Discrete Heat Equation Model with 5-variables

G.Pavithra¹, C.Glorireena², P.Thiruselvi³.
Assistant Professors and Department of mathematics,
Adhiyaman Arts And Science college For Women,

ABSTRACT: We investigate the generalized partial difference equation operator and propose a model of it in discrete heat equation in this paper. The diffusion of heat is studied by the application of newton’s law of cooling in dimension up to five and several solutions are postulated for the same. Through numerical simulations solutions are validated and applications are derived.

KEYWORD: Generalized partial difference equation partial difference, operator and discrete heat.

I. INTRODUCTION

In 1984 Jerzy popenda introduced the difference operator \( \Delta \) defined on \( u(k) \) as

\[
\Delta u(k) = u(k+1) - \alpha u(k).
\]

In 1989, Miller and Rose introduced the discrete analogue of the Riemann-Liouville fractional derivative and proved some properties of the inverse fractional difference operator \( \Delta^{-1} \) several formula on higher order partial sums on arithmetic geometric progressions and products of n-consecutive terms of arithmetic progression have been derived.

In the extended the definition of \( \Delta \) to \( \Delta \) defined equation \( \Delta v(k) = v(k + l) - kv(k) \) equation is established. Here we extend the operator \( \Delta \) to a partial difference operator

\[
\Delta v(k) = v(k_1 + l_1, k_2 + l_2, \ldots, k_n + l_n) - v(k_1, \ldots, k_n)
\]

Partial difference and differential equation play a vital role in heat equations. The generalized difference operator with n-variables.

\[
l = (l_1, l_2, l_3, \ldots, l_n) \neq 0
\]

on a real valued function \( v(k) : n \rightarrow 1 \) is defined as

\[
\Delta v(k) = v(k_1 + l_1, k_2 + l_2, \ldots, k_n + l_n) - v(k_1, \ldots, k_n)
\]

The operator \( \Delta \) becomes generalized partial difference operator if some \( l_i = 0 \). The equation involving \( \Delta \) with at least one \( l_i = 0 \) is called generalized partial difference equation.

A liner generalized partial difference is of the form \( \Delta v(k) = u(k) \). Then the inverse of generalized partial difference equation is

\[
v(k) = \Delta^{-1} u(k)
\]

Where \( \Delta \) is as given in (1)\( l_i = 0 \) and some i and \( u(k) : n \rightarrow 1 \) is given function.

A function \( u(k) : n \rightarrow 1 \) satisfying (2) is called a solution of equation (2) equation (2) has a numerical solution of the form
\[ v(k) - v(k - ml) \sum_{r=1}^{m} u(k - rl) \] ................. (3)

Where \( k = k_1 k_2 k_3 k_4 k_5 \)
\[ k - rl = k_1 - rl_1 k_2 - rl_2, \ldots, k_n - rl_n, \]
m is any positive integer relation (3) is the basic inverse principle with respect to \( \Delta \) [6]. Here, we form partial difference equation for the heat flow transmission in rod, plate and system and obtain its solution.

II. FORMATION OF 5 VARIABLE HEAT EQUATION OF ROD

Consider temperature distribution of a very long rod. Assume that the rod is so long that it can be laid on the top of the set \( R \) of real number. Let \( v(k_1, k_2, k_3, k_4, k_5) \) be the temperature at position \((k_1, k_2, k_3)\). Real time \( k_4 \) and density (or pressure) \( k_5 \). Assume that the diffusion rate \( \alpha \) is constant throughout the rod. Five variable \( > 0 \)

By the fouier law of cooling, the discrete heat equation of the rod is
\[
\Delta v(k) = \alpha \Delta v(k) + \Delta v(k) + \Delta v(k) \\
(\Delta)_{(1,1)}^{(0,0)} (\Delta)_{(1,1)}^{(0,0)} (\Delta)_{(1,1)}^{(0,0)}
\]

Here we derive the temperature formula for \( v(k_1, k_2, k_3, k_4, k_5) \) at the general position \((k_1, k_2, k_3, k_4, k_5)\)

**Theorem 2.1**

Assume that there exists a positive integer \( m \) and a real number \( l_4 > 0 \) and \( l_5 > 0 \) such that
\[ v(k_1, k_2, k_3, k_4 - ml_4, k_5 - ml_5) \]
and
\[ v(k_1, k_2, k_3, k_4, k_5) = \alpha \Delta \ U \\
(k_1, k_2, k_3, k_4, k_5)
\]

Here \( *= k_3, k_4 - l_4, k_5 * l_5 \)

\[ V(k_1, k_2, k_3, k_4, k_5) = \alpha \Delta \ U \\
(k_1, k_2, k_3, k_4, k_5)
\]

**Proof:**
\[ \Delta v(k_1, k_2, k_3, k_4, k_5) = \alpha \Delta \ U \\
(k_1, k_2, k_3, k_4, k_5)
\]

The proof of (5) follows by applying the inverse principle (3) in (6)

**Example 2.2**

From (2) we get,
\[ v(k_1, k_2, k_3, k_4, k_5) = \alpha \Delta \ U \\
(k_1, k_2, k_3, k_4, k_5)
\]

Taking \[ v(k_1, k_2, k_3, k_4, k_5) = e^{k_1 + k_2 + k_3 + k_4 + k_5} \]

\[ (k_4 + k_5) I_4 + I_5 - (k_4 - m I_4) + (k_4 - m I_4) + (k_5 - k_4 - m I_4) + (k_5 - k_4 - m I_4)
\]

\[ + (k_4 + k_5) I_4 + (k_4 + k_5) I_4 + (k_4 + k_5) I_4 + (k_4 + k_5) I_4 = 1 + 1 + 1 + e^{k_4 - rl_4} + e^{k_5 - rl_5} + e^{k_4 - rl_4} + e^{k_5 - rl_5}
\]

The numerical verification of \( m = 1, k_4 = 3, k_5 = 2, l_4 = 4, l_5 = 1, r = 1 \) in (7) we get

7.0861611629 = 7.0861611629

**Theorem 2.3**

Consider (4) and denote \( v(k_1 \pm l_1, k_2, k_3, k_4, l_4, k_5) \)

\[ v(k_1, k_2, k_3, k_4, k_5) = v(k_1 + l_1, k_2, k_3, k_4, k_5) + v(k_1 - l_1, k_2, k_3, k_4, k_5)
\]

Here \( *= k_3, k_4 - l_4, k_5 * l_5 \)

\[ V(k_1, k_2, k_3, k_4, k_5) = \alpha \Delta \ U \\
(k_1, k_2, k_3, k_4, k_5)
\]

Then the following four type's solutions of the equation (4) are equivalent.

a) \[ v(k_1, k_2, k_3, k_4, k_5) = \alpha \Delta \ U \\
(k_1, k_2, k_3, k_4, k_5)
\]

b) \[ v(k_1, k_2, k_3, k_4, k_5) = \alpha \Delta \ U \\
(k_1, k_2, k_3, k_4, k_5)
\]
\[ \sum_{r=1}^\infty \frac{\alpha^2}{(1-6\alpha)^r} \nu \{ (v(k_1+l_1, k_2, k_3, k_5*)} + v(k_1-2l_1, k_2, k_3, k_5*) \}
\]
\[ \sum_{r=1}^\infty \frac{k_5}{(1-6\alpha)^r} v(k_1+l_1, k_2, k_3, k_5*) \]
\[ \sum_{r=1}^\infty \frac{k_5}{(1-6\alpha)^r} v(k_1-2l_1, k_2, k_3, k_5*) \]
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\[ \sum_{r=1}^\infty \frac{k_5}{(1-6\alpha)^r} v(k_1+l_1, k_2, k_3, k_5*) \]
Taking \( k_1=2, \ k_2=3, \ k_3=1, \ l_1 = 3, \ l_2 = 2, \ l_3 = 1, \ m = 1 \) in (a),(b),(c),(d),(e),(f), we get the solution \( 65536=65536 \).

### III. Conclusion

The study of partial difference operator has wide applications in discrete fields and heat equation is one such theorem 2.1 & 2.3 provide the possibility of predicing the temperature either for the past of the future getting the know the temperature at few finite points at present terms. It is also show the nature of transmission of heat for the material study. Thus in conclusion, we can say that the above research help us in reducing any wastage of heat and also enables us in material of optimal choice of material (\( \alpha \)).

### REFERENCES


