Root Square Mean Labeling of Some New Disconnected Graphs

S.S.Sandhya, S.Somasundaram, S.Anusa

1. Department of Mathematics, Sree Ayyappa College for women, Chunkankadai: 629003, Tamil Nadu, India.
2. Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli: 627012, Tamil Nadu, India.
3. Department of Mathematics, Arunachala College of Engineering for Women, Vellichanthai-629203, Tamil Nadu, India.

Abstract

A graph \( G = (V,E) \) with \( p \) vertices and \( q \) edges is called a Root Square Mean graph if it is possible to label the vertices \( x \in V \) with distinct labels \( f(x) \) from \( 1, 2, ..., q + 1 \) in such a way that when each edge \( e = uv \) is labeled with \( f(e = uv) = \sqrt{\frac{(f(u)^2 + f(v)^2)}{2}} \) or \( \sqrt{\frac{(f(u)^2 + f(v)^2)}{2}} \), then the edge labels are distinct. In this case \( f \) is called Root Square mean labeling of \( G \). In this paper we prove that some disconnected graphs are Root Square Mean graphs.

Key Words: Graph, Root Square Mean labeling, Path, Cycle, Comb, Triangular Snake, Quadrilateral Snake.

1. INTRODUCTION

The graph considered here are simple, finite and undirected graphs. Let \( V(G) \) denote the vertex set and \( E(G) \) denote the edge set of \( G \). For detailed survey of graph labeling we refer to Gallian[1]. For all other standard terminology and notations we follow Harary [2]. S.S.Sandhya, S.Somasundaram and S.Anusa introduced the concept of Root Square Mean labeling of graphs in [4] and studied their behavior in [5], [6], [7]. Also we proved that some disconnected graphs are also Root Square Mean graphs in [8] and [9]. In this paper we prove that some new disconnected graphs are Root Square Mean graphs. The following definitions and theorems are necessary for our future study.

Definition 1.1: A graph \( G = (V,E) \) with \( p \) vertices and \( q \) edges is called a Root Square Mean graph if it is possible to label the vertices \( x \in V \) with distinct labels \( f(x) \) from \( 1, 2, ..., q + 1 \) in such a way that when each edge \( e = uv \) is labeled with \( f(e = uv) = \sqrt{\frac{(f(u)^2 + f(v)^2)}{2}} \) or \( \sqrt{\frac{(f(u)^2 + f(v)^2)}{2}} \), then the edge labels are distinct. In this case \( f \) is called Root Square mean labeling of \( G \).

Definition 1.2: The union of two graphs \( G_1 = (V_1,E_1) \) and \( G_2 = (V_2,E_2) \) is a graph \( G = G_1 \cup G_2 \) with vertex set \( V = V_1 \cup V_2 \) and the edge set \( E = E_1 \cup E_2 \).
Definition 1.3: The Corona of two graphs $G_1$ and $G_2$ is the graph $G = G_1 \circ G_2$ formed by taking one copy of $G_1$ and $|V(G_1)|$ copies of $G_2$ where the $i^{th}$ vertex of $G_1$ is adjacent to every vertex in the $i^{th}$ copy of $G_2$.

Definition 1.4: A Triangular Snake $T_n$, is obtained from a path $u_1u_2 \cdots u_n$ by joining $u_i$ and $u_{i+1}$ to a new vertex $v_i$, $1 \leq i \leq n-1$.

Definition 1.5: A Double Triangular Snake $D(T_n)$ consists of two Triangular Snakes that have a common path.

Definition 1.6: A Quadrilateral Snake $Q_n$ is obtained from a path $u_1u_2 \cdots u_n$ by joining $u_i$ and $u_{i+1}$ to two new vertices $v_i$ and $w_i$ respectively and then joining $v_i$ and $w_i$, $1 \leq i \leq n-1$.

Definition 1.7: A Double Quadrilateral Snake $D(Q_n)$ consists of two Quadrilateral Snakes that have a common path.

Theorem 1.8: Triangular Snake $T_n$ is a Root Square Mean graph.

Theorem 1.9: Double Triangular Snake $D(T_n)$ is a Root Square Mean graph.

Theorem 1.10: Quadrilateral Snake $Q_n$ is a Root Square Mean graph.

Theorem 1.11: Double Quadrilateral Snake $D(Q_n)$ is a Root Square Mean graph.

2. MAIN RESULTS

Theorem 2.1: $C_m \cup T_n$ is a Root Square Mean graph.

Proof: Let $u_1u_2 \cdots u_mu_1$ be the cycle $C_m$. Let $v_1v_2 \cdots v_n$ be the path $P_n$. Let $T_n$ be the triangular snake obtained from the path $P_n$ by joining $v_i$ and $v_{i+1}$ to new vertex $w_i$, $1 \leq i \leq n-1$. Let $G = C_m \cup T_n$.

Define a function $f: V(G) \rightarrow \{1, 2, \ldots, q+1\}$ by

$f(u_i) = i , 1 \leq i \leq m$
$f(v_i) = m + 3i - 2 , 1 \leq i \leq n$
$f(w_i) = m + 3i - 1 , 1 \leq i \leq n-1$

Then the edge labels are distinct. Hence $f$ is a Root Square Mean labeling of $G$. 
Example 2.2: Root Square Mean labeling of \( C_7 \cup T_6 \) is given below.

![Figure 1](image1.png)

**Theorem 2.3:** \((C_m \otimes K_1) \cup T_n\) is a Root Square Mean graph.

**Proof:** Let \( u_1 u_2 \cdots u_m u_1 \) be the cycle \( C_m \). Let \( v_i \) be the vertex of \( K_1 \) which is attached to the vertex \( u_i, 1 \leq i \leq m \) of the cycle \( C_m \). Let \( w_1 w_2 \cdots w_n \) be the path \( P_n \). Let \( T_n \) be the triangular snake obtained from \( P_n \) by joining \( w_i \) and \( w_{i+1} \) to a new vertex \( x_i, 1 \leq i \leq n - 1 \). Let \( G = (C_m \otimes K_1) \cup T_n \).

Define a function \( f: V(G) \rightarrow \{1,2,\ldots,q+1\} \) by

\[
\begin{align*}
 f(u_i) &= 2i - 1, 1 \leq i \leq m \\
 f(v_i) &= 2i, 1 \leq i \leq m \\
 f(w_i) &= 2m + 3i - 2, 1 \leq i \leq n \\
 f(x_i) &= 2m + 3i - 1, 1 \leq i \leq n - 1
\end{align*}
\]

Then the edge labels are distinct. Hence \( f \) is a Root Square Mean labeling of \( G \).

Example 2.4: Root Square Mean labeling of \((C_7 \otimes K_1) \cup T_5\) is given below.

![Figure 2](image2.png)
Theorem 2.5: \( C_m \odot D(T_n) \) is a Root Square Mean graph.

Proof: Let \( u_1u_2 \cdots u_mu_1 \) be the cycle \( C_m \). Let \( v_1v_2 \cdots v_n \) be the path \( P_n \). The double triangular snake \( D(T_n) \) is obtained from the path \( P_n \) by joining \( v_i \) and \( v_{i+1} \) to two new vertices \( x_i \) and \( y_i \), \( 1 \leq i \leq n-1 \).

Let \( G = C_m \odot D(T_n) \). Define a function \( f: V(G) \to \{1, 2, \ldots, q + 1\} \) by

\[
\begin{align*}
  f(u_i) &= i, 1 \leq i \leq m \\
  f(v_i) &= m + 5i - 4, 1 \leq i \leq n \\
  f(x_i) &= m + 5i - 3, 1 \leq i \leq n - 1 \\
  f(y_i) &= m + 5i - 2, 1 \leq i \leq n - 1
\end{align*}
\]

Then the edge labels are distinct. Hence \( f \) is a Root Square Mean labeling of \( G \).

Example 2.6: Root Square Mean labeling of \( C_7 \odot D(T_5) \) is given below.

![Figure 3](image)

Theorem 2.7: \( (C_m \odot K_1) \cup (D(T_n)) \) is a Root Square Mean graph.

Proof: Let the cycle \( C_m \) be \( u_1u_2 \cdots u_mu_1 \). Let \( v_i \) be the vertex of \( K_1 \) which is attached to the vertex \( u_i \), \( 1 \leq i \leq m \) of the cycle \( C_m \). Let \( w_1w_2 \cdots w_n \) be the path \( P_n \). The double triangular snake \( D(T_n) \) is obtained by joining \( w_i \) and \( w_{i+1} \) to two new vertices \( x_i \) and \( y_i \), \( 1 \leq i \leq n-1 \).

Let \( G = (C_m \odot K_1) \cup (D(T_n)) \). Define a function \( f: V(G) \to \{1, 2, \ldots, q + 1\} \) by

\[
\begin{align*}
  f(u_i) &= 2i - 1, 1 \leq i \leq m \\
  f(v_i) &= 2i, 1 \leq i \leq m \\
  f(w_i) &= 2m + 5i - 4, 1 \leq i \leq n \\
  f(x_i) &= 2m + 5i - 3, 1 \leq i \leq n - 1
\end{align*}
\]
\[ f(y_i) = 2m + 5i - 2, 1 \leq i \leq n - 1 \]

Then the edge labels are distinct. Hence \( f \) is a Root Square Mean labeling of \( G \).

**Example 2.8:** Root Square Mean labeling of \( (C_6 \odot K_1) \cup (D(T_5)) \) is given below.

![Figure 4](image1.png)

**Theorem 2.9:** \( C_m \cup Q_n \) is a Root Square Mean graph.

**Proof:** Let \( u_1u_2 \cdots u_mu_1 \) be the cycle \( C_m \). Let \( v_1v_2 \cdots v_n \) be the path \( P_n \). Let \( Q_n \) be the Quadrilateral snake obtained by joining \( v_i \) and \( v_{i+1} \) to two new vertices \( x_i \) and \( y_i \), \( 1 \leq i \leq n - 1 \) respectively and then joining \( x_i \) and \( y_i \). Let \( G = C_m \cup Q_n \). Define a function \( f: V(G) \to \{1, 2, \ldots, q + 1\} \) by

\[
\begin{align*}
  f(u_i) &= i, 1 \leq i \leq m \\
  f(v_i) &= m + 4i - 3, 1 \leq i \leq n \\
  f(x_i) &= m + 4i - 2, 1 \leq i \leq n - 1 \\
  f(y_i) &= m + 4i - 1, 1 \leq i \leq n - 1
\end{align*}
\]

Then the edge labels are distinct. Hence \( f \) is a Root Square Mean labeling of \( G \).

**Example 2.10:** The labeling pattern of \( C_7 \cup Q_5 \) is given below.

![Figure 5](image2.png)
**Theorem 2.11:** \((C_m \odot K_1) \cup Q_n\) is a Root Square Mean graph.

**Proof:** Let \(u_1 u_2 \cdots u_m u_1\) be the cycle \(C_m\). Let \(v_i\) be the vertex of \(K_1\) which is attached to the vertex \(u_i, 1 \leq i \leq m\) of the cycle \(C_m\). Let \(w_1 w_2 \cdots w_n\) be the path \(P_n\). Let \(x_i\) and \(y_i, 1 \leq i \leq n - 1\) be the vertices which are joined to \(w_i\) and \(w_{i+1}\) respectively. Join \(x_i\) and \(y_i\).

Let \(G = (C_m \odot K_1) \cup Q_n\).

Define a function \(f: V(G) \to \{1,2,\ldots,q + 1\}\) by

\[
\begin{align*}
    f(u_i) &= 2i - 1, 1 \leq i \leq m \\
    f(v_i) &= 2i, 1 \leq i \leq m \\
    f(w_i) &= 2m + 4i - 3, 1 \leq i \leq n \\
    f(x_i) &= 2m + 4i - 2, 1 \leq i \leq n - 1 \\
    f(y_i) &= 2m + 4i - 1, 1 \leq i \leq n - 1
\end{align*}
\]

Then the edge labels are distinct. Hence \(f\) is a Root Square Mean labeling of \(G\).

**Example 2.12:** The labeling pattern of \((C_7 \odot K_1) \cup Q_5\) is given below.

![Figure 6](image)

**Theorem 2.13:** \(C_m \odot D(Q_n)\) is a Root Square Mean graph.

**Proof:** Let \(u_1 u_2 \cdots u_m u_1\) be the cycle \(C_m\). Let \(v_i, x_i, y_i, x'_i, y'_i\) be the vertices of \(D(Q_n)\).

Let \(G = C_m \odot D(Q_n)\). Define a function \(f: V(G) \to \{1,2,\ldots,q + 1\}\) by

\[
\begin{align*}
    f(u_i) &= i, 1 \leq i \leq m \\
    f(v_i) &= m + 7i - 6, 1 \leq i \leq n
\end{align*}
\]
\[ f(x_i) = m + 7i - 5 \quad 1 \leq i \leq n - 1 \]
\[ f(y_i) = m + 7i - 2 \quad 1 \leq i \leq n - 1 \]
\[ f(x'_i) = m + 7i - 4 \quad 1 \leq i \leq n - 1 \]
\[ f(y'_i) = m + 7i - 1 \quad 1 \leq i \leq n - 1 \]

Then the edge labels are distinct. Hence \( f \) is a Root Square Mean labeling of \( G \).

**Example 2.14**: The labeling pattern of \( C_7 \ominus D(3) \) is given below.

![Figure 7](image)

**Theorem 2.15**: \( (C_m \ominus K_1) \cup D(Q_n) \) is a Root Square Mean graph.

**Proof**: Let \( u_1 u_2 \cdots u_m u_1 \) be the cycle \( C_m \). Let \( v_i \) be the vertex of \( K_1 \) which is attached to the vertex \( u_i \), \( 1 \leq i \leq m \) of the cycle \( C_m \). Let \( w_i, x_i, y_i, x'_i, y'_i \) be the vertices of \( D(Q_n) \).

Let \( G = (C_m \ominus K_1) \cup D(Q_n) \). Define a function \( f : V(G) \to \{1, 2, \ldots, q + 1\} \) by
\[ f(u_i) = 2i - 1 \quad 1 \leq i \leq m \]
\[ f(v_i) = 2i \quad 1 \leq i \leq m \]
\[ f(w_i) = 2m + 7i - 6 \quad 1 \leq i \leq n \]
\[ f(x_i) = 2m + 7i - 5 \quad 1 \leq i \leq n - 1 \]
\[ f(y_i) = 2m + 7i - 2 \quad 1 \leq i \leq n - 1 \]
\[ f(x'_i) = 2m + 7i - 4 \quad 1 \leq i \leq n - 1 \]
\[ f(y'_i) = 2m + 7i - 1 \quad 1 \leq i \leq n - 1 \]
Then the edge labels are distinct. Hence $f$ is a Root Square Mean labeling of $G$.

**Example 2.16:** The labeling pattern of $(C_6\circ K_1) \cup D(Q_5)$ is given below.

![Diagram](image)

**REFERENCES**