On $\alpha sg$ closed sets in Topological spaces

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Abstract

In this paper we introduce the new concept of $\alpha sg$ closed sets in topological spaces and a basic properties of $\alpha sg$-closed sets were obtained.

Mathematics Subject Classification: 54A05

Keywords: $\alpha sg$-closed sets and $\alpha sg$-open sets.

1 Introduction


The aim of this paper is to introduce the new type of closed set called $\alpha sg$ closed set and to continue the study of $\alpha sg$-closed sets thereby contributing new innovation and concept, in the field of topology through analytical as well as research works. The notion of $\alpha sg$-closed sets and its different characterizations are given in this paper.

2 Preliminaries

A subset $A$ of a topological space $X$ is said to be open if $A \in \tau$. A subset $A$ of a topological space $X$ is said to be closed if the set $X - A$ is open. The interior of a subset $A$ of a topological space $X$ is defined as the union of all open sets contained in $A$. It is denoted by $int(A)$. The closure of a
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A subset $A$ of a topological space $X$ is defined as the intersection of all closed sets containing $A$. It is denoted by $cl(A)$.

Throughout this paper $(X, \tau)$ represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let $A \subseteq X$, the closure of $A$ and interior of $A$ will be denoted by $cl(A)$ and $int(A)$ respectively.

Definitions 2.1.

1. A subset $A$ of a space $(X, \tau)$ is said to be **semi open** [7] if $A \subseteq cl(int(A))$ and **semi closed** if $int(cl(A)) \subseteq A$.

2. A subset $A$ of a space $(X, \tau)$ is said to be **α-open** [14] if $A \subseteq int(cl(int(A)))$ and **α-closed** if $cl(int(cl(A))) \subseteq A$.

3. A subset $A$ of a space $(X, \tau)$ is said to be **semi pre-open** [1] if $A \subseteq cl(int(cl(A)))$ and **semi pre-closed** if $int(cl(int(A))) \subseteq A$.

4. A subset $A$ of a space $(X, \tau)$ is said to be **regular-open** [17] if $A = int(cl(A))$ and **regular-closed** if $A = cl(int(A))$.

5. A subset $A$ of a space $(X, \tau)$ is said to be **pre-open** [12] if $A \subseteq int(cl(A))$ and **pre-closed** if $cl(int(A)) \subseteq A$.

The complement of a semi-open (resp.pre-open, α-open) set is called **semi-closed** (resp.pre-closed, α-closed). The intersection of all semi-closed (resp.pre-closed, α-closed) sets containing $A$ is called the **semi-closure** (resp.pre-closure, α-closure) of $A$ and is denoted by $scl(A)$ (resp. $pcl(A)$, $\alpha-cl(A)$) . The union of all semi-open (resp.pre-open, α-open) sets contained in $A$ is called the **semi-interior** (resp.pre-interior, α-interior) of $A$ and is denoted by $sint(A)$ (resp. $pint(A)$, $\alpha-int(A)$). The family of all semi-open (resp.pre-open, α-open) sets is denoted by $SO(X)$ (resp. $PO(X)$, $\alpha-O(X)$). The family of all semi-closed (resp.pre-closed, α-closed) sets is denoted by $Scl(X)$ (resp. $Pcl(X)$, $\alpha-Cl(X)$).

Definitions 2.2.

1. A subset $A$ of a space $(X, \tau)$ is called **generalized-closed set** [8] (briefly $g$-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

2. A subset $A$ of a space $(X, \tau)$ is called **generalized semi-closed set** [2] (briefly $gs$-closed set) if $scl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$. 

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3. A subset $A$ of a space $(X, \tau)$ is called semi-generalized closed set [3] (briefly $sg$-closed set) if $scl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is semi-open in $(X, \tau)$.

4. A subset $A$ of a space $(X, \tau)$ is called $\alpha$ generalized-closed set [10] (briefly $\alpha g$-closed) if $\alpha(cl(A)) \subseteq U$, whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

5. A subset $A$ of a space $(X, \tau)$ is called generalized $\alpha$-closed set [9] (briefly $go$-closed) if $\alpha(cl(A)) \subseteq U$, whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X, \tau)$.

6. A subset $A$ of a space $(X, \tau)$ is called generalized pre-closed set [11] (briefly $gp$-closed) if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

7. A subset $A$ of a space $(X, \tau)$ is called generalized semi-pre closed set [4] (briefly $gsp$-closed) if $spcl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

8. A subset $A$ of a space $(X, \tau)$ is called semi weekly generalized-closed set [13] (briefly $swg$-closed) if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and $U$ is semi-open in $(X, \tau)$.

9. A subset $A$ of a space $(X, \tau)$ is called star generalized-closed set [20] (briefly *g-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $g$-open in $(X, \tau)$.

10. A subset $A$ of a space $(X, \tau)$ is called weekly-closed set [18] (briefly $w$-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is semi-open in $(X, \tau)$.

11. A subset $A$ of a space $(X, \tau)$ is called generalized-closed set [19] (briefly $\hat{g}$-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is semi-open in $(X, \tau)$.

12. A subset $A$ of a space $(X, \tau)$ is called weekly generalized-closed set [13] (briefly $wg$-closed) if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.

13. A subset $A$ of a space $(X, \tau)$ is called $\pi$ generalized-closed set [5] (briefly $\pi g$-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $\pi$-open in $(X, \tau)$.

14. A subset $A$ of a space $(X, \tau)$ is called $\pi$ generalized $\alpha$-closed set [6] (briefly $\pi ga$-closed) if $acl(A) \subseteq U$, whenever $A \subseteq U$ and $U$ is $\pi$-open in $(X, \tau)$.
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15. A subset $A$ of a space $(X, \tau)$ is called a **generalized-closed set** [16] (briefly $mg$-closed) if $cl\left(int\left(A\right)\right) \subseteq U$, whenever $A \subseteq U$ and $U$ is $g$-open in $(X, \tau)$.

16. A subset $A$ of a space $(X, \tau)$ is called a **generalized-closed set** [15] (briefly $\tilde{g}$-closed) if $cl\left(A\right) \subseteq U$, whenever $A \subseteq U$ and $U$ is $\tilde{g}$-open in $(X, \tau)$.

3 $\alpha sg$-Closed sets in Topological Spaces

In this section the notion of a new class of sets called $\alpha sg$-closed sets in topological spaces is introduced and their properties were studied.

**Definition 3.1** A subset $A$ of space $(X, \tau)$ is called **$\alpha sg$-closed** if $int\left(scl\left(A\right)\right) \subseteq U$, whenever $A \subseteq U$ and $U$ is $\alpha$-open in $X$.

The family of all $\alpha sg$-closed subsets of the space $X$ is denoted by $\alpha SGC(X)$.

**Definition 3.2** The intersection of all $\alpha sg$-closed sets containing a set $A$ is called **$\alpha sg$-closure** of $A$ and is denoted by $\alpha sg\text{-}cl(A)$.

A set $A$ is $\alpha sg$-closed set if and only if $\alpha sg\text{-}cl(A) = A$.

**Definition 3.3** A subset $A$ in $X$ is called **$\alpha sg$-open** in $X$ if $A^c$ is $\alpha sg$-closed in $X$.

The family of a $\alpha sg$-open sets is denoted by $\alpha SGO(X)$.

**Definition 3.4** The union of all $\alpha sg$-open sets containing a set $A$ is called **$\alpha sg$-interior** of $A$ and is denoted by $\alpha sg\text{-}Int(A)$.

A set $A$ is $\alpha sg$-open set if and only if $\alpha sg\text{-}Int(A) = A$.

**Theorem 3.5** Every closed set is a $\alpha sg$-closed set.

**Proof:** Let $A$ be a closed set in $X$. Such that $A \subseteq U$, $U$ is $\alpha$-open. Since $A$ is closed, $cl\left(A\right) = A$. For every subset $A$ of $X$, $int\left(scl\left(A\right)\right) \subseteq cl\left(A\right) = A \subseteq U$ and so we have $int\left(scl\left(A\right)\right) \subseteq U$. Hence $A$ is $\alpha sg$-closed.

**Remark 3.6** The converse of the above theorem need not be true as seen from the following example.

**Example 3.7** Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{a\}$ is $\alpha sg$-closed but not a closed set of $(X, \tau)$.

**Theorem 3.8** Every $p$-closed set is a $\alpha sg$-closed set.
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**Proof:** Let $A$ be a $p$-closed set in $X$. Such that $A \subseteq U$, $U$ is $\alpha$-open. Since $A$ is $p$-closed, $pcl (A) = A$. For every subset $A$ of $X$, $int (scl (A)) \subseteq pcl (A) = A \subseteq U$ and so we have $int (scl (A)) \subseteq U$. Hence $A$ is αsg-closed.

**Remark 3.9** The converse of the above theorem need not be true as seen from the following example.

**Example 3.10** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a\}$ is αsg-closed but not a $p$-closed set of $(X, \tau)$.

**Theorem 3.11** Every $\alpha$ closed set is a αsg-closed set.

**Proof:** Let $A$ be a $\alpha$-closed set in $X$. Such that $A \subseteq U$, $U$ is $\alpha$-open. Since $A$ is $\alpha$-closed, $\alpha cl (A) \subseteq A$. For every subset $A$ of $X$, $int (scl (A)) \subseteq \alphacl (A) \subseteq A \subseteq U$ and so we have $int (scl (A)) \subseteq U$. Hence $A$ is αsg-closed.

**Remark 3.12** The converse of the above theorem need not be true as seen from the following example.

**Example 3.13** Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{b\}$ is αsg-closed but not $\alpha$ closed set of $(X, \tau)$.

**Theorem 3.14** Every $r$-closed set is a αsg-closed set.

**Proof:** Let $A$ be a $r$-closed set in $X$. Such that $A \subseteq U$, $U$ is $\alpha$-open. Since $A$ is $r$-closed, $rcl (A) \subseteq A$. For every subset $A$ of $X$, $int (scl (A)) \subseteq rcl (A) \subseteq A \subseteq U$ and so we have $int (scl (A)) \subseteq U$. Hence $A$ is αsg-closed.

**Remark 3.15** The converse of the above theorem need not be true as seen from the following example.

**Example 3.16** Let $X = \{a, b, c, d, e\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{c, d\}, \{a, b, c, d\}\}$. Then $A = \{b, c, e\}$ is αsg-closed but not a $r$-closed set of $(X, \tau)$.

**Theorem 3.17** Every $g\alpha$ closed set is a αsg-closed set.

**Proof:** Let $A$ be a $g\alpha$-closed set in $X$. Such that $A \subseteq U$, $U$ is $\alpha$-open. Since $A$ is $g\alpha$-closed, $gcl (A) \subseteq A$. For every subset $A$ of $X$, $int (scl (A)) \subseteq gcl (A) \subseteq A \subseteq U$ and so we have $int (scl (A)) \subseteq U$. Hence $A$ is αsg-closed.

**Remark 3.18** The converse of the above theorem need not be true as seen from the following example.

**Example 3.19** Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{b\}$ is αsg-closed but not $g\alpha$ closed set of $(X, \tau)$.
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**Theorem 3.20** Every g closed set is a αsg-closed set.

**Proof:** Let $A$ be a $g$-closed set in $X$. Such that $A \subseteq U$, $U$ is $\alpha$-open. Since $A$ is $g$-closed, $\text{cl}(A) \subseteq A$. For every subset $A$ of $X$, $\text{int}(\text{scl}(A)) \subseteq \text{cl}(A) \subseteq A \subseteq U$ and so we have $\text{int}(\text{scl}(A)) \subseteq U$. Hence $A$ is αsg-closed.

**Remark 3.21** The converse of the above theorem need not be true as seen from the following example.

**Example 3.22** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a, c\}$ is αsg-closed but not $g$-closed set of $(X, \tau)$.

**Theorem 3.23** Every $\alpha g$ closed set is a αsg-closed set.

**Proof:** Let $A$ be a $\alpha g$-closed set in $X$. Such that $A \subseteq U$, $U$ is $\alpha$-open. Since $A$ is $\alpha g$-closed, $\alpha\text{cl}(A) \subseteq A$. For every subset $A$ of $X$, $\text{int}(\text{scl}(A)) \subseteq \alpha\text{cl}(A) \subseteq A \subseteq U$ and we have $\text{int}(\text{scl}(A)) \subseteq U$. Hence $A$ is αsg-closed.

**Remark 3.24** The converse of the above theorem need not be true as seen from the following example.

**Example 3.25** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a, d\}$ is αsg-closed but not $\alpha g$ closed set of $(X, \tau)$.

**Theorem 3.26** Every $^*g$-closed set is a αsg-closed set.

**Proof:** Let $A$ be a $^*g$-closed set in $X$. Such that $A \subseteq U$, $U$ is $\alpha$-open. Since $A$ is $^*g$-closed, $\text{cl}(A) \subseteq A$. For every subset $A$ of $X$, $\text{int}(\text{scl}(A)) \subseteq \text{cl}(A) \subseteq A \subseteq U$ and so we have $\text{int}(\text{scl}(A)) \subseteq U$. Hence $A$ is αsg-closed.

**Remark 3.27** The converse of the above theorem need not be true as seen from the following example.

**Example 3.28** Let $X = \{a, b, c\}$ with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $A = \{a\}$ is αsg-closed but not a $^*g$-closed set of $(X, \tau)$.

**Theorem 3.29** Every $w$-closed set is a αsg-closed set.

**Proof:** Let $A$ be a $w$-closed set in $X$. Such that $A \subseteq U$, $U$ is $\alpha$-open. Since $A$ is $w$-closed, $\text{cl}(A) \subseteq A$. For every subset $A$ of $X$, $\text{int}(\text{scl}(A)) \subseteq \text{cl}(A) \subseteq A \subseteq U$ and so we have $\text{int}(\text{scl}(A)) \subseteq U$. Hence $A$ is αsg-closed.

**Remark 3.30** The converse of the above theorem need not be true as seen from the following example.
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**Example 3.31** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a, c\}$ is αsg-closed but not a w-closed set of $(X, \tau)$.

**Theorem 3.32** Every swg-closed set is a αsg-closed set.

**Proof:** Let $A$ be a swg-closed set in $X$. Such that $A \subseteq U$, $U$ is α-open. Since $A$ is swg-closed, $\text{cl}(\text{int}(A)) \subseteq A$. For every subset $A$ of $X$, $\text{int}(\text{scl}(A)) \subseteq \text{cl}(\text{int}(A)) \subseteq A \subseteq U$ and so we have $\text{int}(\text{scl}(A)) \subseteq U$. Hence $A$ is αsg-closed.

**Remark 3.33** The converse of the above theorem need not be true as seen from the following example.

**Example 3.34** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $A = \{a\}$ is αsg-closed but not a swg-closed set of $(X, \tau)$.

**Theorem 3.35** The union of two αsg-closed subsets of $X$ is also αsg-closed set in $X$.

**Proof:** Assume that $P$ and $Q$ are p#g-closed set in $X$. Let $P \cup Q \subseteq U$ and $U$ be α-open in $X$. Since $P \subseteq U$ and $Q \subseteq U$, $U$ is α-open. Then $\text{int}(\text{scl}(P)) \subseteq U$ and $\text{int}(\text{scl}(Q)) \subseteq U$ and we have $\text{int}(\text{scl}(P \cup Q)) \subseteq \text{int}(\text{scl}(P)) \cup \text{int}(\text{scl}(Q)) \subseteq U$. Since $U$ is α-open. Hence $P \cup Q$ is αsg-closed set in $X$.

**Remark 3.36** The intersection of two αsg-closed sets in $X$ is generally not αsg-closed set in $X$.

**Example 3.37** Let $X = \{a, b, c, d\}$ with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. If $P = \{b, c\}$ and $Q = \{b, d\}$, then $P$ and $Q$ are αsg-closed sets in $X$, but $P \cup Q = \{b\}$ is not a αsg-closed set of $X$.

**Theorem 3.38** Every ġ-closed set is a αsg-closed set.

Proof follows from the definition, since every α-open set is semi-open.

**Example 3.39** In example (3.10), the set $\{b, c\}$ is αsg-closed but not a ġ-closed set of $(X, \tau)$.

**Theorem 3.40** Every wg-closed set is a αsg-closed set.

Proof follows from the definition, since every α-open set is open.

**Example 3.41** In example (3.10), the set $\{a, d\}$ is αsg-closed but not a wg-closed set of $(X, \tau)$. 

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**Theorem 3.42** Every $\pi g$-closed set is a $\alpha sg$-closed set.

*Proof* follows from the definition, since every $\alpha$-open set is $\pi$-open.

**Example 3.43** In example (3.13), the set $\{b\}$ is $\alpha sg$-closed but not a $\pi g$-closed set of $(X, \tau)$.

**Theorem 3.44** Every $\pi g\alpha$-closed set is a $\alpha sg$-closed set.

*Proof* follows from the definition, since every $\alpha$-open set is $\pi$-open.

**Example 3.45** In example (3.13), the set $\{a\}$ is $\alpha sg$-closed but not a $\pi g\alpha$-closed set of $(X, \tau)$.

**Theorem 3.46** Every $mg$-closed set is a $\alpha sg$-closed set.

*Proof* follows from the definition, since every $\alpha$-open set is $g$-open.

**Example 3.47** In example (3.16), the set $\{b, c, e\}$ is $\alpha sg$-closed but not a $mg$-closed set of $(X, \tau)$.

**Theorem 3.48** Every $gp$-closed set is a $\alpha sg$-closed set.

*Proof* follows from the definition, since every $\alpha$-open set is open.

**Example 3.49** In example (3.10), the set $\{b\}$ is $\alpha sg$-closed but not a $gp$-closed set of $(X, \tau)$.

So the class of $\alpha sg$-closed sets properly contain the class of $\tilde{g}$-closed set, $wg$-closed set, $\pi g\alpha$-closed set, $\pi g$-closed set, $gp$-closed set and $mg$-closed sets.

**Remark 3.50** The concept of $\alpha sg$-closed set is independent of the following classes of sets namely $gs$-closed set and $\tilde{g}$-closed set.

**Example 3.51** Consider the topological space $X = \{a, b, c, d, e\}$, with topology $\tau = \{X, \phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$. In this space, the set $\{a, b\}$ is $\alpha sg$-closed set but not $gs$-closed set and the set $\{a, c\}$ is $gs$-closed set but not $\alpha sg$-closed set.

**Example 3.52** Consider the topological space $X = \{a, b, c, d\}$, with topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b, c\}, \{a, b, d\}\}$. In this space, the set $\{a\}$ is $\alpha sg$-closed set but not $\tilde{g}$-closed set and the set $\{a, b\}$ is $\tilde{g}$-closed set but not $\alpha sg$-closed set.

**Remark 3.53** Figure 3.1 gives the implication relations of $\alpha sg$-closed sets based on the above results.
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Figure 3.1 Implication of $\alpha$sg-closed set

Where

$\rightarrow$ $\sec B$ represents A implies B
$\leftarrow$ $\sec A$ B represents A does not implies B
$\leftrightarrow$ $\sec A$ $\sec B$ represents B does not implies A

Theorem 3.54 For $x \in X$, the set $X - \{x\}$ is $\alpha$sg-closed or $\alpha$-open.

Proof: Suppose $X - \{x\}$ is not $\alpha$-open. Then $X$ is the only $\alpha$-open set containing $X - \{x\}$. $\Rightarrow \text{int}(\text{scl}(X - \{x\})) \subseteq X$. Then $X - \{x\}$ is $\alpha$sg-closed in $X$

Theorem 3.55 Let $A \subseteq Y \subseteq X$ and suppose that $A$ is $\alpha$sg-closed in $X$, then $A$ is $\alpha$sg-closed relative to $Y$.

Proof: Given that $A \subseteq Y \subseteq X$ and $A$ is $\alpha$sg-closed in $X$. To show that $A$ is $\alpha$sg-closed relative to $Y$. where $U$ is $\alpha$-open in $X$. Since $A$ is $\alpha$sg-closed, $A \subseteq U$, implies that $\text{int}(\text{scl}(A)) \subseteq U$, It follows that $Y \cap \text{int}(\text{scl}(A)) \subseteq Y \cap U$. Thus $A$ is $\alpha$sg-closed relative to $Y$.

Theorem 3.56 If $A$ is $\alpha$sg-closed and $A \subseteq B \subseteq \text{int}(\text{scl}(A))$. Then $B$ is $\alpha$sg-closed.

Proof: Let $U$ be a $\alpha$-open set of $X$, such that $B \subseteq U$. Then $A \subseteq U$ and since $A$ is $\alpha$sg-closed, we have, $\text{int}(\text{scl}(A)) \subseteq U$ Now, $\text{int}(\text{scl}(B)) \subseteq \text{int}(\text{scl}(\text{int}(\text{scl}(A)))) = \text{int}(\text{scl}(A)) \subseteq U$ Hence $B$ is $\alpha$sg-closed set.

Theorem 3.57 If a subset $A$ of $(X, \tau)$ is $\alpha$-open and $\alpha$sg-closed, then $A$ is semi-closed in $(X, \tau)$. 

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**Proof:** If a subset $A$ of $(X, \tau)$ is $\alpha$-open and $\alpha$sg-closed. Then $\text{int}(\text{scl}(A)) \subseteq U \subseteq A$. Hence $A$ is semi-closed in $(X, \tau)$.

**Theorem 3.58** If a set $A$ is $\alpha$sg-closed, then $\text{int}(\text{scl}(A)) - A$ contains no non-empty $\alpha$-closed set.

**Proof:** Let $F$ be a non-empty $\alpha$-closed set such that $F \subseteq \text{int}(\text{scl}(A)) - A$, then $F \subseteq \text{int}(\text{scl}(A))$ and $A \subseteq X - F$, we have $\text{int}(\text{scl}(A)) \subseteq \text{int}(X - F)$. \[\Rightarrow \text{int}(\text{scl}(A)) \subseteq X - \text{cl}(A) \Rightarrow \text{cl}(A) \subseteq X - \text{int}(\text{scl}(A)).\] Therefore $F \subseteq \text{int}(\text{scl}(A)) \cap (X - \text{int}(\text{scl}(A))) = \emptyset$. Hence $\text{int}(\text{scl}(A)) - A$ contains no non-empty $\alpha$-closed set.

**Theorem 3.59** Let $A$ be $\alpha$-closed in $(X, \tau)$, then $A$ is semi-closed iff $\text{int}(\text{scl}(A)) - A$ is $\alpha$-closed.

**Proof:** Necessity: Let $A$ be semi-closed, then $\text{scl}(A) = A$. Hence $\text{int}(\text{scl}(A)) - A = \{\phi\}$. Which is $\alpha$-closed.

Sufficiency: Suppose $\text{int}(\text{scl}(A)) - A$ is $\alpha$-closed. Since $A$ is $\alpha$sg-closed by theorem(), $\text{int}(\text{scl}(A)) - A = \{\phi\}$. Then $\text{int}(\text{scl}(A)) = A$. This means that $A$ is semi-closed.

**References**


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