Reliability of a Multicomponent System Using Pareto Distributions

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Abstract - This paper deals with the stress-strength problem incorporating multi component standby system is considered. The reliability has been derived when strength and stress follow pareto distribution. The general expression for the reliability of a multi component standby system is obtained and the system reliability is computed numerically for different values of the stress and strength parameters.

Key words: Pareto distribution, Reliability, Stress-strength model, Standby system.

I. INTRODUCTION

If X denotes the strength of a component and Y is the stress imposed on it. The component operates as long Y is less than X and the Reliability of the component may therefore be defined as R=(X<Y). The Probability of the failure of a system depends upon the stress and strength of the system. Kapur and Lamberson [1]. The reliability of an n-cascade system with stress attenuation was proposed by Pandit and Sriwastav [2].The reliability for multi component systems when stress-strength follows exponential distributions. Sandhya and Uma mheshwaei [3].Sriwastav and Kokati [4] used cascade system for reliability estimation by considering that the components stress-strength is identically distributed. The purpose of this paper is to study the variations in system reliability for different parameter values in a multicomponent strength-stress based on X and Y being two independent random variables. And considered follow pareto distributions.

II. GENERAL MODEL

Consider a system of n-components, out of which only one is working under the impact of stresses and the remaining (n-1) are standbys. Whenever the working component fails, one of standby components takes the place of a failed component and is subjected to impact of stress then the system works. The system fails when all the components fail. Let \( X_1, X_2, \ldots, X_n \) be the strengths of the n components arranged in order of activation in the system. And let \( Y_1, Y_2, \ldots, Y_n \) are the stresses on the n components respectively then the system reliability \( R_n \) is given by. Where the marginal reliability \( R(n) \) is the reliability of the system by the \( n^{th} \) component then

\[
R(n) = P[X_1 < Y_1, X_2 < Y_2, \ldots, X_{n-1} < Y_{n-1}, X_n > Y_n]
\]

Let \( f_i(x) \) and \( g_i(y) \) are the probability density functions of \( X_i \) & \( Y_i \) \( i = 1, 2, \ldots, n \) respectively and assumed \( X_i \) and \( Y_i \) are independent, then

\[
R(n) = \int_{-\infty}^{\infty} F_1(y) g_1(y) dy \int_{-\infty}^{\infty} F_2(y) g_2(y) dy \cdots \int_{-\infty}^{\infty} F_n(y) g_n(y) dy
\]

\[
F_i(y) = \int_0^y f_i(x) dx \quad F_i(y) = 1 - F_i(y).
\]

Let \( X \) be the Strength and \( Y \) be the Stress of a system with p.d.f.

\[
f_i(x) = \frac{\lambda_i k^\lambda}{x^{\lambda+1}}, k \leq x \leq \infty; \lambda_i, k > 0, i = 1, 2, 3, \ldots, n
\]

\[
g_i(y) = \frac{\mu_i k^{\mu_i}}{y^{\mu_i+1}} k \leq y \leq \infty; \mu_i, k > 0, i = 1, 2, 3, \ldots, n
\]
III. RELIABILITY COMPUTATIONS

A. Strength and stress follow Pareto distribution.

\[ R(1) = \int_0^\infty F_1(y)g_1(y)dy \]

\[ = \int_0^\infty \left( -\frac{k}{y} \right) \frac{\mu_1 k^{\mu_1}}{y^{\mu_1+1}} dy \]

\[ R(1) = \left[ \frac{\mu_1}{\mu_1 + \lambda_1} \right] \]

\[ R(2) = \left[ \int_0^\infty F_1(y)g_1(y)dy \right] \left[ \int_0^\infty F_2(y)g_2(y)dy \right] \]

\[ = \int_0^\infty \left[ 1 - \left( -\frac{k}{y} \right) \frac{\mu_1 k^{\mu_1}}{y^{\mu_1+1}} dy \right] \int_0^\infty \left[ 1 - \left( -\frac{k}{y} \right) \frac{\mu_2 k^{\mu_2}}{y^{\mu_2+1}} dy \right] \]

\[ R(2) = \left[ \frac{\lambda_1}{\mu_1 + \lambda_1} \right] \left[ \frac{\mu_2}{\mu_2 + \lambda_2} \right] \]

\[ R(3) = \left[ \int_0^\infty F_1(y)g_1(y)dy \right] \left[ \int_0^\infty F_2(y)g_2(y)dy \right] \left[ \int_0^\infty F_3(y)g_3(y)dy \right] \]

\[ = \int_0^\infty \left[ 1 - \left( -\frac{k}{y} \right) \frac{\mu_1 k^{\mu_1}}{y^{\mu_1+1}} dy \right] \left[ 1 - \left( -\frac{k}{y} \right) \frac{\mu_2 k^{\mu_2}}{y^{\mu_2+1}} dy \right] \left[ 1 - \left( -\frac{k}{y} \right) \frac{\mu_3 k^{\mu_3}}{y^{\mu_3+1}} dy \right] \]

\[ R(3) = \left[ \frac{\lambda_1}{\mu_1 + \lambda_1} \right] \left[ \frac{\lambda_2}{\mu_2 + \lambda_2} \right] \left[ \frac{\mu_3}{\mu_3 + \lambda_3} \right] \]

In general

\[ R(n) = \left[ \prod_{i=1}^{n-1} \left[ \frac{\lambda_i}{\mu_i + \lambda_i} \right] \right] \left[ \frac{\mu_n}{\mu_n + \lambda_n} \right] \]

B. Strength is Pareto distribution and stress follows mixture of two Pareto distributions.

\[ R(1) = \int_0^\infty F_1(y)g_1(y)dy \]
\[
= \int_{y}^{\infty} \left( p_{1} \frac{\mu_{1} k^{\mu_{1}}}{y^{\mu_{1}+1}} + (1 - p_{1}) \frac{\mu_{2} k^{\mu_{2}}}{y^{\mu_{2}+1}} \right) dy
\]

\[
= \left[ p_{1} \left( \frac{\mu_{1}}{\mu_{1} + \lambda_{1}} \right) \right] + (1 - p_{1}) \left[ \frac{\mu_{2}}{\mu_{2} + \lambda_{1}} \right]
\]

\[
R(2) = \left[ \int_{0}^{\infty} F_{1}(y) g_{1}(y) dy \right] \left[ \int_{0}^{\infty} \overline{F}_{2}(y) g_{2}(y) dy \right]
\]

\[
= \int_{y}^{\infty} \left[ 1 - \left( \frac{k}{y} \right)^{\lambda_{1}} \right] \left[ p_{1} \frac{\mu_{1} k^{\mu_{1}}}{y^{\mu_{1}+1}} + (1 - p_{1}) \frac{\mu_{2} k^{\mu_{2}}}{y^{\mu_{2}+1}} \right] dy \left[ \int_{y}^{\infty} \left( \frac{k}{y} \right)^{\lambda_{2}} \left[ p_{2} \frac{\mu_{2} k^{\mu_{2}}}{y^{\mu_{2}+1}} + (1 - p_{2}) \frac{\mu_{22} k^{\mu_{22}}}{y^{\mu_{22}+1}} \right] dy \right]
\]

\[
R(2) = \left[ 1 - p_{1} \left( \frac{\mu_{1}}{\mu_{1} + \lambda_{1}} \right) \right] - (1 - p_{1}) \left[ \frac{\mu_{2}}{\mu_{2} + \lambda_{1}} \right] \times \left[ p_{2} \left( \frac{\mu_{2}}{\mu_{2} + \lambda_{2}} \right) \right] + (1 - p_{2}) \left[ \frac{\mu_{22}}{\mu_{22} + \lambda_{2}} \right]
\]

\[
R(3) = \left[ \int_{0}^{\infty} F_{1}(y) g_{1}(y) dy \right] \left[ \int_{0}^{\infty} \overline{F}_{2}(y) g_{2}(y) dy \right] \left[ \int_{0}^{\infty} \overline{F}_{3}(y) g_{3}(y) dy \right]
\]

\[
= \int_{y}^{\infty} \left[ 1 - \left( \frac{k}{y} \right)^{\lambda_{1}} \right] \left[ p_{1} \frac{\mu_{1} k^{\mu_{1}}}{y^{\mu_{1}+1}} + (1 - p_{1}) \frac{\mu_{2} k^{\mu_{2}}}{y^{\mu_{2}+1}} \right] dy \left[ \int_{y}^{\infty} \left( \frac{k}{y} \right)^{\lambda_{2}} \left[ p_{2} \frac{\mu_{2} k^{\mu_{2}}}{y^{\mu_{2}+1}} + (1 - p_{2}) \frac{\mu_{22} k^{\mu_{22}}}{y^{\mu_{22}+1}} \right] dy \right] \left[ \int_{y}^{\infty} \left( \frac{k}{y} \right)^{\lambda_{3}} \left[ p_{3} \frac{\mu_{3} k^{\mu_{3}}}{y^{\mu_{3}+1}} + (1 - p_{3}) \frac{\mu_{23} k^{\mu_{23}}}{y^{\mu_{23}+1}} \right] dy \right]
\]

\[
R(3) = \left[ 1 - p_{1} \left( \frac{\mu_{1}}{\mu_{1} + \lambda_{1}} \right) \right] - (1 - p_{1}) \left[ \frac{\mu_{2}}{\mu_{2} + \lambda_{1}} \right] \times \left[ 1 - p_{2} \left( \frac{\mu_{2}}{\mu_{2} + \lambda_{2}} \right) \right] - (1 - p_{2}) \left[ \frac{\mu_{22}}{\mu_{22} + \lambda_{2}} \right] \times \left[ p_{3} \left( \frac{\mu_{3}}{\mu_{3} + \lambda_{3}} \right) \right] + (1 - p_{3}) \left[ \frac{\mu_{23}}{\mu_{23} + \lambda_{3}} \right]
\]

In general

\[
R(n) = \left[ \prod_{i=1}^{n-1} \left[ 1 - p_{1} \left( \frac{\mu_{1i}}{\mu_{1i} + \lambda_{i}} \right) \right] - (1 - p_{1}) \left[ \frac{\mu_{2i}}{\mu_{2i} + \lambda_{i}} \right] \right] \left[ p_{n} \left( \frac{\mu_{1n}}{\mu_{1n} + \lambda_{n}} \right) \right] + (1 - p_{n}) \left[ \frac{\mu_{2n}}{\mu_{2n} + \lambda_{n}} \right]
\]

C. Strength follows Pareto distribution and stress follows mixture of three Pareto distributions.

\[
R(1) = \int_{0}^{\infty} F_{1}(y) g_{1}(y) dy
\]
\[ R(n) = \prod_{i=1}^{n-1} \left[ 1 - p_i \left[ \frac{\mu_i}{\mu_i + \lambda_i} \right] - p_{i+1} \left[ \frac{\mu_{i+1}}{\mu_{i+1} + \lambda_{i+1}} \right] - p_{i+2} \left[ \frac{\mu_{i+2}}{\mu_{i+2} + \lambda_{i+2}} \right] \right] \]

\[ R(1) = \left[ p_1 \left[ \frac{\mu_1}{\mu_1 + \lambda_1} \right] + p_{12} \left[ \frac{\mu_{12}}{\mu_{12} + \lambda_{12}} \right] + p_{13} \left[ \frac{\mu_{13}}{\mu_{13} + \lambda_{13}} \right] \right] \]

\[ R(2) = \left[ \int_0^\infty F_1(y) g_1(y) dy \right] \left[ \int_0^\infty F_2(y) g_2(y) dy \right] \]

\[ R(3) = \left[ \int_0^\infty F_1(y) g_1(y) dy \right] \left[ \int_0^\infty F_2(y) g_2(y) dy \right] \left[ \int_0^\infty F_3(y) g_3(y) dy \right] \]
IV. Numerical Calculations

**Table-1:** Marginal reliabilities and system reliabilities $R_2, R_3$ when strength and stress follow Pareto distribution.

<table>
<thead>
<tr>
<th>$\mu_1$</th>
<th>R(1)</th>
<th>R(2)</th>
<th>R(3)</th>
<th>$R_2$</th>
<th>$R_3$</th>
</tr>
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<td>0.1</td>
<td>0.5</td>
<td>0.25</td>
<td>0.125</td>
<td>0.75</td>
<td>0.875</td>
</tr>
<tr>
<td>0.2</td>
<td>0.666</td>
<td>0.222</td>
<td>0.074</td>
<td>0.888</td>
<td>0.962</td>
</tr>
<tr>
<td>0.3</td>
<td>0.75</td>
<td>0.187</td>
<td>0.046</td>
<td>0.937</td>
<td>0.984</td>
</tr>
<tr>
<td>0.4</td>
<td>0.8</td>
<td>0.16</td>
<td>0.032</td>
<td>0.96</td>
<td>0.992</td>
</tr>
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<td>0.5</td>
<td>0.833</td>
<td>0.138</td>
<td>0.023</td>
<td>0.972</td>
<td>0.995</td>
</tr>
<tr>
<td>0.6</td>
<td>0.857</td>
<td>0.122</td>
<td>0.017</td>
<td>0.979</td>
<td>0.997</td>
</tr>
<tr>
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<td>0.109</td>
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<td>0.984</td>
<td>0.998</td>
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<td>0.082</td>
<td>0.007</td>
<td>0.991</td>
<td>0.999</td>
</tr>
</tbody>
</table>

**Figure-1**

**Table-2:** Marginal reliabilities and system reliabilities $R_2, R_3$ when strength and stress follow Pareto distribution

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>R(1)</th>
<th>R(2)</th>
<th>R(3)</th>
<th>$R_2$</th>
<th>$R_3$</th>
</tr>
</thead>
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<td>0.75</td>
<td>0.875</td>
</tr>
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</tr>
<tr>
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</tr>
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<td>0.3055</td>
<td>0.421</td>
</tr>
</tbody>
</table>
V. CONCLUSION

The reliability of a multi component standby system of stress - strength model is considered. Reliability has been derived for cases. Numerical calculations for $R(r)$, $r=1, 2, 3$ have been obtained for the above particular cases. It has been observed from the graphs, the value of reliability increases when strength parameter decreases & the stress parameter increases.

ACKNOWLEDGMENT


REFERENCES