Changing and Unchanging Domination in Fuzzy Graph

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Abstract - In this paper the concepts of Changing and unchanging domination in Fuzzy Graph and for some standard theorems and examples are discussed.

Keywords - domination in fuzzy graph, changing domination in graph, unchanging domination in graph, changing and unchanging domination in fuzzy graphs.

I. INTRODUCTION


II. PRELIMINARIES

A fuzzy graph G =< σ, μ >is a set with two function σ: V → [0,1] and μ: E → [0,1] such that μ(x,y) ≤ σ(x) ∧ σ(y) for all x, y ∈ V. hereafter we write μ(x,y)or μ(xy).A fuzzy graph H =< τ, ρ > is called a fuzzy subgraph of G if τ(νi) ≤ σ(νί) for all νi ∈ V and ρ(ν,νi) ≤ μ(ν,νi) for all ν, νi ∈ V.

A subset S of V is called a dominating set in G if for every ν ∉ S, there exists u ∈ S such that u dominates ν. The minimum fuzzy cardinality of a dominating set in G is called the domination number of G and is denoted by γ(G).

A dominating set S of a fuzzy graph G is said to be a minimal dominating set if no proper subset of S is a dominating set of G.

The removal of an edge from a graph G can increase by the domination number by at most one and cannot decrease the domination number. γ(G − e) = γ(G) + 1.

The removal of a vertex from a graph G can increase by the domination number by at most one and cannot decrease the domination number. γ(G − v) = γ(G) + 1

Domination color transversal bondage number of a graph G denoted by bbG is defined to be the minimum cardinality of collection of sets E ⊆ E such that γ* = γ( G − E'). If bbG is not defined for K1 and K2.

A graph for which the domination number changes when an vertex is removal (CVR) has V = V − ∪ V + Observed V 0 is never empty for a tree, hence, no tree is in CVR.

V 0 = { v ∈ V; γ(G − v) = γ(G)}; V + = { v ∈ V; γ(G − v) > γ(G)}; V − = { v ∈ V; γ(G − v) < γ(G)}

The domination number is unchanged when an arbitrary vertex is removed class UVR, then V = V 0.

III. CHANGING and UNCHANGING DOMINATION in FUZZY GRAPH
Definition

A Fuzzy graph for which the domination number changes when an arbitrary vertex is removed \((CVR)\) has \(V = V^- \cup V^+\).

\[\gamma_f(G - v) \neq \gamma_f(G) \text{ for all } v \in V\]

A Fuzzy graph for which the domination number is unchanged when an arbitrary vertex is removed, \((UVR)\) has \(V = V^0\).

\[\gamma_f(G - v) = \gamma_f(G) \text{ for all } v \in V\]

Theorem

For any fuzzy tree \(T\) with \(n \geq 2\) there exists a vertex \(v \in V\) such that \(\gamma(T - V) = \gamma(T)\)

Proof

Given that \(T'\) is a fuzzy tree with \(n \geq 2\).

Since \(T'\) is a fuzzy tree if \(G\) is a acyclic with \(n \geq 2\). Let \(\gamma(T)\) be a minimum dominating set in a fuzzy tree \(T'\).

For any vertex \(v \in V\). Now we remove the vertex \(v\) in fuzzy tree \(T'\).

Therefore \(\gamma(T' - V)\) is a minimum dominating set of \(G(T' - V)\).

Hence \(G\) is a acyclic with \(n \geq 2\), then \(\gamma(T') = \gamma(T' - V)\).

Theorem

For any graph \(G, \gamma_f(G) \leq n - \Delta_f(G) - \eta_f(G) + 1\)

Proof
For any fuzzy graph $G$ with $\gamma(G) \geq 2$. Let $\mu = n - \max\{N[S]\}$ for all $S \subset V$ having $|S| = \gamma(G) - 1$. Let $r_f(G)$ to be smallest number of edges which must be added to $G$ to decrease the domination number.

Hence $\gamma_f(G) \leq n - \Delta_f(G) - r_f(G) + 1$.

**Theorem**

If a graph $G \in CVR$ has order $n = (\Delta(G) + 1)(\iota(G) - 1) + 1$. Then $G$ is regular fuzzy graph.

**Proof**

Suppose $G \in CVR$ with $n = (\Delta(G) + 1)(\iota(G) - 1) + 1$

By theorem

Suppose $\iota(G) \neq \iota(G - V)$ for all $v \in V$ then $V^0 = \emptyset$ and so $v \in V^+$ or $v \in V^-$. If $v \in V^+$, then for each $v \in V^P [v, S]$ contains two nonadjacent vertices. These vertices are in $v - S$ and so again if $P_v[v, S] = \{v\}$ then $S - \{v\}$ is an independent fuzzy dominating set of $G - V$ which is a contradiction of $v \in V^+$

Hence $V$ has a private neighbor in $V - S$.

Suppose $< P_v[v, S] >$ is complete. Then $S - \{v\} \cup \{u\}$ for any $u \in P_v[v, S]$ is an $i(G)$ - set of $G$ not containing $v$ again a contradiction.

Hence, $< P_v[v, S] >$ contains at least two nonadjacent vertices they are not in $V^+$. Since $G \in CVR$ we have $V = V^-$. Let $S_u$ denote an $i(G)$ - set of $G - u$. So that $|S_u| = i(G) - 1$.

In order to dominate the $(\Delta(G) + 1)(\iota(G) - 1)$ vertices of $G - u$ each element of $S_u$ must dominate exactly $(\Delta(G) + 1)$ vertices and has degree $\Delta(G)$. Thus no two vertices in $S_u$ have a common neighbor. To prove $u$ is regular. It is enough to prove that for any arbitrary vertex, $x \in S_u$ for some $u$.

Let $r \in S_x$ we prove that $r \in S_{rf}$. Suppose $x \notin S_{rf}$

Since $G \in CVR$, $S_{rf} \cap N[rf] = \emptyset$ each vertex $S_x - \{r\}$ dominates fuzzy unique vertex of $S_x$. So remaining vertex in $S_{rf}$ which is not $x$ must be dominated by $S_x$ and so must be dominated by $rf$ which is a contradiction as $S_{rf} \cap N[rf] = \emptyset$.

Hence $x \in S_{rf}$ and so $G$ is regular fuzzy graph.

**Theorem**

Let $G$ be an unchanging fuzzy graph. If $P_p$ is a path on $P$ vertices. Then $P_p \in UEA$ if and only if $P = 3k + 2(k \geq 1)$.

**Proof**

Suppose $P_p \in UEA$. Let $P = 3k$.

Let $P_p - (1, 2, ..., k)$ that $k > 1$. Consider the edge $e = (2, 5)$. Then, $i(P_p + e) = k + 1$ where $P_p = k$ and so $P_p \notin UEA$ which is a contradiction.

Hence $P \neq 3k (k > 2)$

Similarly,

If $P = 3k + 1$ with $e = (1, 3)$. Then, $i(P_p + e) = k$ where as, $i(P_p) = k + 1$ and so $P_p \notin UEA$ which is a contradiction. Hence $n \neq 3k + 1$ and so either $P = 3$ or $P = 3k + 2$. 

Conversely, 
\[ P = 3k + 2 \quad (k \geq 1) \]. Then \( i(P_p) = k + 1 \)

We prove that \( P_p \in UEA \) by induction on \( k \). When \( k = 1 \), \( P = 5 \) and one can verify that, \( i(P_p + e) = i(P_p) = 2 \) for all \( e \in E(G) \). Assume that the result is true for \( k \).

Now we prove the result for \( P = 3(k + 1) + 2 = 3k + 5 \). Let \( P_{3k+5} = (1,2,\ldots,3k+2) \) by induction hypothesis any edges joining two vertices among \((1,2,\ldots,3k+2)\) will not change \( i(G) \). An adding \( e = (3k + 3,3k + 5) \) we observe that \( i(G + e) = k + 2 \). Also it is easy to verify that any edge joining a vertex of \((1,2,\ldots,3k + 2)\) and a vertex of \((3k + 3,3k + 4,3k + 5)\) does not change \( i(G) \).

Hence \( P_p \in UEA \)

IV. CONCLUSIONS

The concept of changing and unchanging domination are analyzed. In this paper changing and unchanging domination in fuzzy graph are introduced and the some standard theorems are discussed.

REFERENCES