General Infinite integral involving the sequence of functions, a class of polynomials and multivariable Aleph-functions I

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ABSTRACT

In the present paper we evaluate a generalized infinite integral involving the product of the sequence of functions, the multivariable Aleph-functions and general class of polynomials of several variables with general arguments. The importance of the result established in this paper lies in the fact they involve the Aleph-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializing the parameters their in.

Keywords: Multivariable Aleph-function, general class of polynomials, sequence of functions, multivariable I-function, Aleph-function of two variable, I-function of two variables.

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1. Introduction and preliminaries.

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [6], itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occurring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define: $N(z_1, \cdots, z_r) = N_{p_1,p_2,\cdots, p_r}^{n_1, n_2, \cdots, n_r} = \prod_{i=1}^r z_i^{\gamma_{ji}} R^{(i)}(\gamma_{ji}, \tau_{ij}, \alpha_{ij}, \beta_{ij})$

\[
\left[ \left[ \left[ \cdots \left[ \left[ \left[ \delta_j^{(r)}(1) \right]_{1, m_1}, \Gamma_1(\delta_j^{(r)}(1) + \delta_j^{(r)}(1)_{n_1+1, q_1}) \right], \cdots, \left[ \left[ \delta_j^{(r)}(1) \right]_{1, m_r}, \Gamma_1(\delta_j^{(r)}(1) + \delta_j^{(r)}(1)_{n_r+1, q_r}) \right] \right], \cdots, \left[ \left[ \delta_j^{(r)}(1) \right]_{1, m_1}, \Gamma_1(\delta_j^{(r)}(1) + \delta_j^{(r)}(1)_{n_1+1, q_1}) \right] \right], \cdots, \left[ \left[ \delta_j^{(r)}(1) \right]_{1, m_r}, \Gamma_1(\delta_j^{(r)}(1) + \delta_j^{(r)}(1)_{n_r+1, q_r}) \right] \right] \right]
\]

\[
= \frac{1}{(2\pi i)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \cdots, s_r) \prod_{k=1}^r \theta_k(s_k) g_k^{s_k} ds_1 \cdots ds_r
\]

with $\omega = \sqrt{-1}$

\[
\psi(s_1, \cdots, s_r) = \frac{\prod_{j=1}^{n-1} \Gamma(1 - a_j + \sum_{k=1}^r \alpha_{kj}^{(k)} s_k)}{\sum_{i=1}^r \prod_{j=n+1}^{p_i} \Gamma(1 - a_{ji} - \sum_{k=1}^r \alpha_{kj}^{(k)} s_k) \prod_{j=1}^{q_i} \Gamma(1 - b_{ji} + \sum_{k=1}^r \beta_{kj}^{(k)} s_k)}
\]

and $\theta_k(s_k) = \frac{\prod_{j=1}^{m_k} \Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k)}{\sum_{i(k)=1}^{R_{(k)}} \prod_{j=m_k+1}^{q_{(k)}} \Gamma(1 - d_{ji}^{(k)} + \delta_{ji}^{(k)} s_k) \prod_{j=m_k+1}^{n_k} \Gamma(1 - e_{ji}^{(k)} + \gamma_{ji}^{(k)} s_k)}$
Suppose, as usual, that the parameters
\[ a_j, j = 1, \ldots, p; b_j, j = 1, \ldots, q; \]
\[ c_j^{(k)}, j = 1, \ldots, n_k; \]
\[ d_j^{(k)}, j = 1, \ldots, m_k; \]
are complex numbers, and the \( \alpha', \beta', \gamma' \) s and \( \delta' \) s are assumed to be positive real numbers for standardization purpose such that
\[ U_i^{(k)} = \sum_{j=1}^{n} \alpha_j^{(k)} + \tau_i \sum_{j=n+1}^{n_k} \alpha_j^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} + \tau_i \sum_{j=n_k+1}^{n_k} \gamma_j^{(k)} - \tau_i \sum_{j=1}^{m_k} \beta_j^{(k)} - \sum_{j=1}^{m_k} \delta_j^{(k)} \]
\[ -\tau_i^{(k)} \sum_{j=m_k+1}^{q_k} \delta_j^{(k)} \leq 0 \tag{1.4} \]

The reals numbers \( \tau_i \) are positives for \( i = 1 \) to \( R \), \( \tau_i^{(k)} \) are positives for \( \tau_i^{(k)} = 1 \) to \( R^{(k)} \).

The contour \( L_k \) is in the \( s_k \)-p lane and run from \( \sigma - i\infty \) to \( \sigma + i\infty \) where \( \sigma \) is a real number with loop, if necessary, ensure that the poles of \( \Gamma(d_j^{(k)} - \delta_j^{(k)} s_k) \) with \( j = 1 \) to \( m_k \) are separated from those of \( \Gamma(1 - a_j + \sum_{i=1}^{r} c_j^{(k)} s_k) \) with \( j = 1 \) to \( n \) and \( \Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k) \) with \( j = 1 \) to \( n_k \) to the left of the contour \( L_k \).

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as:

\[ \arg z_k < \frac{1}{2} A_i^{(k)} \pi, \]  
where
\[ A_i^{(k)} = \sum_{j=1}^{n} \alpha_j^{(k)} - \tau_i \sum_{j=n+1}^{n_k} \alpha_j^{(k)} - \tau_i \sum_{j=1}^{n_k} \beta_j^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_i \sum_{j=n_k+1}^{m_k} \gamma_j^{(k)} + \sum_{j=1}^{m_k} \delta_j^{(k)} \]
\[ + \sum_{j=m_k+1}^{q_k} \delta_j^{(k)} \tau_i^{(k)} \sum_{j=m_k+1}^{q_k} \delta_j^{(k)} \]  
with \( k = 1 \ldots, r, i = 1, \ldots, R, \tau^{(k)} = 1, \ldots, R^{(k)} \) \tag{1.5}

The complex numbers \( z_k \) are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the asymptotic expansion in the following convenient form:
\[ \Re(z_1, \ldots, z_r) = 0 \left( |z_1|^{\alpha_1}, \ldots, |z_r|^{\alpha_r} \right), \max(|z_1|, \ldots, |z_r|) \to 0 \]
\[ \Re(z_1, \ldots, z_r) = 0 \left( |z_1|^{\beta_1}, \ldots, |z_r|^{\beta_r} \right), \min(|z_1|, \ldots, |z_r|) \to \infty \]

where, with \( k = 1, \ldots, r : \alpha_k = \min \left[ \Re(d_j^{(k)} / \delta_j^{(k)}) \right], j = 1, \ldots, m_k \) and
\[ \beta_k = \max \left[ \Re((c_j^{(k)} - 1) / \gamma_j^{(k)}) \right], j = 1, \ldots, n_k \]
Serie representation of Aleph-function of several variables is given by
\[
N(y_1, \ldots, y_r) = \sum_{G_1, \ldots, G_r=0}^{\infty} \sum_{g_1=0}^{m_1} \cdots \sum_{g_r=0}^{m_r} \frac{(-1)^{g_1+\cdots+g_r}}{G_1! \cdots G_r!} \psi(\eta_{G_1,g_1}, \ldots, \eta_{G_r,g_r}) 
\times \theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r}) y_1^{-\eta_{G_1,g_1}} \cdots y_r^{-\eta_{G_r,g_r}}
\] (1.6)

Where \(\psi(\cdot, \ldots, \cdot), \theta_i(\cdot, i = 1, \ldots, r\) are given respectively in (1.2), (1.3) and
\[
\eta_{G_i,g_1} = \frac{d^{(1)}_{g_1} + G_1}{d^{(1)}_{g_1}}, \ldots, \eta_{G_r,g_r} = \frac{d^{(r)}_{g_r} + G_r}{d^{(r)}_{g_r}}
\]
which is valid under the conditions
\[
d^{(j)}_{g_j}[d^{(i)}_{g_i} + p_i] \neq d^{(j)}_{g_j} + G_i
\] (1.7)

for \(j \neq m_i, m_i = 1, \ldots, G_i, g_i; p_i, n_i = 0, 1, 2, \ldots, y_i \neq 0, i = 1, \ldots, r\) (1.8)

Consider the Aleph-function of \(s\) variables
\[
N(z_1, \ldots, z_s) = \sum_{P_1, Q_1, i_1, \ldots, i_s} \left( \begin{array}{cccc}
Z_1 \\
\vdots \\
\vdots \\
Z_s \\
\end{array} \right) 
\]
\[
= \frac{1}{(2\pi)^s} \int L_1 \cdots L_s \zeta(t_1, \ldots, t_s) \prod_{k=1}^{s} \phi_k(t_k) z_k^{t_k} dt_1 \cdots dt_s
\] (1.9)

with \(\omega = \sqrt{-1}\)

\[
\zeta(t_1, \ldots, t_s) = \frac{\prod_{j=1}^{r} \Gamma(1 - u_j + \sum_{k=1}^{s} \mu^{(k)}_{j} t_k)}{\sum_{i=1}^{r} \left[ t_i \prod_{j=N+1}^{Q_i} \Gamma(u_{ji} - \sum_{k=1}^{s} \mu^{(k)}_{ji} t_k) \prod_{j=1}^{Q_i} \Gamma(1 - v_{ji} + \sum_{k=1}^{s} \nu^{(k)}_{ji} t_k) \right]}
\] (1.10)

and
\[
\phi_k(t_k) = \frac{\prod_{j=1}^{M_k} \Gamma(1 - b^{(k)}_{ji} t_k)}{\sum_{i=k}^{s} \left[ t_i \prod_{j=N+k+1}^{Q_i} \Gamma(1 - b^{(k)}_{ji} t_k) \prod_{j=N+k+1}^{Q_i} \Gamma(1 - a^{(k)}_{ji} + \sum_{k=1}^{s} \alpha^{(k)}_{ji} s_k) \right]}
\] (1.11)

Suppose, as usual, that the parameters
\(u_j, j = 1, \ldots, P; v_j, j = 1, \ldots, Q;\)
with \( k = 1 \ldots s, \ i = 1 \ldots r', \ i^{(k)} = 1 \ldots r^{(k)} \) are complex numbers, and the \( \alpha' \)'s, \( \beta' \)'s, \( \gamma' \)'s and \( \delta' \)'s are assumed to be positive real numbers for standardization purpose such that

\[
U_i^{(k)} = \sum_{j=1}^{N} \mu_j^{(k)} + \sum_{j=N+1}^{P_i} \mu_{ji}^{(k)} \sum_{j=1}^{N_k} \alpha_j^{(k)} + \sum_{j=N_k+1}^{P_i} \alpha_{ji}^{(k)} - \sum_{j=1}^{Q_i} \nu_j^{(k)} + \sum_{j=N_k+1}^{M_k} \beta_j^{(k)} - \sum_{j=M_k+1}^{Q_i} \beta_{ji}^{(k)} \leq 0
\]

The real numbers \( \tau_i \) are positives for \( i = 1, \ldots, r \), \( \tau_i^{(k)} \) are positives for \( i^{(k)} = 1 \ldots r^{(k)} \)

The contour \( L_k \) is in the \( t_k \)-p plane and run from \( \sigma - i\infty \) to \( \sigma + i\infty \) where \( \sigma \) is a real number with loop, if necessary , ensure that the poles of \( \Gamma(b_j^{(k)} - \beta_j^{(k)} t_k) \) with \( j = 1 \) to \( M_k \) are separated from those of \( \Gamma(1 - u_j + \sum_{i=1}^{s} \beta_j^{(k)} t_k) \) with \( j = 1 \) to \( N \) and \( \Gamma(1 - a_j^{(k)} + \alpha_j^{(k)} t_k) \) with \( j = 1 \) to \( N_k \) to the left of the contour \( L_k \). The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

\[
|\arg z_k| < \frac{1}{2} B_i^{(k)} \pi, \text{ where}
\]

\[
B_i^{(k)} = \sum_{j=1}^{N} \mu_j^{(k)} - \sum_{j=N+1}^{P_i} \mu_{ji}^{(k)} - \sum_{j=1}^{Q_i} \nu_j^{(k)} + \sum_{j=1}^{N_k} \alpha_j^{(k)} - \sum_{j=N_k+1}^{P_i} \alpha_{ji}^{(k)} + \sum_{j=1}^{M_k} \beta_j^{(k)} - \sum_{j=M_k+1}^{Q_i} \beta_{ji}^{(k)} > 0, \quad \text{with} \quad k = 1 \ldots s, \ i = 1 \ldots r, \ i^{(k)} = 1 \ldots r^{(k)}
\]

The complex numbers \( z_i \) are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the the asymptotic expansion in the following convenient form :

\[
\Re (z_1, \ldots, z_s) = 0 (|z_1|^{\alpha'_{z_1}}, \ldots, |z_s|^{\alpha'_{z_s}}), \max (|z_1|, \ldots, |z_s|) \to 0
\]

\[
\Re (z_1, \ldots, z_s) = 0 (|z_1|^{\beta'_{z_1}}, \ldots, |z_s|^{\beta'_{z_s}}), \min (|z_1|, \ldots, |z_s|) \to \infty
\]

where, \( k = 1, \ldots, z \) : \( \alpha'_{z_k} = \min [\Re(b_j^{(k)}/\beta_j^{(k)})] \), \( j = 1, \ldots, M_k \) and

\[
\beta'_{z_k} = \max [\Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], \ j = 1, \ldots, N_k
\]

We will use these following notations in this paper
The multivariable Aleph-function write:

\[ U = P_i, Q_i, \tau_i; r' \quad V = M_1, N_1, \cdots, M_s, N_s \]  
\[ \mathbf{w} = P_{(i)}, Q_{(i)}, \tau^{(i)}_1, \cdots, P_{(s)}, Q_{(s)}, \tau^{(s)} \]  
\[ A' = \{(u_{ij}, \mu^{(1)}_{ji}, \cdots, \mu^{(s)}_{ji})_{1, N_j}, \{\tau_i(u_{ij}, \mu^{(1)}_{ji}, \cdots, \mu^{(s)}_{ji})_{N+1, P_i}\} \]  
\[ B = \{\tau_i(v_{j1}, u^{(1)}_{j1}, \cdots, v^{(s)}_{j1})_{M+1, Q_i}\} \]

\[ C = (b^{(1)}_{j1}, \alpha^{(1)}_{ji}, \cdots, b^{(s)}_{j1}, \alpha^{(s)}_{ji})_{1, N_j}, \{\tau_i(b^{(1)}_{ji}, \alpha^{(1)}_{ji}, \cdots, b^{(s)}_{ji}, \alpha^{(s)}_{ji})_{N+1, P_i}\} \]

The multivariable Aleph-function is given in the following manner:

\[ \mathbb{N}(z_1, \cdots, z_s) = \mathbb{N}_{U, W}^{0, N, V} \left( \begin{array}{c}
\vdots \quad A' \vdots \quad C \\
\vdots \\
\vdots \\
\vdots \\
\vdots \quad B \vdots \quad D
\end{array} \right) \]

The generalized polynomials defined by Srivastava [9], is given in the following manner:

\[ S_{N_1, \cdots, N_k}^{M_1, \cdots, M_k} \left[ y_1, \cdots, y_t \right] = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_t=0}^{[N_t/M_t]} \frac{(-N_1)_{M_1} K_1}{K_1!} \cdots \frac{(-N_t)_{M_t} K_t}{K_t!} \]

\[ A[N_1, K_1; \cdots; N_t, K_t] y_1^{K_1} \cdots y_t^{K_t} \]

Where \( M_1, \cdots, M_s \) are arbitrary positive integers and the coefficients \( A[N_1, K_1; \cdots; N_t, K_t] \) are arbitrary constants, real or complex. In the present paper, we use the following notation:

\[ a_1 = \frac{(-N_1)_{M_1} K_1}{K_1!} \cdots \frac{(-N_t)_{M_t} K_t}{K_t!} A[N_1, K_1; \cdots; N_t, K_t] \]

In the document, we note:

\[ G(\eta_{G_1, g_1}, \cdots, \eta_{G_r, g_r}) = \phi(\eta_{G_1, g_1}, \cdots, \eta_{G_r, g_r}) \theta_1(\eta_{G_1, g_1}) \cdots \theta_r(\eta_{G_r, g_r}) \]

where \( \phi(\eta_{G_1, g_1}, \cdots, \eta_{G_r, g_r}), \theta_1(\eta_{G_1, g_1}), \cdots, \theta_r(\eta_{G_r, g_r}) \) are given respectively in (1.2) and (1.3)

2. Sequence of functions

Agarwal and Chaubey [1], Salim [5] and several others have studied a general sequence of functions. In the present document we shall study the following useful series formula for a general sequence of functions.

\[ R^n_{\alpha, \beta} \left[ x; E; F, g, h; p, q; \gamma; \delta; e^{-sz^T} \right] = \sum_{w, v, u, t, e, k_1, k_2=0}^{\infty} \psi(w, v, u, t, e, k_1, k_2) x^{R} \]  
\[ \sum_{w, v, u, t, e, k_1, k_2=0}^{\infty} = \sum_{w=0}^{\infty} \sum_{v=0}^{\infty} \sum_{u=0}^{\infty} \sum_{t=0}^{\infty} \sum_{e=0}^{\infty} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \]
and the infinite series on the right side (2.1) is absolutely convergent, \( R = ln + qv + pt + rw + k_1 r + k_2 q \)

and \( \psi(w, v, u, t, e, k_1, k_2) = \frac{(-)^{t+w+k_2}(-v)u(-t)e(\alpha)_t}{w!v!u!t!e!K_nk_1k_2!} \frac{s^{w+k_2}e^{\gamma_n-t}}{(1-\alpha-t)_e}(\alpha-\gamma_n)e \)

\[ (-\beta - \delta_n)v^k \delta_n - v - k_2(v - \delta_n)k_2 e^t \left( \frac{pt + rw + \lambda + qn}{l} \right)^n \]  

(2.3)

where \( K_n \) is a sequence of constants.

By suitably specializing the parameters involving in (2.1), a general sequence of function reduced to generalized polynomials set studied by Raizada [4], a class of polynomials introduced by Fujiwara [2] and several others authors.

3. Required integral

We have the following result, see Marichev et al ([3], 2.2.11, eq.23 page 316)

**Lemma**

\[
\int_0^{+\infty} \frac{x^{\alpha-1}}{\sqrt{(x+z)^2 - x^2y^2}} \, dx = 2^{-\mu}z^{\alpha-\mu-1}B(\alpha, \mu - \alpha + 1)
\]

\[
\times_2F_1 \left[ \frac{\alpha}{2}, 1 + \frac{\alpha + 1}{2}; 1 + \mu; y^2 \right]
\]

(3.1)

where : \(|y| < 1, |\arg z| < \pi; 0 < Re(\alpha) < Re(\mu) + 1\)

4. main integral

Let \( X_{\alpha, \mu} = \frac{x^\alpha}{(x + y + \sqrt{(x+z)^2 - y^2})^{\mu}} \), We have the following general integral

**Theorem**

\[
\int_0^{+\infty} \frac{x^{\alpha-1}}{\sqrt{(x+z)^2 - x^2y^2}} \left( E, F, g, h, p, q, \gamma'; \delta'; e^{-s(x'X_{\gamma, \delta})^t} \right) \, dx = 2^{-\mu}z^{\alpha-\mu-1}
\]

\[
S_{N_1, \ldots, N_t}^{M_1, \ldots, M_r} \left( \begin{array}{c} y_1X_{\gamma_1, \mu_1} \\ \vdots \\ y_tX_{\gamma_t, \mu_t} \end{array} \right) N_{0, n; v}^{0, N; v} \left( \begin{array}{c} z_{1X_{\alpha_1, \beta_1}} \\ \vdots \\ z_{sX_{\alpha_s, \beta_s}} \end{array} \right) N_{u; W}^{0, N; V} \left( \begin{array}{c} Z_{1X_{\eta_1, e_1}} \\ \vdots \\ Z_{sX_{\eta_s, e_s}} \end{array} \right) \, dx = 2^{-\mu}z^{\alpha-\mu-1}
\]

\[
\sum_{G_1, \ldots, G_r} \sum_{g_1=0}^{m_1} \sum_{g_r=0}^{m_r} \sum_{n'=0}^{\infty} \sum_{[N_1/M_1]} \sum_{[N_t/M_t]} \sum_{R_1=0}^{\infty} \sum_{w, v, u, t, e, k_1, k_2} \frac{(-)^{g_1+\cdots+g_r}}{g_1! \cdots g_r!} G(\eta g_1, g_1, \ldots, \eta g_r, g_r) a_1^2 \frac{n'}{n!}
\]

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\[ \psi(w, v, u, t, e, k_1, k_2) = \prod_{i=1}^{P_1} x_i P^{\gamma_1} \prod_{i=1}^{P_s} \frac{z}{\lambda_1} \prod_{i=1}^{P_{s^*}} z_i^{\gamma_{s^*}} \prod_{i=1}^{P_{s^*}} y_i^{\lambda_i} \prod_{i=1}^{Q_1} \frac{z}{\lambda_1} \prod_{i=1}^{Q_s} z_i^{\gamma_{s^*}} \prod_{i=1}^{Q_{s^*}} y_i^{\lambda_i} \prod_{i=1}^{Q_{s^*}} z_i^{\gamma_{s^*}} \]

\[ z^{(\gamma - \delta)RA + \sum_{i=1}^{t} K_i(\gamma_i - \mu_i) + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i - \beta_i); \, \epsilon_1 - \eta_1, \cdots, \epsilon_s - \eta_s,} \]

\[ (\alpha - \mu + (\gamma - \delta)RA + \sum_{i=1}^{t} K_i(\gamma_i - \mu_i) + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i - \beta_i); \, \epsilon_1 - \eta_1, \cdots, \epsilon_s - \eta_s) \]

\[ (1-n_{-1})^{\frac{1}{2}} (\alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i); \, \eta_1, \cdots, \eta_s) \]

\[ (1-\frac{1}{2}) (\alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i); \, \eta_1, \cdots, \eta_s) \]

\[ \left( 1 - \frac{n}{2} - \frac{1}{2} \right) \left( \alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i); \, \frac{\eta_1}{2}, \cdots, \frac{\eta_s}{2} \right) \]

\[ \left( \alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i); \, \frac{\eta_1}{2}, \cdots, \frac{\eta_s}{2} \right) \]

\[ (1 - \frac{n}{2} - \frac{1}{2}) (\alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i); \, \frac{\eta_1}{2}, \cdots, \frac{\eta_s}{2} \right) \]

\[ (1 - \frac{n}{2} - \frac{1}{2}) (\alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i); \, \frac{\eta_1}{2}, \cdots, \frac{\eta_s}{2} \right) \]

\[ (1 - \frac{n}{2} - \frac{1}{2}) (\alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i); \, \frac{\eta_1}{2}, \cdots, \frac{\eta_s}{2} \right) \]

\[ \left( 1 - \frac{n}{2} - \frac{1}{2} \right) \left( \alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i); \, \frac{\eta_1}{2}, \cdots, \frac{\eta_s}{2} \right) \]

\[ \left( 1 - \frac{n}{2} - \frac{1}{2} \right) \left( \alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i); \, \frac{\eta_1}{2}, \cdots, \frac{\eta_s}{2} \right) \]

\[ \left( 1 - \frac{n}{2} - \frac{1}{2} \right) \left( \alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i); \, \frac{\eta_1}{2}, \cdots, \frac{\eta_s}{2} \right) \]

\[ \left( 1 - \frac{n}{2} - \frac{1}{2} \right) \left( \alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta_{g_i,g_i}(\alpha_i); \, \frac{\eta_1}{2}, \cdots, \frac{\eta_s}{2} \right) \]

\[ A' = C \]

\[ B = D \]

\[ (4.1) \]

where \( U_{43} = P_i + 4, Q_i, R_i; r' \)

Provided that

a) \( \min\{A, \gamma, \delta, \rho_i, \delta_i, \gamma_j, \mu_j, \alpha_k, \beta_k, \eta_k, \epsilon_i \} > 0, i = 1, \cdots, s, j = 1, \cdots, t, k = 1, \cdots, \tau, l = 1, \cdots, R \)

b) \( 0 < \text{Re}(\alpha + RA\gamma) + \sum_{i=1}^{r} \alpha_i \min_{1 \leq j \leq m_i} \text{Re} \left( \frac{d_j(i)}{\delta_j(i)} \right) + \sum_{i=1}^{R} \eta_i \min_{1 \leq j \leq M_i} \text{Re} \left( \frac{b_j(i)}{\beta_j(i)} \right) \)

\[ < \text{Re}(\mu + RA\delta) + \sum_{i=1}^{r} \beta_i \min_{1 \leq j \leq m_i} \text{Re} \left( \frac{d_j(i)}{\delta_j(i)} \right) + \sum_{i=1}^{R} \epsilon_i \min_{1 \leq j \leq M_i} \text{Re} \left( \frac{b_j(i)}{\beta_j(i)} \right) + 1 \]

c) \( |arg z_k| < \frac{1}{2} A_i^{(k)} \pi, \text{ where } A_i^{(k)} \text{ is defined by (1.5); } i = 1, \cdots, r \)

d) \( |arg Z_k| < \frac{1}{2} B_i^{(k)} \pi, \text{ where } B_i^{(k)} \text{ is defined by (1.13); } i = 1, \cdots, s \)

e) The series occurring on the right-hand side of (3.1) is absolutely and uniformly convergent.
Proof

First, expressing the sequence of functions \( R_n^{\alpha,\beta} \left[ z X^A_{\gamma,\delta}; E, F, g, h; p, q; \gamma, \delta, e^{-s(zX^A_{\gamma,\delta})^T} \right] \) in multiple series with the help of equation (2.1), the Aleph-function of \( r \) variables in series with the help of equation (1.6), the general class of polynomial of several variables \( S_{N_1,\cdots,N_r}^{M_1,\cdots,M_r} \) with the help of equation (1.22) and the Aleph-function of \( s \) variables in Mellin-Barnes contour integral with the help of equation (1.9), changing the order of integration and summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process) and then evaluating the resulting integral with the help of equation (3.1) and expressing the Gauss hypergeometric function \( _2F_1 \) in serie, use the following relations \( \Gamma(a)(a_n) = \Gamma(a+n) \) and \( a = \frac{\Gamma(a+1)}{\Gamma(a)} \) with \( \text{Re}(a) > 0 \). Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

5. Multivariable I-function

If \( \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_r \rightarrow 1 \), the Aleph-function of several variables degenerates to the I-function of several variables. The simple integral have been derived in this section for multivariable I-functions defined by Sharma et al [6].

Corollary 1

\[
\int_0^{+\infty} \frac{x^{n-1}}{\sqrt{(x+z)^2 - x^2y^2}} \prod_{i=1}^{r} \frac{R_n^{\alpha,\beta} \left[ z X^A_{\gamma,\delta}; E, F, g, h; p, q; \gamma, \delta, e^{-s(zX^A_{\gamma,\delta})^T} \right]}{z^\alpha z^\mu - 1} \prod_{j=1}^{r} S_{N_j,\cdots,N_j}^{M_1,\cdots,M_j} \left( \prod_{k=1}^{m_1} \frac{y_j^{X}_{\chi,\mu}}{y_k^{X}_{\chi,\mu}} \right) \prod_{l=1}^{r} \frac{f_{U_j}^{N_j,\cdots,N_j} \left( Z_j^{X}_{\eta,\epsilon} \right)}{Z_j^{X}_{\eta,\epsilon} \eta^s} \right) dx = 2^{-\mu} z^{\alpha-\mu-1} \]

\[
\sum_{G_1,\cdots,G_r=0}^{g_1=0} \sum_{G_r=0}^{g_r=0} \sum_{K_0=0}^{\infty} \sum_{K_1=0}^{\infty} \frac{(-\delta)G_1+\cdots+G_r}{\delta g_1! \cdots \delta g_r G_r} \sum_{\eta G_1,\cdots,\eta G_r,\eta_0} \frac{y^{2n}}{n!} 
\]

\[
\psi(w, v, u, t, e, k_1, k_2) \left( x_1^{p_1} \cdots x_s^{p_s} \frac{\eta_1 g_1 \cdots \eta_r g_r}{\gamma_1 \cdots \gamma_r} z^{K_1} \cdots z^{K_r} R_{A_2}^{-(\delta RA+\sum_{i=1}^{r} K_i(\eta_i-\mu_i)+\sum_{i=1}^{r} \eta G_i, g_i(\alpha_i-\beta_i))} \sum_{i=1}^{r} \frac{Z_i^{\eta_i-\epsilon_i} 2^{-\epsilon_i}}{Z_s^{\eta_s-\epsilon_s} 2^{-\epsilon_s}} \right) 
\]

\[
(\alpha - \mu + \delta) \sum_{i=1}^{r} K_i(\gamma_i - \mu_i) + \sum_{i=1}^{r} \eta G_i, g_i(\alpha_i - \beta_i); \epsilon_1 - \eta_1, \cdots, \epsilon_s - \eta_s, \cdots 
\]

\[
(1-n^{\frac{1}{2}}(\alpha + RA + \sum_{i=1}^{r} K_i \gamma_i + \sum_{i=1}^{r} \eta G_i, g_i(\alpha_i); \frac{\eta_i}{2}, \cdots, \frac{\eta_k}{2}), 
\]

\[
(1-\frac{1}{2}(\alpha + RA + \sum_{i=1}^{r} K_i \gamma_i + \sum_{i=1}^{r} \eta G_i, g_i(\alpha_i); \frac{\eta_i}{2}, \cdots, \frac{\eta_k}{2}) 
\]
\[
\frac{1}{2}n^\frac{1}{2}(\alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta \alpha_i, \eta_1, \cdots, \eta_n), \quad \frac{1}{2}n^\frac{1}{2}(\alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta \alpha_i, \eta_1, \cdots, \eta_n),
\]

\[
(1 - \alpha - \gamma RA - \sum_{i=1}^{t} K_i \gamma_i - \sum_{i=1}^{r} \eta \alpha_i, \eta_1, \cdots, \eta_n, \alpha', \cdots, \alpha', \cdots, \alpha'), \quad C : \quad B : D
\]

\[
(1 - \alpha - \gamma RA - \sum_{i=1}^{t} K_i \gamma_i - \sum_{i=1}^{r} \eta \alpha_i, \eta_1, \cdots, \eta_n, \alpha', \cdots, \alpha'), \quad C : \quad B : D
\]

under the same notation and conditions that \( (\text{4.1}) \) with \( \epsilon_i, \epsilon_i(1), \cdots, \epsilon_i(s) \rightarrow 1 \).

6. Aleph-function of two variables

If \( s = 2 \), we obtain the Aleph-function of two variables defined by K. Sharma [8], and we have the following simple integrals.

**Corollary 2**

\[
\int_{0}^{\infty} \frac{x^{\alpha-1}}{\sqrt{(x+z)^2 - x^2y^2}} \left( x + y + \sqrt{(x+z)^2 - x^2y^2} \right)^n R^{\alpha, \beta}_{n, \eta} \left[ z' X_{\gamma, \delta}^{\alpha, \beta}, E, F, g, h, p, q; \gamma', \delta'; e^{-s(x' X_{\gamma, \delta}^{\alpha, \beta})} \right]
\]

\[
S^{M_1, \cdots, M_t}_{N_1, \cdots, N_t} \left( \begin{array}{c}
Y_1 X_{\gamma_1, \mu_1} \\
\vdots \\
Y_t X_{\gamma_t, \mu_t}
\end{array} \right) N^{0, n, \eta}_{u, v} \left( \begin{array}{c}
Z_1 X_{\gamma_1, \beta_1} \\
\vdots \\
Z_t X_{\gamma_t, \beta_t}
\end{array} \right) N^{0, N, \psi}_{u, v, \eta} \left( \begin{array}{c}
Z_1 X_{\eta_1, \epsilon_1} \\
\vdots \\
Z_2 X_{\eta_2, \epsilon_2}
\end{array} \right) dx = 2^{-\mu} e^{-\alpha - 1 - \frac{\text{one integral}}{n!}}
\]

\[
\sum_{G_1 = \text{one integral}}^{\infty} \sum_{G_r = \text{one integral}}^{\infty} \sum_{r = \text{one integral}}^{\infty} \sum_{k = \text{one integral}}^{\infty} \sum_{p = \text{one integral}}^{\infty} \sum_{q = \text{one integral}}^{\infty} (-G_1 \cdots G_r \delta_g_1 \cdots \delta_g_r G \eta_{\gamma_1, \beta_1}, \cdots, \eta_{\gamma_r, \beta_r}, \eta_{\gamma, \beta})
\]

\[
\psi(w, v, u, l, \epsilon, k_1, k_2) x_{1}^{\gamma_1} \cdots x_{s}^{\gamma_s} y_{1}^{\gamma_1} \cdots y_{t}^{\gamma_t} z' \left( \left( \delta_{RA} + \sum_{i=1}^{t} K_i \mu_i \right) + \sum_{i=1}^{r} \eta \alpha_i, \beta_i \right)
\]

\[
Z^{(\gamma - \delta) RA + \sum_{i=1}^{t} K_i (\gamma_i - \mu_i) + \sum_{i=1}^{r} \eta \alpha_i, \beta_i} N^{0, N+4, \psi}_{u, v, \eta} \left( \begin{array}{c}
Z_1 x_{1}^{\eta_1} - \epsilon_1, 2 - \epsilon_1 \\
\vdots \\
Z_s x_{s}^{\eta_s} - \epsilon_s, 2 - \epsilon_s
\end{array} \right)
\]

\[
(\alpha - \mu + (\gamma - \delta) RA + \sum_{i=1}^{t} K_i (\gamma_i - \mu_i) + \sum_{i=1}^{r} \eta \alpha_i, \beta_i) \epsilon_1 \eta_1, \epsilon_2 - \eta_2, \cdots
\]
(1-n^{-1/2}(\alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta G_i, g_i, \alpha_i) \mid \eta_{\frac{1}{2}}, \eta_{\frac{3}{2}})\),

(1-\frac{1}{2}(\alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta G_i, g_i, \alpha_i) \mid \eta_{\frac{1}{2}}, \eta_{\frac{3}{2}})\),

(1-\frac{1}{2}(\alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta G_i, g_i, \alpha_i) \mid \eta_{\frac{1}{2}}, \eta_{\frac{3}{2}})\),

\(\frac{1}{2} - \frac{1}{2}(\alpha + \gamma RA + \sum_{i=1}^{t} K_i \gamma_i + \sum_{i=1}^{r} \eta G_i, g_i, \alpha_i) \mid \eta_{\frac{1}{2}}, \eta_{\frac{3}{2}})\),

(1-\alpha - \gamma RA - \sum_{i=1}^{t} K_i \gamma_i - \sum_{i=1}^{r} \eta G_i, g_i, \alpha_i; \eta_1, \eta_2), A' : C\) \(\quad \ldots \quad (6.1)\)

(-n^{-1} - \delta RA - \sum_{i=1}^{t} K_i \mu_i - \sum_{i=1}^{r} \eta G_i, g_i, \beta_i; \epsilon_1, \epsilon_2), B : D\)

under the same notation and conditions that (4.1) with \(s = 2\)

7. I-function of two variables

If \(t_i, t_i', t_i'' \rightarrow 1\), then the Aleph-function of two variables degenerates in the I-function of two variables defined by Sharma et al [7] and we obtain the same formula with the I-function of two variables.

Corollary 3

\[
\int_{0}^{+\infty} \frac{x^{\alpha-1}}{\sqrt{(x + z)^2 - x^2} \sqrt{y^2 (x + y + \sqrt{(x + z)^2 - x^2 y})}} \mu \rho^{\alpha, \beta} \left[ z X_{g_1, g_1}^A, E, F, g, h, p, q, r, \delta', e^{-s(z X_{g_1, g_1})^r} \right] \]

\[
S_{N_1, \ldots, N_t}^{M_1, \ldots, M_t} \left( \begin{array}{c}
y_1 X_{\alpha_1, \mu_1} \\
\ldots \\
y_t X_{\alpha_t, \mu_t}
\end{array} \right) \eta_{\frac{0}{w}, \frac{V}{w}} \left( \begin{array}{c}
z_1 X_{\alpha_1, \beta_1} \\
\ldots \\
z_t X_{\alpha_t, \beta_t}
\end{array} \right) \iota_{U,V,W}^{\eta_1, \eta_2} \left( \begin{array}{c}
Z_1 X_{\eta_1, \epsilon_1} \\
\ldots \\
Z_t X_{\eta_t, \epsilon_t}
\end{array} \right)
\]

\[
\int_{0}^{+\infty} \frac{x^{\alpha-1}}{\sqrt{(x + z)^2 - x^2} \sqrt{y^2 (x + y + \sqrt{(x + z)^2 - x^2 y})}} \mu \rho^{\alpha, \beta} \left[ z X_{g_1, g_1}^A, E, F, g, h, p, q, r, \delta', e^{-s(z X_{g_1, g_1})^r} \right]
\]

\[
= \sum_{G_1, \ldots, G_t = 0}^{\infty} \sum_{g_1 = 0}^{\infty} \cdots \sum_{g_t = 0}^{\infty} \sum_{\eta_1 = 0}^{\infty} \cdots \sum_{\eta_t = 0}^{\infty} \frac{(-)^{G_1 + \cdots + G_t}}{\delta_{g_1}! \cdots \delta_{g_t}!} G_{G_1, \eta_1, \ldots, G_t, \eta_t} \delta x_{\eta_1, \eta_2, \ldots, \eta_t}
\]

\[
\psi(w, v, u, t, e, k_1, k_2) x_1^{p_1} \cdots x_t^{p_t} z_1^{\eta_1, g_1, \ldots, \eta_t, g_t} y_1^{K_1} \cdots y_t^{K_t} z^{2(\gamma - \delta)RA + \sum_{i=1}^{t} K_i (\gamma_i - \mu_i) + \sum_{i=1}^{r} \eta G_i, g_i, \alpha_i - \beta_i) \mid \eta_{\frac{1}{2}}, \eta_{\frac{3}{2}}) \]

\[
\begin{array}{c}
Z_1 z^{\eta_1 - \epsilon_1} \\
\ldots \\
Z_t z^{\eta_t - \epsilon_t} \\
\end{array} \]

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\[(\alpha - \mu + (\gamma - \delta)R A + \sum_{i=1}^{t} K_{i}(\gamma_{i} - \mu_{i}) + \sum_{i=1}^{r} \eta_{G_{i}, g_{i}}(\alpha_{i} - \beta_{i}); \epsilon_{1} - \eta_{1}, \epsilon_{2} - \eta_{2}), \]

\[ (1-\eta)^{-1}2(\alpha + \gamma R A + \sum_{i=1}^{t} K_{i} \gamma_{i} \sum_{i=1}^{r} \eta_{G_{i}, g_{i}}(\alpha_{i}); \frac{\eta_{1}}{2}, \frac{\eta_{2}}{2}), \]

\[ (1-\frac{1}{2}(\alpha + \gamma R A + \sum_{i=1}^{t} K_{i} \gamma_{i} + \sum_{i=1}^{r} \eta_{G_{i}, g_{i}}(\alpha_{i}); \frac{\eta_{1}}{2}, \frac{\eta_{2}}{2}), \]

\[ (\frac{1}{2}-\frac{1}{2}(\alpha + \gamma R A + \sum_{i=1}^{t} K_{i} \gamma_{i} + \sum_{i=1}^{r} \eta_{G_{i}, g_{i}}(\alpha_{i}); \frac{\eta_{1}}{2}, \frac{\eta_{2}}{2}), \]

\[ (1-\alpha - \gamma R A - \sum_{i=1}^{t} K_{i} \gamma_{i} - \sum_{i=1}^{r} \eta_{G_{i}, g_{i}}(\alpha_{i}; \eta_{1}, \eta_{2}), A^{1} : C \]

\[ (-\mu - \delta R A - \sum_{i=1}^{t} K_{i} \mu_{i} - \sum_{i=1}^{r} \eta_{G_{i}, g_{i}}(\beta_{i}; \epsilon_{1}, \epsilon_{2}), B : D \) \]

under the same notation and conditions that (4.1) with \( s = 2 \) and \( \iota_{i}, \iota_{i}', \iota_{i}'' \to 1 \)

8. Conclusion

In this paper we have evaluated a unified generalized infinite integral involving the multivariable Aleph-functions, a class of polynomials of several variables and the sequence of functions and general arguments. The integral established in this paper is of very general nature as it contains Multivariable Aleph-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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