Some Results on Mean Cordial Graphs

A. Nellai Murugan 1 and G. Esther 2

Department of Mathematics, V.O. Chidambaram College, Tuticorin, Tamilnadu (INDIA)

Abstract

Let \( G = (V, E) \) be a simple graph. \( G \) is said to be a mean cordial graph if \( f : V(G) \rightarrow \{0, 1, 2\} \) such that for each edge \( uv \) the induced map \( f^* \) defined by \( f^*(uv) = \left\lfloor \frac{f(u) + f(v)}{2} \right\rfloor \) where \( \lfloor x \rfloor \) denote the least integer which is \( \leq x \) and \( |e_f(0) - e_f(1)| \leq 1 \) where \( e_f(0) \) is no. of edges with zero label. \( e_f(1) \) is no. of edges with one label.

The graph that admits a mean cordial labeling is called a mean cordial graph (MCG).

In this paper, we proved that \( D_2[C_n] \), \( D_2[K_{1,n}] \), \( D_2[P_n + K_1] \), \( D_2[P_n] \) are mean cordial graphs.

Key words: Mean cordial labeling, Mean cordial graph.

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1. INTRODUCTION:

A graph \( G \) is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of \( G \) which is called edges. Each \( e = \{uv\} \) of vertices in \( E \) is called an edge or a line of \( G \). For graph theoretical Terminology we follow

2. PRELIMINARIES:

We define the concept of mean cordial labeling as follows.

Let \( G = (V, E) \) be a simple graph. \( G \) is said to be a mean cordial graph if \( f : V(G) \rightarrow \{0, 1, 2\} \) such that for each edge \( uv \) the induced map \( f^* \) defined by \( f^*(uv) = \left\lfloor \frac{f(u) + f(v)}{2} \right\rfloor \) where \( \lfloor x \rfloor \) denote the least integer which is \( \leq x \) and \( |e_f(0) - e_f(1)| \leq 1 \) where \( e_f(0) \) is no. of edges with zero label. \( e_f(1) \) is no. of edges with one label.

The graph that admits a mean cordial labeling is called a mean cordial graph (MCG).

In this paper, we proved that \( D_2[C_n] \), \( D_2[K_{1,n}] \), \( D_2[P_n + K_1] \), \( D_2[P_n] \) are mean cordial graphs.

DEFINITION 2.1 (SHADOW GRAPH)

Let \( G \) be a connected Graph. A Graph, constructed by taking two copies of \( G \) say \( G_1 \) and \( G_2 \) and joining each vertex \( u \) in \( G_1 \) to the neighbours of the corresponding vertex \( v \) in \( G_2 \), that is for every vertex \( u \) in \( G_1 \) there exists \( v \) in \( G_2 \) such that \( N(u) = N(v) \). The resulting Graph is known as Shadow Graph and it is denoted by \( D_2(G) \).

DEFINITION 2.2 (CYCLE)

A closed path is called a cycle and a cycle of length \( k \) is denoted by \( C_k \).

DEFINITION 2.3 (STAR)

Let \( S_{m,n} \) \((n > 2)\) is a star with \( n \) spokes in which each spoke is a path of length \( m \).

DEFINITION 2.4 (FAN)

The join \( G_1 + G_2 \) of \( G_1 \) and \( G_2 \) consists of \( G_1 \cup G_2 \) and all lines joining \( V_1 \) with \( V_2 \) as vertex set \( V(G_1 \cup G_2) = V(G_1) \cup V(G_2) \) and edges \( E(G_1 \cup G_2) = E(G_1) \cup E(G_2) \cup \{ uv : u \in V(G_1) \text{ and } v \in V(G_2) \} \). The graph \( P_n + K_1 \) is called a Fan and \( P_n + 2K_1 \) is called the Doublefan.

DEFINITION 2.5 (PATH)

If all the vertices in a walk are distinct, then it is called a path and a path of length \( k \) is denoted by \( P_{k+1} \).
3. MAIN RESULTS ON MEAN CORDIAL GRAPH

**Theorem 3.1**

$D_2(C_n)$ is a Mean Cordial Graph.

**Proof:**

Let $G = (V, E)$

Let $G$ be $[D_2(C_n)]$

Let $V[D_2(C_n)] = \{u_i, v_i: 1 \leq i \leq n\}$

Let $E[D_2(C_n)] = \{[(u_i, u_{i+1}) \cup (v_i, v_{i+1}) : 1 \leq i \leq n-1] \cup \{(u_i, u_{i+1}) \cup (v_i, v_{i+1}) \cup (u_{i+1}, u_{i}) \cup (v_{i+1}, v_{i}) : 1 \leq i \leq n-1\}\}$

Define $f : V(G) \rightarrow \{0, 1, 2\}$ by

$f(u_i) = 1$

$f(v_i) = 1$

$f(u_i) = \begin{cases} 0 & \text{if } i \equiv 0 \mod 2 \\ 1 & \text{if } i \equiv 1 \mod 2 \end{cases}$, $2 \leq i \leq n$

$f(v_i) = \begin{cases} 1 & \text{if } i \equiv 0 \mod 2 \\ 0 & \text{if } i \equiv 1 \mod 2 \end{cases}$, $2 \leq i \leq n$

The induced edge labeling are

$f^*(u_i, u_{i+1}) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases}$, $1 \leq i \leq n-1$

$f^*(v_i, v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \mod 2 \\ 0 & \text{if } i \equiv 0 \mod 2 \end{cases}$, $1 \leq i \leq n-1$

$f^*(u_i, v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \mod 2 \\ 0 & \text{if } i \equiv 0 \mod 2 \end{cases}$, $2 \leq i \leq n-1$

$f^*(u_i, v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \mod 2 \\ 0 & \text{if } i \equiv 0 \mod 2 \end{cases}$, $2 \leq i \leq n-1$

$f^*(v_i, u_{i+1}) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases}$, $2 \leq i \leq n-1$

$f^*(v_i, u_{i+1}) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases}$, $2 \leq i \leq n-1$

When $n$ is even, $f^*(u_i, v_{i+1}) = 1$

When $n$ is odd, $f^*(v_i, u_{i+1}) = 1$

Here $e_i(0) = e_i(1)$ for all $n$.

It satisfies the condition $|e_i(0) - e_i(1)| \leq 1$.

Hence, $D_2(C_n)$ is a mean cordial graph.

For example the graph $D_2(C_4)$ and $D_2(C_5)$ are shown in the figure 1 and figure 2.
Theorem 3.2

\( D_2[K_{1,n}] \) is a Mean cordial Graph.

**Proof:**

Let \( G = (V, E) \)

Let \( G \) be \( D_2[K_{1,n}] \)

Let \( V[D_2(K_{1,n})] = \{u, v, (u_i, v_i) : 1 \leq i \leq n\} \)

Let \( E[D_2(K_{1,n})] = \{(u, u_i) \cup (u, v_i) \cup (v, u_i) \cup (v, v_i) : 1 \leq i \leq n\} \)

Define \( f : V(G) \rightarrow \{0, 1, 2\} \) by

**Case (i):** \( n \) is even

- \( f(u) = 1 \)
- \( f(v) = 1 \)
- \( f(u_i) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 2 & \text{if } i \equiv 0 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)
- \( f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 2 & \text{if } i \equiv 0 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)

The induced edge labeling are

- \( f^*(u, u_i) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)
- \( f^*(u, v_i) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)
- \( f^*(v, u_i) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)
- \( f^*(v, v_i) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)

Here \( e_i(0) = e_i(1) \)

It satisfies the condition \( |e_i(0) - e_i(1)| \leq 1 \).

Hence \( [D_2(K_{1,n})] \) \( (n \) is even) is a mean cordial graph.

For example \( D_2(K_{1,2}) \) is shown in the figure 3.

**Case (ii):** \( n \) is odd

- \( f(u) = 1 \)
- \( f(v) = 1 \)
- \( f(u_i) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 2 & \text{if } i \equiv 0 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)
- \( f(v_i) = \begin{cases} 0 & \text{if } i \equiv 0 \mod 2 \\ 2 & \text{if } i \equiv 1 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)

The induced edge labeling are

- \( f^*(u, u_i) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)
- \( f^*(u, v_i) = \begin{cases} 0 & \text{if } i \equiv 0 \mod 2 \\ 1 & \text{if } i \equiv 1 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)
- \( f^*(v, u_i) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)
- \( f^*(v, v_i) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2 \\ 1 & \text{if } i \equiv 0 \mod 2 \end{cases} \), \( 1 \leq i \leq n \)

Here \( e_i(0) = e_i(1) \)

It satisfies the condition \( |e_i(0) - e_i(1)| \leq 1 \).

Hence \( [D_2(K_{1,n})] \) \( (n \) is odd) is a mean cordial graph.

For example \( D_2(K_{1,3}) \) is shown in the figure.
**Theorem 3.3**

Graph $D_2[P_n + K_1]$ is a Mean Cordial Graph.

**Proof:**

Let $G = (V, E)$

Let $G$ be $D_2[P_n + K_1]$

Let $V[D_2(P_n + K_1)] = \{u, v, u_i, v_i : 1 \leq i \leq n\}$

Let $E[D_2(P_n + K_1)] = \{(u, u_i) \cup (v, v_i) \cup (v_i, v_{i+1}) \cup (u_i, u_{i+1}) : 1 \leq i \leq n-1\}$

Define $f: V(G) \rightarrow \{0, 1, 2\}$ by

$f(u) = 2$

$f(v) = 0$

$f(u_i) = \begin{cases} 1 & \text{if } i \equiv 1 \mod 2 \\ 0 & \text{if } i \equiv 0 \mod 2 \end{cases}, \ 1 \leq i \leq n$

$f(v_i) = 1$

The induced edge labeling are

$f^*(u, u_i) = 1$

$f^*(v, v_i) = 1$

$f^*(v, u_i) = 0$

$f^*(v, v_{i+1}) = 0$

$f^*(u_i, u_{i+1}) = 0$

$f^*(v_i, v_{i+1}) = 1$

Hence $e_f(0) = e_f(1)$ for all $n$. It satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. Hence $D_2(P_n + K_1)$ is a mean cordial graph.

For example, $D_2(P_5 + K_1)$ is shown in the figure 5.

**Theorem 3.4**

Graph $D_2(P_n)$ is a Mean Cordial Graph.

**Proof:**

Let $G = (V, E)$

Let $G$ be $D_2(P_n)$

Let $V[D_2(P_n)] = \{u_i, v_i : 1 \leq i \leq n\}$

Let $E[D_2(P_n)] = \{(u_i, v_i) : 1 \leq i \leq n\}$

Define $f: V(G) \rightarrow \{0, 1, 2\}$ by

$f(u_i) = \begin{cases} 2 & \text{if } i \equiv 1 \mod 2 \\ 0 & \text{if } i \equiv 0 \mod 2 \end{cases}, \ 1 \leq i \leq n$

$f(v_i) = \begin{cases} 0 & \text{if } i \equiv 0 \mod 2 \\ 1 & \text{if } i \equiv 1 \mod 2 \end{cases}, \ 1 \leq i \leq n$
The induced edge labeling are
\[
f^* (u_i u_{i+1}) = \begin{cases} 
1, & 1 \leq i \leq n-1 \\
0, & 1 \leq i \leq n-1
\end{cases}
\]
\[
f^* (v_i v_{i+1}) = \begin{cases} 
0, & 1 \leq i \leq n-1 \\
1, & 1 \leq i \leq n-1
\end{cases}
\]
\[
f^* (v_i u_{i+1}) = \begin{cases} 
0, & 1 \leq i \leq n-1 \\
1, & 1 \leq i \leq n-1
\end{cases}
\]
\[
f^* (u_i v_{i+1}) = \begin{cases} 
1, & 1 \leq i \leq n-1 \\
0, & 1 \leq i \leq n-1
\end{cases}
\]

It satisfies the condition
\[
e_f(0) = e_f(1) \text{ for all } n
\]
Hence, \(D_2(P_n)\) is a mean cordial graph.

For example, the mean cordial graph of \(D_2(P_4)\) is shown in the figure 6.

\[
\begin{array}{c}
2(u_1) & 1 & 0(u_2) & 1 & 2(u_3) & 1 & 0(u_4)\\
1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array}
\]

\[
\begin{array}{c}
l(v_1) & 0 & 0(v_2) & 0 & l(v_3) & 0 & 0(v_4)
\end{array}
\]

Figure 6

4. Conclusion

Graph labeling place a vital role not only in the theoretical aspect but also in many practical application problems. There are number of labeling such as magic labeling, graceful labeling, mean labeling and super-mean labeling.

Particularly cordial related labeling such as mean cordial, divisor cordial, mean-square cordial labeling etc. place an important role in digital technology

5. References


