Role Model Service Rendered to Orphans by Using Fuzzy Soft Matrices

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Abstract—In this paper, we define fuzzy soft matrices and coin their properties by establishing with examples. Finally, we extend our concept in application of these matrices in (Role model service to orphans) decision making problem.

Keywords—Soft set, Fuzzy soft set (FSS), Fuzzy soft matrices (FSM), fuzzy soft complement matrices, Addition of fuzzy soft matrices.

I. INTRODUCTION


In 2010, Cagman et al [2] defined soft matrix which is representation of soft set, to make operations in theoretical studies in soft set more functional. This representation has several advantages. In 2011, Yong et al [13] successfully applied the proposed notion of fuzzy soft matrix in certain decision making problems. In 2011, Neog and sut [11] have defined the “addition operation” for fuzzy soft matrices and an attempt has been made to apply our notion in solving a decision problem.


In 2014, Dr. N. Sarala and Rajkumari [8] introduced intuitionistic fuzzy soft matrices (IFSM) in Agriculture and also issued [9] IFSM in Medical diagnosis.

In this paper, we proposed fuzzy soft matrices in decision making problem which will yield fruitful results in this field.

II. PRELIMINARIES

In this we section, We recall some basic essential notion of fuzzy soft set theory.

2.1 Soft Set [7]

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power Set of U. Let AG E. A pair (F_A,E) is called a soft set over U, where F_A is a mapping given by F_A: E → P(U) Such that F_A(e) = φ if e ∈ A.

Here F_A is called approximate function of the soft set (F_A,E). The set F_A(e) is called e-approximate value set which consist of related objects of the parameter e∈E. In other words, a soft set over U is a parameterized family of subsets of the universe U.

Example 2.1:

Let U={u_1, u_2, u_3, u_4} be a set of four type of sarees and E={Nylon(e_1), Cotton(e_2), Silk(e_3)} be a set of parameters. If A={e_2, e_3} ⊆ E. Let F_A(e_2)={u_1, u_2, u_4} and F_A(e_3)={u_1, u_3, u_4} then we write the soft set (F_A,E) ={(e_2, {u_1, u_2, u_4}),(e_3, {u_1, u_3, u_4})} over U which describe the “Variety of sarees” Which Mr.Z is going to buy.

We may represent the soft set in the following form:

<table>
<thead>
<tr>
<th>U</th>
<th>Nylon(e_1)</th>
<th>Cotton(e_2)</th>
<th>Silk(e_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>u_2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>u_3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>u_4</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

TABLE 2.1.1

2.2 Fuzzy Soft Set [4]

Let U be an initial universe set and E be a set of parameters. Let AG E, A pair (F_A,E) is called a fuzzy soft set (FSS) over U, where F_A is a mapping given by, F_A:E → 2^U, where 2^U denotes the collection of all fuzzy subsets of U.

Example 2.2:

Consider the example 2.1., here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1.
which associate with each element a real number in the interval [0,1]. Then
\[
(\bar{F}_A(E)) = \{ \bar{F}_A(e_2) = ((u_1,0.5),(u_2,0.3),(u_3,0.7),(u_4,0.6)), \\
               \bar{F}_A(e_3) = ((u_1,0.4),(u_2,0.8),(u_4,0.9)) \}
\]
is the fuzzy soft set representing the “Variety of sarees” which Mr. Z is going to buy.

We may represent the fuzzy soft set in the following form:

<table>
<thead>
<tr>
<th>U</th>
<th>Nylon(e_1)</th>
<th>Cotton(e_2)</th>
<th>Silk(e_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_1</td>
<td>0.0</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>u_2</td>
<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
</tr>
<tr>
<td>u_3</td>
<td>0.0</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>u_4</td>
<td>0.0</td>
<td>0.6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

**TABLE 2.2.2**

2.3 Fuzzy Soft Class[7]

Let U be an initial universe set and E be the set of attributes. Then the pair (U,E) denotes the collection of all fuzzy soft sets on U with attributes from E and is called a fuzzy soft class.

2.4 Fuzzy Soft Sub Set[7]

For two fuzzy soft sets \((\bar{F}_A,E)\) and \((\bar{G}_B,E)\) over a common universe U, we have \((\bar{F}_A,E) \subseteq (\bar{G}_B,E)\) if \(A \subseteq B\) and \(\forall e \in A, \bar{F}_A(e)\) is a fuzzy subset of \(\bar{G}_B(e)\). i.e., \((\bar{F}_A,E)\) is a fuzzy soft subset of \((\bar{G}_B,E)\).

2.5 Fuzzy Soft complement set[11]

The complement of fuzzy soft set \((\bar{F}_A,E)\) denoted by \((\bar{F}_A,E)^c\) is defined by \((\bar{F}_A,E)^c = \langle \bar{F}_A^c,(E^c)\rangle\), where \(\bar{F}_A^c:E \rightarrow [0,1]^U\) is a mapping given by \(\bar{F}_A^c(e) = \bar{F}_A(e)^c, \forall e \in E\).

2.6 Fuzzy Soft Null Set[7]

A fuzzy soft set \((\bar{F}_A,E)\) over U is said to be null fuzzy soft set with respect to the parameter set E, denoted by \(\bar{N}\), if \(\bar{F}_A(e) = \bar{N}, \forall e \in E\).

III. FUZZY SOFT MATRICES THEORY

3.1 Fuzzy Soft Matrices(FSM):

Let \(U = \{u_1, u_2, u_3, \ldots, u_m\}\) be the universal set and E be the set of parameters given by \(E = \{e_1, e_2, \ldots, e_n\}\). Then the fuzzy soft set \((\bar{F}_A,E)\) can be expressed in matrix form as \(\bar{A} = [a_{ij}]_{m \times n}\) or simply by \([a_{ij}^A]\), \(i=1,2,3,\ldots,m; j=1,2,3,\ldots,n\) and \([a_{ij}^A] = [(\mu_i^A, v_i^A)]\); where \(\mu_i^A\) and \(v_i^A\) represent the fuzzy membership function and fuzzy reference function respectively of U in the fuzzy set \(\bar{F}_A(e)\). We shall identify a fuzzy soft set with its fuzzy soft matrix and use these two concepts interchangeable. The set of all m×n fuzzy soft matrices over U will be denoted by \(\text{FSM}_{m \times n}\). For usual fuzzy sets with fuzzy reference function 0, it is obvious to see that \(a_{ij}^A = [(\mu_i^A, 0)]\) \(\forall i,j\).

Example 3.1:

Let \(U = \{u_1, u_2, u_3\}\) be the universal set and E be the set of parameters given by \(E = \{e_1, e_2, e_3\}\)

we consider a fuzzy soft set
\[
(\bar{F}_A,E) = \{\bar{F}_A(e_1) = ((u_1,0.5),(u_2,0.3),(u_3,0.7)), \\
               \bar{F}_A(e_2) = ((u_1,0.6),(u_2,0.8),(u_3,0.4)), \\
               \bar{F}_A(e_3) = ((u_1,0.1),(u_2,0.2),(u_3,0.9)) \}
\]

We would represent this fuzzy soft set in matrix form as

\[
[a_{ij}^A]_{3 \times 3} = \begin{bmatrix}
0.5,0 & 0.6,0 & 1.0,0 \\
0.3,0 & 0.8,0 & 0.2,0 \\
0.7,0 & 0.4,0 & 0.9,0
\end{bmatrix}_{3 \times 3}
\]

3.2 Membership Value Matrix:

The membership value matrix corresponding to the matrix \(\bar{A}\) as \(\text{MV(A)} = [\delta_{ij}]_{m \times n}\), where \(\delta_{ij}^A = \mu_i^A - v_i^A\) \(\forall i=1,2,3,\ldots,m; j=1,2,3,\ldots,n\) where \(\mu_i^A\) and \(v_i^A\) represent the fuzzy membership function and fuzzy reference function respectively of U in the fuzzy set \(\bar{F}_A(e)\).

3.3 Fuzzy soft Complement Matrix:

Let \(\bar{A} = [a_{ij}^A]_{m \times n}\), then complement of \(\bar{A}\) is denoted by \(\bar{A}^c = [(\bar{C}_{ij})]\); where \(\bar{C}_{ij} = 1 - a_{ij}^A\) for all I and j.

3.4 Addition of fuzzy Soft Matrices:

Let \(U = \{u_1, u_2, u_3, \ldots, u_m\}\) be the universal set and E be the set of parameters given by \(E = \{e_1, e_2, e_3, \ldots, e_n\}\). Let the set of all m×n fuzzy soft matrices over U be \(\text{FSM}_{m \times n}\). Let \(\bar{A}, \bar{B} \in \text{FSM}_{m \times n}\). Where \(\bar{A} = [a_{ij}^A]_{m \times n}\), \(\bar{B} = [b_{ij}^B]_{m \times n}\), \(\bar{C} = [c_{ij}^C]_{m \times n}\) be the fuzzy soft matrix. To avoid degenerate cases we assume that \(\max((\mu_i^A, v_i^A)) \geq \max((\mu_i^B, v_i^B))\) for all i and j. We define the operation ‘addition(+’ between \(\bar{A}\) and \(\bar{B}\) as \(\bar{A} + \bar{B} = \bar{C}\), where \(\bar{C} = [c_{ij}^C]_{m \times n}\), \(c_{ij}^C = (\min(\mu_i^A, \mu_i^B), \min(\mu_i^B, \mu_i^A))\).

3.5 Proposition:

Let \(\bar{A}, \bar{B}, \bar{C} \in \text{FSM}_{m \times n}\). Then the following results hold.
Example 3.5:


\[
\begin{pmatrix}
(0.5,0.0) & (0.7,0.0) & (0.9,0.0) \\
(0.9,0.0) & (0.7,0.0) & (0.5,0.0) \\
(0.7,0.0) & (0.5,0.0) & (0.6,0.0) \\
(0.8,0.0) & (0.6,0.0) & (0.7,0.0)
\end{pmatrix}_{4\times3}
\]

Hence \( \hat{A} + \hat{B} = \hat{A} + \hat{B} \).

Let \( \hat{A} = \begin{pmatrix} (0.5,0.0) & (0.7,0.0) & (0.9,0.0) \\ (0.8,0.0) & (0.4,0.0) & (0.3,0.0) \\ (0.6,0.0) & (0.5,0.0) & (0.2,0.0) \\ (0.3,0.0) & (0.6,0.0) & (0.7,0.0) \end{pmatrix}_{4\times3} \)

\( \hat{B} = \begin{pmatrix} (0.4,0.0) & (0.6,0.0) & (0.8,0.0) \\ (0.9,0.0) & (0.7,0.0) & (0.5,0.0) \\ (0.7,0.0) & (0.2,0.0) & (0.6,0.0) \\ (0.8,0.0) & (0.4,0.0) & (0.3,0.0) \end{pmatrix}_{4\times3} \)

\( \hat{C} = \begin{pmatrix} (0.8,0.0) & (0.9,0.0) & (0.7,0.0) \\ (0.6,0.0) & (0.4,0.0) & (0.8,0.0) \\ (0.5,0.0) & (0.3,0.0) & (0.7,0.0) \\ (0.4,0.0) & (0.8,0.0) & (0.6,0.0) \end{pmatrix}_{4\times3} \)

Then \( \hat{A} + \hat{B} = \begin{pmatrix} (0.5,0.0) & (0.7,0.0) & (0.9,0.0) \\ (0.9,0.0) & (0.7,0.0) & (0.5,0.0) \\ (0.7,0.0) & (0.5,0.0) & (0.6,0.0) \\ (0.8,0.0) & (0.6,0.0) & (0.7,0.0) \end{pmatrix}_{4\times3} \) and \( \hat{A} + \hat{B} = \begin{pmatrix} (0.5,0.0) & (0.7,0.0) & (0.9,0.0) \\ (0.9,0.0) & (0.7,0.0) & (0.5,0.0) \\ (0.7,0.0) & (0.5,0.0) & (0.6,0.0) \\ (0.8,0.0) & (0.6,0.0) & (0.7,0.0) \end{pmatrix}_{4\times3} \)

3.6 Score Matrix:

Let \( \hat{A}, \hat{B} \in \text{FSM}_{\text{max}} \). Let the corresponding membership value matrices be \( \text{MV}(\hat{A}) = [\delta^A_{ij}]_{\text{max}} \) and \( \text{MV}(\hat{B}) = \)
[\delta]_{inm} = 1,2,3,\ldots,m; j= 1,2,3,\ldots.n. \text{ Then the score matrix } S_{(A,B)} \text{ would be defined as } S_{(A,B)}=[\rho_{ij}]_{n\times m} \text{ where } \rho_{ij} = \delta^{A}_{ij} - \delta^{B}_{ij}.

3.7 Total Score Matrix:

Let \( \tilde{A}, \tilde{B} \in FSM_{inm} \). Let the corresponding membership value matrices be MV(\( \tilde{A} \)) = \([\delta]_{inm}\) and MV(\( \tilde{B} \)) = \([\delta]_{inm}\) respectively and the score matrix be \( S_{(A,B)}=[\delta^{A}_{ij} - \delta^{B}_{ij}] \), \( i=1,2,3,\ldots,m; j=1,2,3,\ldots.n \). Then the total score for each \( u_i \) in \( U \) would be calculated by the formula \( S_i = \sum_{j=1}^{n}[\delta^{A}_{ij} - \delta^{B}_{ij}] = \sum_{j=1}^{n}([\mu^{A}_{ij} - \mu^{B}_{ij}]^{--}) \).

METHODOLOGY:

Suppose \( U \) is the set of certain number of orphanages. \( E \) is a set of parameters related to highest service rendered to orphans by the orphanages. We construct a fuzzy soft set \((\tilde{F}_A,E)\) over \( U \) representing the best service hospitality and showed by the orphanages. Where \( \tilde{F}_A \) is a mapping \( \tilde{F}_A:E \rightarrow I^U \). \( I^U \) is the set of all fuzzy subset of \( U \). We further construct another fuzzy soft set \((\tilde{G}_B,E)\) over \( U \) denoting the best hospitality and need of service focus to the orphans by the organization. The matrices \( \tilde{A} \) and \( \tilde{B} \) corresponding to the fuzzy soft sets \((\tilde{F}_A,E)\) and \((\tilde{G}_B,E)\) are constructed. We compute the complements \((\tilde{F}_A,E)^c\) and \((\tilde{G}_B,E)^c\) and write the matrices \( \tilde{A}^c \) and \( \tilde{B}^c \) corresponding to \((\tilde{F}_A,E)^c\) and \((\tilde{G}_B,E)^c\) respectively. Using definition 3.4, we compute \( \tilde{A}+\tilde{B} \), which represents the maximum membership of best service and hospitality rendered to the orphans by the orphanages and then compute \( \tilde{A}^c+\tilde{B}^c \), which represents the maximum membership of less service showed to the orphan by the orphanages. Using definition 3.2, we compute MV(\( \tilde{A}+\tilde{B} \)) and MV(\( \tilde{A}^c+\tilde{B}^c \)). The score matrix \( S_{(\tilde{A}+\tilde{B})} \) is constructed. Using definition 3.6 and the total score \( S_i \) for each \( u_i \) in \( U \) is calculated using definition 3.7. Finally, we would find \( S_k = \max_i(S_i) \), then we conclude that the orphanage \( u_k \) has the maximum service rendered between the orphanages. If \( S_k \) has more than one value the process is repeated by reassessing the parameters for choosing the role model organization.

IV. ALGORITHM

1. Input the fuzzy soft matrices \((\tilde{F}_A,E)\) and \((\tilde{G}_B,E)\). Also write the fuzzy soft matrices \( \tilde{A} \) and \( \tilde{B} \) commensurate to \((\tilde{F}_A,E)\) and \((\tilde{G}_B,E)\) respectively.

2. Write the fuzzy soft matrices \((\tilde{F}_A,E)^c\) and \((\tilde{G}_B,E)^c\). Also write the fuzzy soft matrices \( \tilde{A} \) and \( \tilde{B} \) corresponding to \((\tilde{F}_A,E)^c\) and \((\tilde{G}_B,E)^c\) respectively.

3. Compute \( \tilde{A}+\tilde{B} \) and MV(\( \tilde{A}+\tilde{B} \)).

4. Compute \( \tilde{A}^c+\tilde{B}^c \) and MV(\( \tilde{A}^c+\tilde{B}^c \)).

5. Compute the score matrix \( S_{(\tilde{A}+\tilde{B})} \).

IV. CASE STUDY

Let \( (\tilde{F}_A,E) \) and \( (\tilde{G}_B,E) \) be two fuzzy soft sets representing the orphanages has the maximum score value between the four orphanages \( U = \{u_1,u_2,u_3,u_4\} \) respectively.

Step1:

Let us consider \( E = \{e_1,e_2,e_3,e_4\} \) as the set of parameter for choosing the service rendered to orphans by the orphanages.

\( e_1 \) is the orphanage having only physically handicapped people.

\( e_2 \) is the orphanage having only mentally disorder people.

\( e_3 \) is the orphanage having only old aged people.

\( e_4 \) is the orphanage having only orphan children.

\( (\tilde{F}_A,E)=[\tilde{F}_A(e_1)=[\{u_1,0.8,0.0),(u_2,0.5,0.0),(u_3,0.9,0.0),(u_4,0.4,0.0)\] \)

\( \tilde{F}_A(e_2)=[\{u_1,0.6,0.0),(u_2,0.3,0.0),(u_3,0.7,0.0),(u_4,0.1,0.0)\] \)

\( \tilde{F}_A(e_3)=[\{u_4,0.4,0.0),(u_2,0.2,0.0),(u_3,0.6,0.0),(u_4,0.5,0.0)\] \)

\( (\tilde{G}_B,E)=[\tilde{G}_B(e_1)=[\{u_1,0.7,0.0),(u_2,0.6,0.0),(u_3,0.8,0.0),(u_4,0.9,0.0)\] \)

\( \tilde{G}_B(e_2)=[\{u_1,0.5,0.0),(u_2,0.4,0.0),(u_3,0.9,0.0),(u_4,0.6,0.0)\] \)

\( \tilde{G}_B(e_3)=[\{u_4,0.3,0.0),(u_2,0.5,0.0),(u_3,0.7,0.0),(u_4,0.8,0.0)\] \)

These two fuzzy soft sets are represented by the following fuzzy matrices respectively.

\( \tilde{A}=[\begin{bmatrix}0.8,0.0 \& 0.6,0.0 \& 0.4,0.0 \\0.5,0.0 \& 0.3,0.0 \& 0.2,0.0 \\0.9,0.0 \& 0.7,0.0 \& 0.6,0.0 \\0.4,0.0 \& 0.1,0.0 \& 0.5,0.0\end{bmatrix}] \)
Step 2:

The fuzzy soft sets representing the less service showed to the orphans of four orphanages \( U = \{u_1, u_2, u_3, u_4\} \) respectively are given by

\[
(F_A, E) = \{(u_1, 1.0, 0.8), (u_2, 1.0, 0.5), (u_3, 1.0, 0.9), (u_4, 1.0, 0.4)\}
\]

\[
(F_B, E) = \{(u_1, 1.0, 0.6), (u_2, 1.0, 0.3), (u_3, 1.0, 0.7), (u_4, 1.0, 0.1)\}
\]

\[
(G_A, E) = \{(u_1, 1.0, 0.7), (u_2, 1.0, 0.6), (u_3, 1.0, 0.8), (u_4, 1.0, 0.9)\}
\]

\[
(G_B, E) = \{(u_1, 1.0, 0.5), (u_2, 1.0, 0.4), (u_3, 1.0, 0.9), (u_4, 1.0, 0.6)\}
\]

These two fuzzy soft sets are represented by the following fuzzy soft complement matrices in order.

\[
\bar{A} = \begin{bmatrix}
1.0 & 1.0 & 1.0 \\
1.0 & 1.0 & 1.0 \\
1.0 & 1.0 & 1.0 \\
1.0 & 1.0 & 1.0 \\
\end{bmatrix}
\]

\[
\bar{B} = \begin{bmatrix}
1.0 & 1.0 & 1.0 \\
1.0 & 1.0 & 1.0 \\
1.0 & 1.0 & 1.0 \\
1.0 & 1.0 & 1.0 \\
\end{bmatrix}
\]

Step 3:

Then the fuzzy soft matrix \( \bar{A} + \bar{B} \) represents the maximum membership function of best orphanage among the orphanages.

The membership value matrix \( \text{MV}(\bar{A} + \bar{B}) \) gives the respective membership value for best role model orphanage among the orphanages by considering the nature of service.

\[
\text{MV}(\bar{A} + \bar{B}) = \begin{bmatrix}
0.8 & 0.6 & 0.4 \\
0.6 & 0.4 & 0.5 \\
0.9 & 0.9 & 0.7 \\
0.9 & 0.6 & 0.8 \\
\end{bmatrix}
\]

Step 4:

Again the fuzzy soft matrix \( \bar{A}^o + \bar{B}^o \) represents the maximum membership function of less service rendered among the orphanages.

The membership value matrix \( \text{MV}(\bar{A}^o + \bar{B}^o) \) gives the respective membership value for less service rendered among the orphanages.

\[
\text{MV}(\bar{A}^o + \bar{B}^o) = \begin{bmatrix}
1.0 & 1.0 & 1.0 \\
1.0 & 1.0 & 1.0 \\
1.0 & 1.0 & 1.0 \\
1.0 & 1.0 & 1.0 \\
\end{bmatrix}
\]

Step 5:

We now calculate the score matrix \( S_{(\bar{A} + \bar{B})} \) and total score provided for the best role model orphanage of each orphanage.

\[
S_{(\bar{A} + \bar{B})} = \begin{bmatrix}
0.5 & 0.1 & -0.3 \\
0.1 & -0.3 & -0.3 \\
0.7 & 0.6 & 0.3 \\
0.3 & -0.3 & 0.3 \\
\end{bmatrix}
\]
Step 6: Total score for the best service oriented orphanage:

<table>
<thead>
<tr>
<th>S</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_1</td>
<td>0.3</td>
</tr>
<tr>
<td>S_2</td>
<td>0.5</td>
</tr>
<tr>
<td>S_3</td>
<td>1.6</td>
</tr>
<tr>
<td>S_4</td>
<td>0.3</td>
</tr>
</tbody>
</table>

We see that S_3, orphanage act as a real refuge to the orphans with regard to the nature of service and has the maximum value and thus come to a conclusion that the orphanage S_3 has secured the highest total. Hence the said orphanage is selected as role model organization which render valuable multifarious activities and best service given among the orphanages.

VI. Conclusion

In this paper, we have applied the motto of fuzzy soft matrices and complement of fuzzy soft sets in decision making problem. Finally, we attribute our contribution would enhance this study on fuzzy soft sets and also matrices which will give a note worthy result in this field.

REFERENCES


