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Abstract - The present research article is to investigate the magnetohydrodynamic (MHD) free convective heat and mass transfer flow of viscous incompressible electrically conductive fluid over an inclined stretching sheet with viscous dissipation and constant heat flux with the help of homotopy perturbation technique. This paper gives the description of the effect of flow parameters on velocity, temperature and concentration, which is graphically represented in figures.

Keywords - MHD flow, Heat and Mass Transfer, inclined stretching sheet, concentration, angle of inclination.

I. INTRODUCTION

In past decades the magnetohydrodynamic (MHD) flow over an inclined stretching sheet with viscous dissipation and heat generation gained lots of interest. It has importance in liquid metals, electrolytes and ionized gases. The presence of strong magnetic field effect, the conduction mechanism in ionized gases is differing from that in metallic substance. Generally in the ionized gases the electric current carried out by the electrons, which undergo the successive collision with other particles may be charged or neutral particles. In the presence of strong electric field, the conductivity is affected by a magnetic field. In many applications like glass blowing, continuous casting, paper production, hot rolling, drawing in plastic films, wire drawing, polymer extrusion, metal spinning and spinning of fibres are based on MHD laminar boundary layer flow over stretching sheet. During the process of manufacturing a stretched sheet interacts with the fluid thermally and mechanically. Kinematics of stretching and the simultaneous heating or cooling during this kind of process has a decisive influence on the quality of the final product. The sheet is stretched some times in the extrusion of a polymer sheet from a die. Drawing such a sheet in a viscous fluid, the rate of cooling can be controlled and the final product achieved with desired characteristics.


Therefore the present work give emphasis on the heat and mass transfer MHD flow over an inclined stretching sheet with viscous dissipation and constant heat flux in the presence of magnetic field using similarity solution and homotopy perturbation technique.

II. FORMULATION OF THE PROBLEM

Let the flow considered to be two dimensional laminar MHD viscous incompressible electrically conducting fluid along an inclined stretching sheet. The leading edge of the inclined stretching sheet is along the X direction and Y is normal to the X axis. A magnetic field \( B_0 \) is applying normal to the direction of the flow and \( T_w (T > T_v) \) is the uniform temperature of the plate, where \( T_v \) is the temperature of the fluid away from the plate. Let \( \alpha \) be the angle of inclination. Let \( u \) and \( v \) are the velocities along X and Y axis respectively. Using general boundary layer approximation and given assumptions the governing equations under the influence of external imposed magnetic field are given as:

Equation of continuity:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

Equation of momentum:
\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta (T - T_v) \cos \alpha + \frac{\sigma B_0^2}{\rho} \frac{\partial u}{\partial y} \tag{2}
\]

Equation of energy:
\[
u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 \theta}{\partial y^2} + \frac{\nu}{C_p} \left( \frac{\partial u}{\partial y} \right)^2 \tag{3}
\]

Equation of concentration:
\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{u_0 K_f}{\rho} \frac{\partial^2 \phi}{\partial y^2} \tag{4}
\]

with the boundary conditions
\[
y = 0: \quad u = U_0, v = 0, \frac{\partial \theta}{\partial y} = -\frac{q}{k}, \frac{C}{C_w} \tag{5}
\]
\[
y \rightarrow \infty: \quad u = 0, \theta = T_v, C = C_w \tag{5}
\]

where \( T, T_w \) and \( T_v \) are the fluid temperature, the stretching sheet temperature and the free stream temperature respectively, \( C, C_w \) and \( C_v \) are corresponding concentration, \( k \) is thermal conductivity, \( C_p \) is specific heat with constant pressure, \( \alpha = \frac{k}{\rho C_p} \) is thermal diffusivity, \( \mu \) is viscosity, \( \nu \) is kinematic viscosity, \( \sigma \) is electrical conductivity, \( \rho \) is density, \( \beta \) is thermal expansion coefficient, \( \beta^* \) is concentration expansion coefficient, \( B_0 \) is magnetic field intensity, \( U_0 \) is stretching sheet parameter, \( g \) is acceleration due to gravity, \( q \) is constant heat flux per unit area, \( D_m \) is coefficient of mass diffusivity, \( K_f \) is thermal diffusion ratio, \( T_m \) is mean fluid temperature.

Now introducing the stream function \( \Psi(x,y) \) such as
\[
u \frac{\partial \psi}{\partial x} \tag{6}
\]
Again introducing following similarity transformation:
\[
\Psi = x \sqrt{2 \nu U_0 \phi (\eta)}, \eta = \frac{y}{\sqrt{2 \nu \frac{U_0}{\nu_0}}}, \theta(\eta) = \frac{k (T - T_v)}{q} \sqrt{\frac{2 \nu}{\nu_0}} \phi(\eta) = \frac{C - C_w}{C_w - C_v} \tag{7}
\]

Using above similarity transformations in equation (2) to equation (4) we get the non dimensional, nonlinear and coupled differential equations as
\[
f^{'''} + 2f f^{''} - 2f^2 - M f' + \frac{\sigma B_0^2}{\rho} \cos \alpha \frac{\sigma B_0^2}{\rho} \frac{\partial u}{\partial y} = 0 \tag{8}
\]
\[
\theta^{'''} + 2 Pr f \theta' + \frac{\nu}{C_p} \left( \frac{\partial u}{\partial y} \right)^2 = 0 \tag{9}
\]
\[
\phi^{'''} + 2Sc f \phi' + \frac{\nu}{C_p} \phi' = 0 \tag{10}
\]
Where $f'$, $\theta$ and $\varphi$ are non-dimensional velocity, temperature and concentration and $\eta$ is similarity variable. And the Magnetic Parameter is $M = \frac{2g\beta(t - C_c)}{\mu u_0^2}$, the Grashof Number is $Gr = \frac{\beta \rho_0 T}{\mu^2}$, the Eckert Number is $Ec = \frac{k/\eta t^2}{T/2v}$. And the transformed boundary conditions are

$$
\eta = 0: f'(\eta) = 1, f(\eta) = 0, \theta(\eta) = -1, \varphi(\eta) = 1 \quad (11)
$$

$$
\eta \to \infty: f'(\eta) = 0, \theta(\eta) = 0, \varphi(\eta) = 0
$$

The homotopy for the above equations defined as

$$
H(f, \theta) = \left( 1 - \eta \right) \left( f'' - (3 - \eta) e^{-\eta} + M(1 - \eta)e^{-\eta} \right) + p[f'' + 2f' + 2f^2 - Mf' + Gr\theta \cos \gamma + Gm\varphi \cos \gamma] = 0 \quad (12)
$$

$$
H(\theta, \varphi) = \left( 1 - \eta \right) [\theta'' - e^{-\eta}] + p[\theta'' + 2Prf\theta' + EcPrf'^2] = 0 \quad (13)
$$

$$
H(\varphi, \theta) = \left( 1 - \eta \right) [\varphi'' - e^{-\eta}] + p[\varphi'' + 2Sc\varphi' + ScS_0\theta'] = 0 \quad (14)
$$

It is considered that

$$
f = f_0 + pf_1 + p^2f_2 + \ldots \quad (15)
$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + \ldots
$$

$$\varphi = \varphi_0 + p\varphi_1 + p^2\varphi_2 + \ldots
$$

Substituting these assumptions in equation (12) to equation (14) and comparing the coefficients of like powers of $p$, we get

$$
p^0: f_0'' - Mf_0' - [(3 - \eta) - M(1 - \eta)]e^{-\eta} = 0 \quad (16)
$$

$$
p^1: f_1'' - Mf_1' + [(3 - \eta) - M(1 - \eta)]e^{-\eta} + 2f_0f_0'' - 2f_0^2 = Gr\cos \gamma \theta_0 + Gm\cos \gamma \varphi_0 = 0 \quad (17)
$$

$$
p^0: \theta_0'' - e^{-\eta} = 0 \quad (18)
$$

$$
p^1: \theta_1'' + e^{-\eta} + 2Prf_0\theta_0 + EcPrf_0'^2 = 0 \quad (19)
$$

$$
p^0: \varphi_0'' - e^{-\eta} = 0 \quad (20)
$$

$$
p^1: \varphi_0'' + e^{-\eta} + 2Scf_0\varphi_0 + ScS_0\theta_0'' = 0 \quad (21)
$$

And the corresponding boundary conditions are

$$
\eta = 0 \quad \left\{ \begin{array}{l}
 f_0 = 0, f_1 = 0, \ldots \\
 f_0' = 1, f_1' = 0, \ldots \\
 \theta_0 = -1, \theta_1 = 0, \ldots \\
 \varphi_0 = 1, \varphi_1 = 0, \ldots
 \end{array} \right. \quad (22)
$$

$$
\eta \to \infty \quad \left\{ \begin{array}{l}
 f_0 = 0, f_1 = 0, \ldots \\
 \theta_0 = 0, \theta_1 = 0, \ldots \\
 \varphi_0 = 0, \varphi_1 = 0, \ldots
 \end{array} \right.
$$

Solution of above equations (16) to (21) under the boundary conditions (22) are obtained as follows:

$$
f_0 = \eta e^{-\eta} \quad (23)
$$

$$
f_1 = \frac{1}{\sqrt{M-1}} \left( M - 2 \right) \left( 1 - e^{-\eta} \right) + \frac{1}{\sqrt{M}} \left( Gr + Gm \right) \left( 1 - e^{-\eta} \right) \cos \gamma + \frac{1}{M-4} \left( e^{-2\eta} - 1 \right) + \left( Gr + Gm \right) \left( e^{-\eta} - 1 \right) \cos \gamma - \eta e^{-\eta} \quad (24)
$$

$$
\theta_0 = e^{-\eta} \quad (25)
$$

$$
\theta_1 = \frac{1}{2} Pr(\eta + 1)e^{-2\eta} - \frac{1}{8} EcPr(2\eta^2 - 4\eta + 3)e^{-2\eta} - e^{-\eta} + \left( \frac{1}{2} Pr - \frac{1}{8} EcPr - 1 \right) \eta \quad (26)
$$

$$
\varphi_0 = e^{-\eta} \quad (27)
$$

$$
\varphi_1 = \frac{1}{2} Sc(\eta + 1)e^{-2\eta} - ScS_0 e^{-\eta} - e^{-\eta} + 1 - \frac{1}{2} Sc + ScS_0 \quad (28)
$$
The values of \( f, \theta \) and \( \varphi \) are obtained as
\[
f = \lim_{n \to 1} f = f_0 + f_1 + f_2 + \cdots
\]
\[
\theta = \lim_{n \to 1} \theta = \theta_0 + \theta_1 + \theta_2 + \cdots
\]
\[
\varphi = \lim_{n \to 1} \varphi = \varphi_0 + \varphi_1 + \varphi_2 + \cdots
\]

Therefore we obtained
\[
f = \frac{1}{\sqrt{M}} \left( \frac{M-2}{M-4} \right) \left( 1 - e^{-\sqrt{M}\eta} \right) + \frac{1}{\sqrt{M}} \left( \frac{Gr+Gm}{1-M} \right) \left( 1 - e^{-\sqrt{M}\eta} \right) \cos \gamma + \frac{1}{M-4} \left( e^{-\varphi} - 1 \right) e^{-\varphi} \cos \gamma
\]

\[
\theta = \frac{1}{2} Pr(\eta + 1)e^{-\varphi} - \frac{1}{6} EcPr(2\eta^2 - 4\eta + 3)e^{-\varphi} + \left( \frac{1}{2} Pr - \frac{1}{6} EcPr - 1 \right) \eta \quad \text{...(30)}
\]
\[
\varphi = \frac{1}{2} Sc(\eta + 1)e^{-\varphi} - ScS_0e^{-\varphi} + 1 - \frac{1}{2} Sc + ScS_0 \quad \text{...(31)}
\]

Hence the velocities of the flow are given as
\[
u = xU_0 \left[ \left( \frac{M-2}{M-4} \right) e^{-\sqrt{M} \frac{u_0}{2v}} + \left( \frac{Gr+Gm}{1-M} \right) \left( 1 - e^{-\sqrt{M} \frac{u_0}{2v}} \right) \cos \gamma + \frac{1}{M-4} \left( e^{-\varphi} - 1 \right) \cos \gamma \right]
\]
\[

The temperature is given as
\[
T = \frac{q}{k} \sqrt{\frac{2v}{u_0}} \left( \frac{u_0}{2v} + 1 \right) e^{-\sqrt{M} \frac{u_0}{2v}} - \frac{1}{6} EcPr \left( 2\frac{u_0^2}{2v} - 4 \frac{u_0}{2v} + 3 \right) e^{-2\sqrt{M} \frac{u_0}{2v}} + \left( \frac{1}{2} Pr - \frac{1}{6} EcPr - 1 \right) \sqrt{\frac{u_0}{2v}} \right) + T_\infty
\]

The concentration of the flow is given as
\[
C = (C_w - C_0) \left[ \frac{1}{2} Sc \left( \frac{u_0}{2v} + 1 \right) e^{-2\sqrt{M} \frac{u_0}{2v}} - ScS_0 e^{-\sqrt{M} \frac{u_0}{2v}} + 1 - \frac{1}{2} Sc + ScS_0 \right] + C_0
\]

The important physical quantities are skin friction \( C_f \) and local Sherwood number \( S_h \) are defined as
\[
C_f \propto f''(0)
\]
\[
C_f \propto -\sqrt{M} \left( \frac{M-2}{M-4} \right) + \frac{4}{M-4} + \left( \frac{Gr+Gm}{1-M} \right) \cos \gamma
\]

\[
S_h \propto -\theta (0)
\]
\[
S_h \propto 1 - \frac{2}{8} EcPr
\]
III. RESULTS AND DISCUSSIONS

<table>
<thead>
<tr>
<th>M</th>
<th>Gr</th>
<th>Gm</th>
<th>$-f''(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.2</td>
<td>-0.2</td>
<td>1.3107</td>
</tr>
<tr>
<td>2</td>
<td>-0.2</td>
<td>-0.2</td>
<td>1.9137</td>
</tr>
<tr>
<td>2</td>
<td>-0.3</td>
<td>-0.2</td>
<td>2.1609</td>
</tr>
<tr>
<td>2</td>
<td>-0.2</td>
<td>-0.3</td>
<td>2.1609</td>
</tr>
<tr>
<td>3</td>
<td>-0.2</td>
<td>-0.2</td>
<td>2.3817</td>
</tr>
<tr>
<td>5</td>
<td>-0.2</td>
<td>-0.2</td>
<td>2.5823</td>
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<tr>
<td>10</td>
<td>-0.2</td>
<td>-0.2</td>
<td>3.4142</td>
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</table>

Table 1: Skin friction coefficient $f''(0)$ for different values of M, Gr and Gm with $\gamma = 81^\circ$

<table>
<thead>
<tr>
<th>Ec</th>
<th>Pr</th>
<th>$-\theta'(0)$</th>
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<tbody>
<tr>
<td>0.5</td>
<td>0.71</td>
<td>-0.6213</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>-0.8750</td>
</tr>
<tr>
<td>1</td>
<td>0.71</td>
<td>-0.8875</td>
</tr>
</tbody>
</table>

Table 2: Rate of heat transfer (Nusselt Number) $-\theta'(0)$ for different values of Ec and Pr

<table>
<thead>
<tr>
<th>Sc</th>
<th>So</th>
<th>$-\varphi'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>0.5531</td>
</tr>
<tr>
<td>0.22</td>
<td>0.2</td>
<td>0.5543</td>
</tr>
<tr>
<td>0.2</td>
<td>0.3</td>
<td>0.5446</td>
</tr>
</tbody>
</table>

Table 3: Rate of Mass Transfer (Sherwood Number) $-\varphi'(0)$ for different values of Sc and So

<table>
<thead>
<tr>
<th>By Shooting Method</th>
<th>By Homotophy Perturbation Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-f''(0)$</td>
<td>$-f''(0)$</td>
</tr>
<tr>
<td>1.912023 (M=2)</td>
<td>0.554767 (Sc=0.2,So=0.2)</td>
</tr>
<tr>
<td>2.57678 (M=5)</td>
<td>0.543557 (Sc=0.22,So=0.2)</td>
</tr>
<tr>
<td>3.422358 (M=10)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Comparison of Skin Friction Coefficient and Local Sherwood number for different Values of M, Sc and So
Figure 1: Velocity $f'$ distribution versus $\eta$ for different values of $M$, $Gr$ and $Gm$

Figure 2: Velocity $f'$ distribution versus $\eta$ for different values of $\gamma$ the angle of inclination
Numerical observation for Skin Friction coefficient, Nusselt Number and Local Sherwood Number for different values of non-dimensional parameters are carried out. It is observed from Table 1 that coefficient of skin friction increases due to increase of magnetic parameter. Table 2 shows the effect of Prandtl number and Eckert Number on the rate of heat transfer. It is observed that Nusselt Number decreases due to increase of Prandtl number and Eckert Number. It is shown in the Table 3 that Local Sherwood Number increase due to increase of Schmidt Number. Table 4 shows the comparison between Shooting Method and Homotopy Perturbation Technique. It is observed from the table that result obtained by Schooting Method and HPM are in good agreement.

The observations and calculations for the velocity distribution, temperature and concentration profiles across the boundary layer for various values of parameters are carried out. Figure 1 displays the effect of Magnetic Parameter M, Grashof Number Gr and modified Grashof Number Gm on the velocity. It is observed that the velocity decreases due to increase in magnetic parameter and decreases due to decrease of Grashof Number. From the Figure 2 it is observed that velocity increases due to increase in angle of inclination.
Figure 3 shows the effect of Eckert Number and Prandtl Number on temperature. It is found that the temperature increase due to increase of Eckert Number and Prandtl number both. Figure 4 displays the concentration profile obtains by the variation in non-dimensional parameters. It is observed that in certain interval of \( \eta \), the concentration profile decreased and then increased due to increase of Schmidt Number and Soret Number.

IV. REFERENCES


