Magneto Hydro Dynamic Dissipative Flow Over a Stretching/Shrinking Surface with Cross Diffusion and Non-Uniform Heat Source/Sink

Angeline Kavitha M*1, Vijayaragavan R*2

*1Department of Mathematics, Thiruvalluvar University, Vellore-632115, Tamilnadu, India.

Abstract

This paper reports the magnetohydrodynamic dissipative fluid flow towards a stretching/shrinking surface with Joule heating, non-uniform heat source/sink, Brownian motion, thermophoresis and cross diffusion effects. The transformed governing equations are explored numerically by shooting technique. The consequences for different pertinent parameters on the flow, thermal and concentration distributions are explained with the assistance of tables and graphs. The wall friction, local Sherwood and reduced Nusselt number is also computed and analyzed. The solutions are exhibited for Casson fluid case. It is found that an increase in the Prandt number enhances the heat and mass transfer rate.

Keywords: MHD, Boundary layer, Brownian motion, thermophoresis, Soret and Dufour.

I. INTRODUCTION

The fluid dynamics due to stretching surface is important in manufacturing processes. Examples are numerous and they include the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, paper production, glass blowing, metal spinning and drawing plastic films.

The quality of the final product depends on the rate of heat transfer at the stretching surface. The flow due to stretching surface was first investigated by Crane [1].Ananth et al. [2] carried out the study on stagnation flow of a casson fluid over a stretching surface in the presence of induced magnetic field and nonlinear thermal radiation effects. Sandeep and Sulochana [3] established the study on an unsteady mixed convection flow of MHD micropolar fluid towards a stretching/shrinking sheet with chemical reaction, mass transfer and heat source/sink. The study on MHD nanofluid flow towards a stretching sheet with thermal radiation was reported by Sandeep et al. [4].

Jayachandra Babu and Sandeep [5] investigated the study of non-aligned MHD stagnation point flow of a nano-fluid past a stretching sheet with variable viscosity and nonlinear thermal radiation. The study on radiative ferrofluid across a slandering stretching sheet in the presence of frictional heating and velocity slip was discussed by Ramana Reddy et al. [6]. The study on heat and mass transfer behavior of MHD nanofluid flow towards an inclined stretching sheet with radiation, volume fraction of Dust and Nanoparticles was showed by Sandeep and Jagadeesh kumar [7]. Satish kumar et al. [8] presented the behavior of the MHD bio-convective flow towards a stretching/shrinking sheet with suction/injection. The character of Jeffrey , Maxwell and Oldroyd-B nanofluids over a stretching sheet with thermal radiation and transverse magnetic field was illustrated by Sandeep and Sulochana [9]. The study on 3D MHD slip flow of a nanofluid past a slandering stretching sheet with thermophoresis and Brownian motion effects was examined by Jayachandra Babu and Sandeep [10]. Jayachandra Babu and Sandeep [11] performed the cross-diffusion effects on the MHD non-Newtonian fluid flow towards a slandering stretching sheet. In this study they used Runge-Kutta based shooting process. The study on Mhd force convective flow of a nanofluid across a slandering stretching sheet in porous medium with thermal radiation and slip effects was exhibited by Sulochana and Sandeep [12].

Sandeep and Sulochana [13] studied thermophoretic effects of the MHD nanofluid flow on an exponentially stretching sheet by considering heat source/sink. The effect of Nanofluid flow through a heated stretching sheet with thermal radiation was analysed by Kalidas et al.[14]. The unsteady MHD mixed convective nanofluid flow on an exponentially stretching surface in porous media was discovered by Anwar Beg et al.[15]. The stagnation point flow of a linear viscoelastic fluid bounded by a stretching/ shrinking surface was illustrated by Khan et al. [16]. The study on MHD convection flow on a stretched vertical flat plate in the presence of viscous dissipation, Hall current and thermal radiation was investigated by Gnaneswara Reddy and...
Machireddy [17]. Hayat et al. [18] presented the effects of stretched flow of tangent hyperbolic nanofluid in the presence of variable thickness. The behavior of radiative nonlinear heat transfer and fluid flow past a stretching surface was discussed by Cortell [19]. The effect of magnetic field on 2D flow of Casson fluid over an exponentially stretched surface with non-Fourier flux theory was discovered by Bilal et al. [20].

Kumaran et al. [21] analyzed the MHD chemically reacting Casson and Marwell fluids towards a stretching surface with cross diffusion. Animasaun et al. [22] investigated the free convective MHD Casson fluid flow past an exponentially stretching surface by using homotopy analysis method. Chemically reaction and radiation effects on MHD Williamson fluid over a stretching sheet with nanoparticles was presented by Krishnamurthy et al. [23]. Kayalvizhi et al. [24] studied the velocity slip effects on MHD viscous Ohmic dissipative flow toward a stretching sheet with thermal radiation. The study on MHD stagnation point flow of a magnetic-micropolar fluid past a stretching sheet with viscous dissipation and chemical reaction by using shooting technique was illustrated by Surya Narayana Reddy et al. [25]. The combine effects of thermal radiation and thermo-diffusion on Williamson nanofluid toward a porous stretching sheet was discussed by Bhatti and Rashidi [26]. Sathish kumar et al. [27] analyzed the flow and heat transfer of MHD dissipative carreau nanofluid flow past a stretching surface with space and heat source/sink. In this method they used Runge-Kutta and Newton's method. The study on MHD 3-D flow of an Oldroyd-B nanofluid flow over a stretching sheet with heat source/sink was exhibited by Tasawar Hayat et al. [28]. Swati Mukhopadhyay et al. [29] analyzed the unsteady 2D Casson fluid flow towards a stretching sheet. In this study they used shooting method. The discussion on the stagnation point flow past a stretching/shrinking sheet can be found in the work of Bhattacharyya [30]. Sarojamma et al. [31] were investigated the effects of a MHD Casson fluid in a Vertical channel with stretching walls using shooting method.

In these studies they found very interesting solutions as the non-uniform heat source/sink parameter has tendency to control the temperature profiles also the non-Newtonian fluids are regulating the temperature profiles of the flow. The transformed governing equations are explored numerically by shooting technique.

II. FORMULATION OF THE PROBLEM

Consider an unaltering 2D incompressible fluid past a stretching surface is located at \( y = 0 \) is shown in Fig. 1. The sheet is separated into two equal parts and \( x \)-axis along the opposite force at the fixed point with the velocity \( U_c = ax \). The fluid velocity \( U_c(x) = bx \) is located in outside of the boundary layer. \( H \) is the uniform magnetic field applied perpendicular to the flow direction. Joule heating and viscous dissipation effects considered in this study. The joint impact of the thermophoresis, Brownian moment and cross diffusion are examined.

With the above supposition, the governing equations are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \\
(1)
\]
\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_x + v \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma H^2}{\rho_f} (u - u_\infty),
\]
(2)

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2} + v \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial^2 u}{\partial y^2}\right)^2 + \frac{\sigma H^2}{\rho_f} (u - u_\infty)^2 + \tau \partial_x \left[ \frac{\partial T}{\partial y} \left(\frac{\partial u}{\partial y}\right) \right] + \frac{D_s K_f \partial^2 C}{C C_P} \frac{\partial^2 T}{\partial y^2} + \frac{1}{(\rho C_v)} \partial^2 T.
\]
(3)

\[
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_n \frac{\partial^2 C}{\partial y^2} + \frac{D_s}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2}\right)^2 + \frac{D_s K_f \partial^2 C}{C C_P} \frac{\partial^2 T}{\partial y^2}.
\]
(4)

The boundary conditions are

\[
u = U_\infty (x) = ax, v = 0 \text{ at } y = 0
\]
\[
u \to u_\infty (x) = bx \text{ as } y \to \infty
\]
\[-k \frac{\partial T}{\partial y} = h (T - T_\infty), C = C_\infty \text{ at } y = 0
\]
\[T \to T_\infty, C \to C_\infty \text{ as } y \to \infty
\]
(5)

Now, we are initiating the following quantities

\[
\eta = \sqrt{\frac{a}{v_f}}, \nu = ax f (\eta), \nu = -\sqrt{av_f} f (\eta).
\]
\[
\theta (\eta) = \frac{T - T_\infty}{T_\infty - T_\infty}, \phi (\eta) = \frac{C - C_\infty}{C_\infty - C_\infty}
\]
(6)

By maleiy use of Eq. (6), Eqs. (2) - (4) transformed as

\[
\left(1 + \frac{1}{\beta}\right) f'' + \lambda (\lambda - f') = 0,
\]
(7)

\[
\frac{1}{Pr} \theta'' + f \theta' + Nb \theta' \phi + Nt \theta' + Ec \left(1 + \frac{1}{\beta}\right) f'' + M \theta' (\lambda - f')^2 + Du \phi'' = 0,
\]
(8)

\[
\phi'' + Le \phi' + \frac{Ns}{Nf} \theta'' + Le Sr \theta'' = 0,
\]
(9)

along with transformed boundary condition

\[
f (0) = 0, f' (0) = 1, \theta (0) = -\gamma [1 - \theta (0)], \phi (0) = 1,
\]
\[f' (\infty) \to \lambda, \theta (\infty) \to 0, \phi (\infty) \to 0
\]
(10)
The non-dimensionless parameters are $M = \frac{\sigma H^2}{\rho f}$ is the magnetic parameter, $\lambda = \frac{h}{a}$ stretching ratio parameter, $Pr = \frac{v_f}{\alpha}$ - Prandtl number, $\gamma = h/k \sqrt{v_f/\alpha}$ - Biot number, $Nb = \tau D_b (C_v - C_x)/v_f$ - Brownian moment parameter, $Nt = \tau D_f (T_v - T_x)/v_f$ - thermophoresis parameter, $Ec = u^2 / C_f (T_v - T_x)$ - local Eckert number, $Sr = \frac{D_a K_f (T_v - T_x)}{T_v v_f (C_v - C_x)}$ - Soret number, $Du = \frac{D_a K_f (C_v - C_x)}{C_p C_f (T_v - T_x)}$ - Dufour number, $Le = \frac{aT}{D_e}$. Lewis number.

Physical amounts of engineering interest reduced Nusselt number, reduced Sherwood number and wall friction are given by

$$Nu_x = \frac{xq_u}{k(T_f - T_x)}, Sh_x = \frac{xJ_u}{D_b(C_f - C_x)}$$

$$\frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0) = Nu r, \frac{Sh_x}{\sqrt{Re_x}} = -\phi'(0) = Shr, \sqrt{Re_x} = \left(1 + \frac{1}{\beta}\right) f'(0)$$

Where $Re_x = U_x(x)/v$ - local Reynolds number.

### III. RESULTS AND DISCUSSION

In order to get a clear insight of this mathematical model, the influence of various pertinent parameters on flow, heat and mass transfer characteristics have been analysed and the results are furnished in form of Figs. 1-15 and Table. The following choice of values for governing parameters are adopted based on the previous investigations. $\beta=0.5$, $M=0.5$, $pr=6$, $Nb=0.3$, $Nt=0.2$, $Ec=0.6$, $Du=0.3$, $le=0.4$, $Sr=0.3$, $ga=0.3$. These values are kept as common in entire study except the variations in respective figures and tables.

![Fig.2. Velocity profile for different values of M.](image_url)
Fig. 3. Concentration profile for different values of $\beta$.

Fig. 4. Velocity profile for different values of $\beta$.

Figs. 2 and 4 render the velocity profiles for different values of $M$ and $\beta$ respectively. Figs. 2 and 4 elucidates that velocity profiles decrease with an increase in the values of $M$ and $\beta$. The rise in $\beta$ is used to dilute the strength of the Casson fluid which enhances the value of plastic dynamic viscosity and causing the resistance in fluid flow. Fig. 3 depicts the concentration profiles for different values of $\beta$. It is clear that increasing the values of $\beta$ enhance the concentration profile.
Fig. 5. Concentration profile for various values of Nb.

Fig. 6. Concentration profile for different values of Nt.
**Fig. 7.** Concentration profile for different values of $\gamma$.

**Fig. 8.** Concentration profile for different values of $Sr$. 

\[ \gamma = 0.10, 0.15, 0.20, 0.25 \]

\[ Sr = 1.0, 1.5, 2.0, 2.5 \]
Fig. 9. Concentration profile for different values of $\lambda$.

Fig. 10. Concentration profile for different values of $E_c$. 
Fig. 11. Concentration profile for different values of $Du$.

Fig. 12. Concentration profile for different values of $Le$. 
Figs. 5-13 are plotted to show the influence of Nb, Nt, γ, Sr, λ, Ec, Du, Le, and Pr, on concentration profiles, respectively. It is apparent from Fig. 5. shows that an increasing the values of Nb, decreases the concentration profile. Fig. 6. represents that the concentration profile increases for increasing values of Nt. Figs. 7-10 and 12, it is observed that the increasing values of Biot number, Soret number, Stretching ratio parameter, Local Eckert number and Lewis number depreciates the concentration profiles. Figs. 11 and 13 represents the influence of Dufour number and Prandtl number on concentration profiles, respectively. It is clear from Figs. 11 and 13. that increasing the values of Dufour number and Prandtl number, enhances the concentration profiles.

The effects of Ec, γ and Pr on temperature profiles are presented in Figs. 14-16, respectively. It is observed from Figs. 14 and 16 that the increasing values of Ec and Pr, enhances the temperature profiles. Fig. 15 represents that the temperature profiles diminish for increasing values of γ.
Fig. 15. Temperature profile for different values of $\gamma$.

Fig. 16. Temperature profile for different values of $Pr$.

IV. TABLE I Physical parameters of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for Casson fluid.

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<th>$M$</th>
<th>$\beta$</th>
<th>$Nb$</th>
<th>$Nt$</th>
<th>$\gamma$</th>
<th>$Sr$</th>
<th>$\lambda$</th>
<th>$Ec$</th>
<th>$Du$</th>
<th>$Le$</th>
<th>$Pr$</th>
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<th>$-\phi'(0)$</th>
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V. CONCLUSIONS

In this paper we analyzed the magnetohydrodynamic dissipative fluid flow towards a stretching/shrinking surface with Joule heating, non-uniform heat source/sink, Brownian motion, thermophoresis and cross diffusion effects. The transformed governing equations are explored numerically by shooting technique. The consequences for different pertinent parameters on the flow, thermal and concentration distributions are explained with the assistance of tables and graphs. The wall friction, local Sherwood and reduced Nusselt number is also computed and analyzed.

- Magnetic field parameter have tendency to control and decrease the fluid flow.
- Biot numbers have tendency to control the heat and mass transfer rate.
- The concentration profiles fall for increasing the Soret number and Eckert number.
- An increase in Casson fluid parameter decreases the fluid flow.
- Rise in the values of Prandtl number enhances the heat and mass transfer rate.

REFERENCES

The homotopy analysis method.


