)). \)

**Definition 2.8:** [11] An IF A of an IFTS \((X, \tau)\) is an
(i) intuitionistic fuzzy semipre closed set (IFSPCS for short) if there exists an IFCS B such that \( B \subseteq A \subsetneq B, \)
(ii) intuitionistic fuzzy semipre open set (IFSPSO for short) if there exists an IFPOS B such that \( B \subsetneq A \subseteq \text{cl}(B). \)

Definition 2.9: [8] An IF A of an IFTS \((X, \tau)\) is called an intuitionistic fuzzy W-closed set (IFWCS in short) if \( \text{cl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is an IFPOS in \((X, \tau). \) An IF A of an IFTS \((X, \tau)\) is called an intuitionistic fuzzy W-open set (IFWOS in short) if \( A^c \) is an IFWCS in \((X, \tau). \)

**Definition 2.10:** [10] An IF A in an IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy semipre generalized closed set (IFSPGCS for short) if \( \text{spcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is an IFPOS in \((X, \tau). \) Every IFCS, IFSCS, IFWCS, IF\(\alpha\)CS, IFPCS, IFSPCS is an IFSPGCS but the converses are not true in general.

**Definition 2.11:** [9] The complement \( A^c \) of an IFSPGCS A in an IFTS \((X, \tau)\) is called an intuitionistic fuzzy semipre generalized open set (IFSPGOS for short) in \(X\).

The family of all IFSPGOS of an IFTS \((X, \tau)\) is denoted by IFSPGOS(\(X).\) Every IFOS, IFPOS, IFWOS, IF\(\alpha\)OS, IFPOS is an IFSPGOS but the converses are not true in general.

**Definition 2.12:** [6] Let A be an IF in an IFTS \((X, \tau). \) Then
(i) \( \text{spint}(A) = U \{ G / G \text{ is an IFPOS in } X \text{ and } G \subseteq A \}, \)
(ii) \( \text{spcl}(A) = \cap \{ K / K \text{ is an IFPCS in } X \text{ and } A \subseteq K \}. \)

Note that for any IF A in \((X, \tau), \) we have \( \text{spcl}(A^c) = (\text{spint}(A))^c \) and \( \text{spint}(A^c) = (\text{spcl}(A))^c. \)

**Result 2.13:** [2] For an IF A in an IFTS\((X, \tau), \) we have \( \text{spcl}(A) \supseteq A \cup \text{int}(\text{cl}(\text{int}(A))). \)

**Definition 2.14:** [9] If every IFSPGCS in \((X, \tau)\) is an IFSPCS in \((X, \tau), \) then the space can be called as an intuitionistic fuzzy semipre T\(_{1/2}\) (IF\(\text{SP}\)T\(_{1/2}\) for short) space.

**Definition 2.15:** [7] A map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy closed mapping (IFCM for short) if \( f(A) \) is an IFCS in \(Y\) for each IFCS \(A\) in \(X. \)

**Definition 2.16:** [7] A map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called an
(i) intuitionistic fuzzy semiopen mapping (IFSO\(M\) for short) if \( f(A) \) is an IFOS in \(Y\) for each IFOS \(A\) in \(X. \)
(ii) intuitionistic fuzzy \(\alpha\)-open mapping (IF\(\alpha\)OM for short) if \( f(A) \) is an IF\(\alpha\)OS in \(Y\) for each IF\(\alpha\)OS \(A\) in \(X. \)
(iii) intuitionistic fuzzy preopen mapping (IFP\(\alpha\)OM for short) if \( f(A) \) is an IFPOS in \(Y\) for each IFOS \(A\) in \(X. \)

### III. INTUITIONISTIC FUZZY SEMIPRE GENERALIZED CLOSED MAPPINGS

In this paper we introduce intuitionistic fuzzy semipre generalized closed mappings and intuitionistic M-semipre generalized closed mappings and study some of their properties.

**Definition 3.1:** A mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy semipre generalized closed mapping (IFSPGCM for short) if \( f(A) \) is an IFSPGCS in \(Y\) for each IFCS \(A\) in \(X. \)

For the sake of simplicity, we shall use the notation \( A = (x, (\mu, \mu), (\nu, \nu)) \) instead of \( A = (x, (a/\mu, b/\mu), (a/\nu, b/\nu)) \) in all the examples used in this paper. Similarly we shall use the notation \( B = (x, (\mu, \mu), (\nu, \nu)) \) instead of \( B = (x, (u/\mu, v/\mu), (u/\nu, v/\nu)) \) in the following examples.

**Example 3.2:** Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = (x, (0.4, 0.3), (0.6, 0.4)), G_2 = (y, (0.7, 0.8), (0.3, 0.2)), G_3 = (y, (0.6, 0.7), (0.4, 0.3)) \). Then \( \tau = \{0, \}, G_1, 1, \} \) and \( \sigma = \{0, G_2, G_3, 1, \} \) are IFTs on \(X\) and \(Y\) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v. \) Then \( f \) is an IFSPGCM.

**Theorem 3.3:** Every IFCM is an IFSPGCM but not conversely.

**Proof:** Assume that \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IFCM. Let A be an IFCS in \(X. \) Then \( f(A) \) is an IFCS in \(Y. \) This implies that \( f(A) \) is an IFSPGCS in \(Y. \) Hence \( f \) is an IFSPGCM.

**Example 3.4:** In Example 3.2, \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IFSPGCM but not an IFCM.

**Theorem 3.5:** Every IF\(\alpha\)CM is an IFSPGCM but not conversely.

**Proof:** Assume that \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IF\(\alpha\)CM. Let A be an IFCS in \(X. \) Then \( f(A) \) is an IF\(\alpha\)CS in \(Y. \) This implies that \( f(A) \) is an IFSPGCS in \(Y. \) Hence \( f \) is an IFSPGCM.

**Example 3.6:** In Example 3.2, \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IFSPGCM but not an IF\(\alpha\)CM.

**Theorem 3.7:** Every IFSCM is an IFSPGCM but not conversely.
Example 3.8: In Example 3.2, \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IFSPGCM but not an IFSCM.

Theorem 3.9: Every IFWCM is an IFSPGCM but not conversely.

Proof: Assume that \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IFWCM. Let \( A \) be an IFCS in \( X \). Then \( f(A) \) is an IFWCS in \( Y \). Hence \( f \) is an IFSPGCM.

Example 3.10: In Example 3.2, \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IFSPGCM but not an IFWCM.

Theorem 3.11: Every IFPCM is an IFSPGCM but not conversely.

Proof: Assume that \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IFPCM. Let \( A \) be an IFCS in \( X \). Then \( f(A) \) is an IFPCM in \( Y \). Hence \( f \) is an IFSPGCM.

Example 3.12: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \{(x, (0.4, 0.5)), (0.6, 0.5)\}, G_2 = \{(y, (0.3, 0.2)), (0.6, 0.6)\} \). Then \( \tau = \{0., G_1, 1.\} \) and \( \sigma = \{0., G_2, 1.\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFSPGCM but not an IFPCM.

Definition 3.13: A mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be an intuitionistic fuzzy semipre generalized open mapping (IFSPGOM for short) if \( f(A) \) is an IFSPGCS in \( Y \) for every IFSPGCS \( A \) in \( X \).

Definition 3.14: A mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be an intuitionistic fuzzy M-semipre generalized closed mapping (IFSPGMC for short) if \( f(A) \) is an IFSPGCS in \( Y \) for every IFSPGCS \( A \) in \( X \).

Definition 3.15: A mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be an intuitionistic fuzzy M-semipre generalized open mapping (IFSPGOM for short) if \( f(A) \) is an IFSPGCS in \( Y \) for every IFSPGCS \( A \) in \( X \).

Example 3.16: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \{(x, (0.5, 0.6)), (0.5, 0.4)\}, G_2 = \{(y, (0.3, 0.2)), (0.7, 0.8)\} \). Then \( \tau = \{0., G_1, 1.\} \) and \( \sigma = \{0., G_2, 1.\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFSPGCM.

Example 3.17: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \{(x, (0.3, 0.2)), (0.7, 0.8)\}, G_2 = \{(y, (0.5, 0.6)), (0.5, 0.4)\} \). Then \( \tau = \{0., G_1, 1.\} \) and \( \sigma = \{0., G_2, 1.\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFSPGCM.

Theorem 3.18: Every IFMSPGCM is an IFSPGCM but not conversely.

Proof: Assume that \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IFMSPGCM. Let \( A \) be an IFCS in \( X \). Then \( A \) is an IFSPGCS in \( X \). By hypothesis \( f(A) \) is an IFSPGCS in \( Y \). Hence \( f \) is an IFSPGCM.

Example 3.19: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \{(x, (0.5, 0.4)), (0.5, 0.6)\}, G_2 = \{(y, (0.6, 0.7)), (0.4, 0.3)\} \). Then \( \tau = \{0., G_1, 1.\} \) and \( \sigma = \{0., G_2, 1.\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFSPGCM but not an IFMSPGCM.

The relation between various types of intuitionistic fuzzy closed mappings is given by:

\[
\text{IFPCM} \rightarrow \text{IFWCM} \quad \text{IFSCM} \rightarrow \text{IFSPGCM} \leftarrow \text{IFMSPGCM}
\]

The reverse implications are not true in general in the above diagram.

Theorem 3.20: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a mapping. Then the following statements are equivalent if \( Y \) is an IFSPT_{1/2} space:

(i) \( f \) is an IFSPGCM,
(ii) \( \text{spcl}(f(A)) \subseteq f(\text{cl}(A)) \) for each IFS \( A \) of \( X \).

Proof: (i) \( \Rightarrow \) (ii) Let \( A \) be any IFS in \( X \). Then \( f(A) \) is an IFCS in \( Y \). Since \( Y \) is an IFSPT_{1/2} space, \( f(\text{cl}(A)) \) is an IFSPCS in \( Y \). Therefore \( \text{spcl}(f(A)) \subseteq f(\text{cl}(A)) \). Hence \( \text{spcl}(f(A)) \subseteq f(\text{cl}(A)) \) for each IFS \( A \) of \( X \).

(ii) \( \Rightarrow \) (i) Let \( A \) be any IFSCS in \( X \). Then \( f(A) \) is an IFPCM. (ii) implies that \( \text{spcl}(f(A)) \subseteq f(\text{cl}(A)) \). But \( f(A) \subseteq \text{spcl}(f(A)) \). Therefore \( f(\text{cl}(A)) = f(A) \). This implies \( f(A) \) is an IFSPCS in \( Y \). Since every IFSPCS is an IFSPGCS, \( f(A) \) is an IFSPGCS in \( Y \). Hence \( f \) is an IFSPGCM.

Theorem 3.21: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a bijection. Then the following statements are equivalent if \( Y \) is an IFSPT_{1/2} space:

(i) \( f \) is an IFSPGCM,
(ii) \( \text{spcl}(f(A)) \subseteq f(\text{cl}(A)) \) for each IFS \( A \) of \( X \),
(iii) \( f^{-1}(\text{spcl}(B)) \subseteq \text{cl}(f^{-1}(B)) \) for every IFS \( B \) of \( Y \).

Proof: (i) \( \iff \) (ii) is obvious from Theorem 3.20.

(ii) \( \iff \) (iii) Let \( B \) be an IFS in \( Y \). Then \( f^{-1}(B) \) is an IFS in \( X \). Since \( f \) is onto, \( \text{spcl}(B) = \text{spcl}(f^{-1}(B)) \) and (ii) implies \( \text{spcl}(f^{-1}(B)) \subseteq f(\text{cl}(f^{-1}(B))) \). Therefore \( f(\text{cl}(f^{-1}(B))) \subseteq f(\text{cl}(f^{-1}(B))) \). Now \( f^{-1}(\text{spcl}(B)) \subseteq f^{-1}(f(\text{cl}(f^{-1}(B)))) = f(\text{cl}(f^{-1}(B))) \), since \( f \) is one to one. Hence \( f^{-1}(\text{spcl}(B)) \subseteq f(\text{cl}(f^{-1}(B))) \).
Theorem 3.22: If \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IFCM and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) is an IFSPGCM then \( g \circ f : (X, \tau) \rightarrow (Z, \eta) \) is an IFSPGCM.

Proof: Let \( A \) be any IFCS in \( X \). Then \( f(A) \) is an IFCS in \( Y \), by hypothesis. Since \( g \) is an IFSPGCM, \( g(f(A)) \) is an IFPGCS in \( Z \). Therefore \( g \circ f \) is an IFSPGCM.

Theorem 3.23: Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a mapping where \( Y \) is an IFSP\( T_{1/2} \) space. Then the following statements are equivalent.

(i) \( f \) is an IFSPGCM,
(ii) \( f(B) \) is an IFSPGOS in \( Y \) for every IFOS \( B \) in \( X \),
(iii) \( f(\text{int}(B)) \subseteq \text{cl}(\text{int}(f(B))) \) for every IFOS \( B \) in \( X \).

Proof: (i) \( \Leftrightarrow \) (ii) is obvious.

(ii) \( \Rightarrow \) (iii) Let \( B \) be an IFCS in \( X \). Then \( \text{int}(B) \) is an IFOS in \( X \). By hypothesis \( f(\text{int}(B)) \) is an IFSPGOS in \( Y \). Since \( Y \) is an IFSP\( T_{1/2} \) space, \( f(\text{int}(B)) \) is an IFSPGOS in \( Y \). Therefore \( f(\text{int}(B)) \subseteq \text{cl}(\text{int}(f(B))) \subseteq \text{cl}(\text{cl}(f(B))) \).

(iii) \( \Rightarrow \) (i) Let \( A \) be an IFCS in \( X \). Then \( A \) is an IFOS in \( X \). By hypothesis, \( f(A) \subseteq \text{cl}(\text{int}(f(A))) \subseteq \text{cl}(f(A)) \). This implies \( f(A) \) is an IFPGCS in \( Y \) and hence an IFSPGCS in \( Y \). Therefore \( f \) is an IFSPGCM.

Theorem 3.24: If \( f : (X, \tau) \rightarrow (Y, \sigma) \) is a mapping. Then the following are equivalent if \( Y \) is an IFSP\( T_{1/2} \) space

(i) \( f \) is an IFSPGCM
(ii) \( f(\text{int}(A)) \subseteq \text{spint}(f(A)) \) for each IF A of X
(iii) \( f^{-1}(\text{spint}(B)) \subseteq f^{-1}(\text{spint}(B)) \) for every IF B of Y.

Proof: (i) \( \Rightarrow \) (ii) Let \( f \) be an IFSPGCM. Let \( A \) be any IFCS in \( X \). Then \( \text{int}(A) \) is an IFOS in \( X \). (i) implies that \( f(\text{int}(A)) \) is an IFSPGOS in \( Y \). Since \( Y \) is an IFSP\( T_{1/2} \) space, \( f(A) \) is an IFSPGOS in \( Y \). Therefore \( f(\text{int}(A)) = \text{int}(f(A)) \). Now \( f(\text{int}(A)) = \text{spint}(f(\text{int}(A))) \subseteq \text{spint}(f(A)) \).

(ii) \( \Rightarrow \) (iii) Let \( B \) be an IFCS in \( Y \). Then \( f^{-1}(B) \) is an IFCS in \( X \). By (ii) \( f(\text{int}(B)) \subseteq \text{spint}(f(f^{-1}(B))) \subseteq \text{spint}(B) \). Therefore \( f^{-1}(B) \subseteq f^{-1}(\text{spint}(B)) \). (iii) \( \Rightarrow \) (i) Let \( A \) be an IFCS in \( X \). Then \( \text{int}(A) = A \) and \( f(A) \) is an IFCS in \( Y \). By (iii) \( \text{int}(f^{-1}(f(A))) \subseteq f^{-1}(\text{spint}(f(A))) \). This implies \( \text{spint}(f(A)) \subseteq f(A) \) is an IFSPGOS in \( Y \) and hence an IFSPGOS in \( Y \). Thus \( f \) is an IFSPGCM.

Theorem 3.25: A mapping \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an IFSPGCM if \( f(\text{spint}(A)) \subseteq \text{spint}(f(A)) \) for every \( A \subseteq X \).
Theorem 3.29: If \( f: (X, \tau) \to (Y, \sigma) \) is a mapping where \( X \) and \( Y \) are IFSPPT_{1/2} space, then the following statements are equivalent:

(i) \( f \) is an IFMSPGCM,
(ii) \( f(A) \) is an IFSPGOS in \( Y \) for every IFSPGOS \( A \) in \( X \),
(iii) \( spint(f(B)) \subseteq f(spint(B)) \) for every IFS \( B \) in \( X \),
(iv) \( spcl(f(B)) \subseteq f(spcl(B)) \) for every IFS \( B \) in \( X \).

Proof: (i) \( \Rightarrow \) (ii) is obvious.
(ii) \( \Rightarrow \) (iii) Let \( B \) be any IFS in \( X \). Since \( spint(B) \) is an IFSPPOS, it is an IFSPGOS in \( X \). Then by hypothesis, \( f(spint(B)) \) is an IFSPGOS in \( Y \). Since \( Y \) is an IFSPPT_{1/2} space, \( f(spint(B)) \) is an IFSPOS in \( Y \). Therefore \( f(spint(B)) = spint(f(spint(B))) \subseteq spint(f(B)) \).
(iii) \( \Rightarrow \) (iv) is obvious by taking complement in (iii).
(iv) \( \Rightarrow \) (i) Let \( A \) be an IFSPGCS in \( X \). By Hypothesis, \( spcl(f(A)) \subseteq f(spcl(A)) \). Since \( X \) is an IFSPPT_{1/2} space, \( A \) is an IFSPCS in \( X \). Therefore \( spcl(f(A)) \subseteq f(spcl(A)) = f(A) \subseteq f(spcl(A)) \). Hence \( f(A) \) is an IFSPCS in \( Y \) and hence an IFSPGCS in \( Y \). Thus \( f \) is an IFMSPGCM.

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