The Investigation of Exact Traveling Wave Solutions of the (2+1)-Dimensional Burger Equation using the Generalized Kudryashov Methods

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Abstract In this paper, using generalized kudryashov method, we present some new explicit formulas of exact traveling the (2+1)-Dimensional Burger equation. Three exact traveling wave solutions in terms exponential function are found from the investigation. It is shown that the generalized kudryashov method is a very effective and powerful mathematic tool for solving nonlinear evolution equations in mathematical physics and engineering.

Keywords — The generalized kudryashov method; the (2+1)-Dimensional Burger equation; traveling wave solutions.

1. INTRODUCTION

Most scientific problems and physical phenomena occur nonlinearly. Except in a limited number of these problems, finding the exact analytical solutions of such problems are rather difficult. Recently, many kinds of powerful methods have been proposed to find exact solutions of nonlinear partial differential equations, e.g., the homogeneous balance method [1], homotopy analysis method [2, 3], three-wave method [4], extended homoclinic test approach [5], the \((G'/G)\)-expansion method [6, 10] and the exp-function method [11-13] and so on. Hellal and Mehanna [14] used a semi-analytical method, that is the Adomian decomposition and the tanh method to handle the foam drainage equation. Roshid et al. [15-16] investigated solitary wave solutions for some nonlinear evolution equations via exp-function and Exp\((-\varphi(\xi))\)-expansion method. Demiray et al. [17] investigate exact solutions of nonlinear time fractional Klein–Gordon equation by using generalized Kudryashov. The modified simple equation method is performed in [18], multi-soliton solutions are found in [19].

In this paper, we will apply the the generalized Kudryashov method on the (2+1)-Dimensional Burger equation.

II. FUNDAMENTAL PROPERTIES OF THE GENERALIZED KUDRYASHOV METHOD

The basic properties of the generalized Kudryashov method is explained in this section. In order to apply the methods, we consider the nonlinear evolution equation of the following form:

\[
\Omega(u, u_t, u_x, u_{xx}, u_{xxx}, \cdots) = 0,
\]

where \(\Omega\) is a polynomial of the function \(u = u(x,t)\) and its partial derivatives with respect to the spatial variable \(x, y\) and the time variable \(t\). Let us combine the real variables \(x, y\) and \(t\) by a compound variable \(\eta\) as

\[
u(x, y, t) = u(\eta);
\]

\(\eta = \kappa x + \beta y + \omega t\) which convert the Eq.(1) into an ODE for \(u = u(\eta)\):

\[
\Psi(u, u_t, u_x, u_{xx}, \cdots) = 0. \tag{2}
\]

Step 1: Suppose that the solution of Eq. (2) has the following form:

\[
u(\eta) = \sum_{i=0}^{N}\sum_{j=0}^{M}A_i \rho^i(\eta)B_j \rho^j(\eta) \tag{3}
\]

where \(A_i \ (i = 0, 1, 2, \cdots, N)\) and \(A_j \ (j = 0, 1, 2, \cdots, M)\) are constants to be determined afterward such that \(A_n \neq 0\) and \(B_m \neq 0\), and \(\rho = \rho(\eta)\) satisfies the following auxiliary differential equation:

\[
\frac{d\rho(\eta)}{d\eta} = \rho'(\eta) - \rho(\eta). \tag{4}
\]

Equation Eq. (4) has the solution

\[
\rho(\eta) = \frac{1}{1 + C \exp(\eta)}, \tag{5}
\]

where \(C\) is a constant of integration.

Step 2: The positive integers \(N\) and \(M\) appearing in Eq. (3) can be determined by define the degree of \(u(\eta)\) as \(D(u(\eta)) = N - M\) which gives rise to the degree of other expression as follows:

\[
D\left(\frac{d^q u}{d\eta^q}\right) = N - M + q,
\]

\[
D\left((N - M)p + (N - M + q)s\right) = (N - M) p + (N - M + q)s,
\]

where \(p, q, s\) are integer numbers. In this regard, we can find the value of \(N\) and \(M\) in Eq. (3).
Step 3: Inserting Eq.(3) with Eq.(4) into Eq.(2) provides a polynomial in \( \rho(\eta) \). We equate the coefficients of all terms of the same powers of \( \rho(\eta) \) to zero, this procedure yields a system of equations which can be solved to find unknown parameters in the trial solution.

### III. THE (2+1)-DIMENSIONAL BURGER EQUATION

In this section, we apply the generalized Kudryashov method to construct the traveling wave solutions for the (2+1)-Dimensional Burger equation in the form:

\[
u_u - uu_u - u_{uu} = 0 \tag{6}
\]

Upon using the transformation

\[
u = \nu(\eta) ; \eta = kx + ly - wt
\]

where \( k, l \) and \( w \) are constants, eq. (6) is transferred to

\[(k^2 + l^2)\nu'' + k\nu' + uu' = 0 \tag{8}\]

Integrating eq. (8) with respect to \( \eta \), we have

\[(k^2 + l^2)\nu'' + k\nu' + uu' + wu + C = 0 \tag{9}\]

where the prime denotes differentiation with respect to \( \eta \). Taking homogeneous balance between the highest order derivative term \( \nu' \) and highest nonlinear term \( u' \) yields a set of algebraic equations about \( A_1, B_1, w \) as follows:

\[
\begin{align*}
ka_1^2 + 2i^2 A_1 B_1 + 2k^2 A_1 B_1 &= 0, \\
2wA_1 B_1 + 2kA_1 A_1 - 2k^2 A_1 B_1 + 4l^2 A_1 B_1 + 2k^2 A_1 B_1 &= 0, \\
2CB_1 - 4kl A_1 B_1 - 4l^2 A_1 B_1 - 2l^2 A_1 B_1 + 2wA_1 B_1 + 2k^2 A_1 B_1 &= 0, \\
+ 2wA_1 B_1 - 2kA_1 A_1 + 4l^2 A_1 B_1 &= 0, \\
2lA_1 B_1 + 2l^2 A_1 B_1 - 2kA_1 B_1 - 2wA_1 B_1 &= 0, \\
+ 2wA_1 B_1 + 2kA_1 A_1 + 4CB_1 B_1 &= 0, \\
kA_1^2 + 2CB_1 &= 0.
\end{align*}
\]

Then the solution of the system of equations, we achieve the solutions sets:

**Set-1:**

\[
C = \frac{A_1(8k^2 B_1 + 8l^2 B_1 + kA_1)}{8B_1^2}, \quad w = \frac{k^2 B_1 + l^2 B_1 - kA_1}{B_1},
\]

\[
A_1 = \frac{2(kA_1 + l^2 B_1 + k^2 B_1)}{k}, \quad A_j = \frac{4(k^2 + l^2)B_1}{k}, \quad B_1 = -2B_1, \quad A_1 = const., B_1 = const.
\]

**Set-2:**

\[
C = \frac{A_1(8k^2 B_1 + 8l^2 B_1 + kA_1)}{8B_1^2}, \quad w = \frac{4k^2 B_1 + 4l^2 B_1 + kA_1}{2B_1}, \quad A_1 = \frac{4B_1(k^2 + l^2)}{k}, \quad B_1 = -2B_1, \quad A_1 = const., B_1 = const.
\]

### IV. CONCLUSIONS

We successfully implemented the generalized Kudryashov method to construct the traveling wave solutions for the (2+1)-Dimensional Burger equation. In fact, we have presented three new solutions for the nonlinear Burger equation. Results of the current work illustrates that the generalized Kudryashov method is indeed powerful analytical technique for
most types of nonlinear problems and several such problems in scientific studies and engineering may be solved by this method.

REFERENCES


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