Properties of Connected Semirings and B-lattice Semirings

D. Mrudula Devi #1, G. Shobha Latha #2, T. Padma Praveen #3

Professor, Aditya college of Engineering and technology, Suram palem, Andhra Pradesh, India #1
Professor, S.K.D. University, Ananthapur Andhra Pradesh, ndia #2
Assistant Professor, Aditya college of Engineering and technology, Suram palem, Andhra Pradesh, India #3

Abstract — This paper contains some results on connected semirings and b-lattice semirings. We consider a connected semiring \((s,+,0)\) satisfying the identity \(1+y=y+1=1\) for all \(y\) in \(s\) in which \(s\) is a variant of semi group \((s,a)\) is I-Medial, I-Semi Medial, Quasive separative, weakly separative. \((S,.)\) is singular if \((S,.)\) is rectangular band. Again we consider the same identity with \((S,.)\) is commutative if \((S,a)\) is L- Commutative. On the other hand \((S,+,.)\) be a b-lattice semiring then\((S,+)\) is diagonal if \((S,+)\) is singular and the b-lattice semiring satisfying the identity \(a+b+ab=ab\) for all \(a,b\) in \(S\) then \((S,+)\) is semimedial if \((S,+)\) \((S,.)\) are singular

Keywords — connectedsemiring, I-medial, I-semimedial, rectangularband, quasive separative, weakly separative, b-lattice semi ring.

I. INTRODUCTION

The notation of semiring was introduced by Vandiver in 1934. The structure of semirings has been studied by so many authors. The theory of semirings and the theory of semigroups have considerable impact on the developments of the theory of semirings. It is observed that many researchers studied on different structures of semigroups. First of all we study variants of semigroup, it is developed by Hickey J.B [2] and extend this vital information into the semirings. This approach construct a new algebraic structure of connected semirings. In this paper we presented that results on connected semirings and b-lattice semirings. The motivation of this paper due to the results of Hickey J.B[2], Howie J.M. [3], Golan J.S [1].

I. PRILIMINARIES

1.1 Definition : Let \((S,.)\) be a semigroup and for any \(a\) in \(S\), we define a binary operation (sandwich operation) ‘\(o\)’ on the set \(S\) by \(xoy = xay\) where \(x,y \in S\). Then \(S\) becomes a semigroup with respect to this operation. We denote it by \((S,a)\) and we refer to \((S,a)\) as a variant of \((S,.)\) (or) a-connected semigroup.

1.2 Definition : Let \((S,+,.)\) be a semiring. For any \(a \in S\) consider the semigroup \((S,a)\) defined as \(xoy = xay\) for \(x,y \in S\) then \((S,+,o)\) is a semiring If (1) \((S,a)\) is a semigroup.
(2) ‘\(o\)’ distributes over addition i.e., \(xo(b+c) = xob+xoc\) and \((b+c)ox = box+cox\)
Here \((S,+,o)\) is called a connected semiring.

1.3 Definition : A semigroup \((S,.)\) is said to be I-semimedial if \(aabc = abac\) for all \(a,b,c \in S\)

1.4 Definition : A semigroup \((S,.)\) is said to be L-commutative if \(abc = acb\) for all \(a,b,c \in S\)

1.5 Definition : A semigroup \((S,.)\) is said to be I-medial if \(abcd = acbd\) for all \(a,b,c,d \in S\)

1.6 Definition : A semigroup \(S\) is called quasi separative if \(x^2 = xy = yx = y^2\) implies \(x = y\) for all \(x,y \in S\).

1.7 Definition : A semigroup is called weakly separative if for any \(x,y \in S\) \(x^2 = xy = y^2\) implies \(x = y\).

1.8 Definition : A semiring \((s,+,.)\) is called a b-lattice, if \((s,.)\) is a band and \((s,+)\) is a semilattice

1.9 Definition : A semigroup \((S,.)\) is said to be rectangular band if it satisfies the identity \(aba = a\) for all \(a,b \in S\).
1.10 Theorem: Let $(S, +, ., o)$ be a connected semiring and satisfies the identity $1+y = y+1 = y$. If $(S, o)$ is a rectangular band then $(S, a)$ is I-medial and I-semi medial.

Proof: Let $(S, +, ., o)$ be a connected semiring in which $(S, .)$ is rectangular band. Then $aba = a$ for all $a, b \in S$ Since $S$ satisfies the condition $1+y = y+1 = y$ for $y \in S$

We have $1+y = y 
\Rightarrow xo(1+y) = xoy 
\Rightarrow xo1+xoy = xoy 
\Rightarrow xa+xay = xay$

Now we prove that $(S, a)$ is I-medial then $xoyozot = xozoyot$

\[ xoyozot = xayaz 
= xaya(1+z) a(1+t) 
= xay(a+az)(a+at) 
= (xaya+xayaz)(a+at) 
= (xa+az)(a+at) 
= xaz(a+at) 
= xaza+xzat 
= xaxaya+xaxayaz 
= xaxaya(1+z) 
= xaxay \]

\[ \Rightarrow xoyozot = xozoyot \]

Hence $(S, a)$ is I-medial

(2) To prove that $(S, a)$ is I-semi medial.

i.e $xoxoyo = xoyoxoz$

now $xoxoyo = xaxayaz$

\[ = xaya 
= xa(1+y) a(1+z) 
= (xa+xay)(a+az) 
= xay(a+az) 
= xaya+xayaz 
= xaxaya+xaxayaz 
= xaxaya(1+z) 
= xaxay \]

\[ \Rightarrow xoxoyo = xoyoxoz \]

Hence $(S, a)$ is I-semi medial.

1.11 Theorem: Let $(S, +, ., o)$ be a connected semiring satisfies the identity $1+y = y+1 = y$ and $(S, .)$ is commutative then $(S, a)$ is L-commutative.

Proof: Given that $(S, +, ., o)$ is a connected semiring with the property $1+y = y+1 = y$ for all $y \in S$

Let $(S, .)$ be a commutative.

To prove that $(S, a)$ is L-commutative for any $a \in S$ i.e $xoyoz = xozoy$

Consider $xoyoz = xayaz$

\[ = xazay 
= xa(1+z)ay 
= xa(1+z) a(1+y) 
= (xa+xaz)(a+ay) 
= xaz(a+ay) 
= xaza+xazay 
= xaxa(1+y) 
= xaxay \]
\[ x_0y_0z_0 = x_0z_0y_0 \]

Hence \((S, a)\) is \(L\)-commutative.

**Note**: Similarly we prove \((S, a)\) is \(R\)-commutative.

1.12 **Theorem**: Let \((S, \cdot, o)\) be a connected semiring satisfying \(1+y = y+1 = y\) for \(y \in S\). If \((S, \cdot)\) is rectangular band then

1. \((S, \cdot)\) is singular.
2. \((S, a)\) is quasi separative and
3. \((S, a)\) is weakly separative.

**Proof**: Assume that \(S\) satisfies the identity \(1+y = y+1 = y\) for \(y \in S\)

Let \((S, +, \cdot, o)\) be a semiring in which \((S, \cdot)\) is a rectangular band

Now \(1+y = y\)
\[
\begin{align*}
  x(1+y) &= xy \\
  \Rightarrow y(x+xy) &= yxy \\
  \Rightarrow xy+xy &= yxy \\
  \Rightarrow yx+y &= y \\
  \Rightarrow yx &= y
\end{align*}
\]

Again \(1+y = y\)
\[
\begin{align*}
  (1+y)x &= yx \\
  \Rightarrow x+yx &= yx \\
  \Rightarrow xy+yxy &= yxy \\
  \Rightarrow xy+y &= y \\
  \Rightarrow xy &= y
\end{align*}
\]

\[
\therefore (S, \cdot) \text{ is left and right singular.}
\]

Hence \((S, \cdot)\) is singular.

Now we show that \((S, a)\) is weakly separative
\[
\begin{align*}
  xox &= xoy \\
  \Rightarrow xox &= xay \\
  \Rightarrow x &= xy
\end{align*}
\]

\[
\begin{align*}
  xoy &= yoy \\
  \Rightarrow xay &= yay \\
  \Rightarrow xy &= y
\end{align*}
\]

\[
\therefore xox = xoy = yox = yoy \Rightarrow x = y
\]

Hence \((S, a)\) is quasi separative

1.13 **Theorem**: Let \((S, +, \cdot, o)\) be a connected semiring satisfies the identity \(x+y+xoy = x\) if \((S, +)\) is singular then \((S, a)\) is singular.

**Proof**: Given that \((S, +, \cdot, o)\) is a connected semiring.

Now \(S\) satisfies the identity \(x+y+xoy = x\)

Clearly \((S, +)\) is singular i.e \(x+y = x\)

If \(x+y+xoy = x\)
\[
\begin{align*}
  x+y+xay &= x \\
  x+xy &= x \\
  xay &= x \\
  \quad (1+ay = ay) \\
  xoy &= x \\
  \quad (x+xay = xay)
\end{align*}
\]
Hence \((S, a)\) is singular

**1.14 Theorem:** Let \((S, +, \cdot)\) be a \(b\)-lattice semiring satisfying the identity \(a + b + ab = a\) for all \(a, b\) in \(S\). If \((S, +)\), \((S, \cdot)\) are singular then \((S, +)\) is semi medial.

**Proof:** Given that \(S\) be a \(b\)-lattice semiring then \((S, +)\) is a semilattice and \((S, \cdot)\) is a band.

\[
\begin{align*}
&\Rightarrow a + b + ab = a \\
&\Rightarrow a + b = a
\end{align*}
\]

\[
\begin{align*}
&(a+b) a = a^2 \\
&a^2 + ba = a^2
\end{align*}
\]

\[
\begin{align*}
&a^2 + ab + b + c = a^2 + b + c \\
&a^2 + ab + b + a + c = a^2 + b + c \\
&a + b + a + c = a^2 + b + c \\
&(a^2 = a, b + b = b)
\end{align*}
\]

\[
\begin{align*}
&(a^2 = a, b + b = b)
\end{align*}
\]

\[
\begin{align*}
&(S, +)\text{ is semi media.}
\end{align*}
\]

**1.15 Theorem:** Let \((S, +, \cdot)\) be a \(b\)-lattice semiring. If \((S, +)\) is singular then \((S, +)\) is diagonal.

**Proof:** Given that \((S, +, \cdot)\) be a \(b\)-lattice semiring.

To show that \((S, +)\) is diagonal.

For this we have to show that \(a + a = a\) and \(a + b + c = a + c\) for all \(a, b, c\) in \(S\).

Since \(S\) is \(b\)-lattice semiring we have \(a + a = a\)

\[
\begin{align*}
&a + b + c = a + (c + b) \\
&a + c
\end{align*}
\]

\[
\begin{align*}
&(a + c)\text{ (\(S, +\) commutative)} \\
&= a + c
\end{align*}
\]

\[
\begin{align*}
&\Rightarrow a + b + c = a + c
\end{align*}
\]

\[
\begin{align*}
&(S, +)\text{ is singular)
\end{align*}
\]

\[
\begin{align*}
&\Rightarrow a + b + c = a + c
\end{align*}
\]

\[
\begin{align*}
&\Rightarrow \text{\((S, +)\) is diagonal.}
\end{align*}
\]

**REFERENCES**

