Mathematical Model for Unemployment Control-A Numerical Study

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Abstract: The paper presented a nonlinear mathematical model for unemployment using system of dynamic differential equations. This paper analyzed the situation of job competition between native unemployed and new migrant workers. We observed the effect of efforts for creating new vacancies made by government and private sector with delay and without delay as well as efforts of native unemployed and new migrant workers to become self-employed. We studied the stability of equilibrium points and carried out numerical simulation to compare with analytical result.

Key words: Employed persons, unemployed persons, self-employment, migration, dynamic variables, delay.

I. INTRODUCTION

Unemployment is one of the annoying problem of the world which is spread day by day. Many people leave their home town, their country and go to other country for getting job. Because of this, government realize more burden of unemployment of native unemployed and new migrant workers.

Nikolopoulos and Tzanetis ([6]) presented a model for a housing allocation of homeless families due to natural disaster. Using some concept of this paper, Misra and Singh ([1, 2]) developed a nonlinear mathematical model for unemployment. In ([2]) the model considered three dynamic variables number of unemployed persons, employed persons and newly created vacancies by government intervention. Inspired by this paper G.N.Pathan and P.H.Bhathawala ([7]) developed a mathematical model for unemployment with effect of self-employment. In ([4]) M. Neamtu presented a model for unemployment basedon some concept of ([2]) with adding two new variables said number of present jobs in the market and number of immigrants.

Based on concept of above models G.N.Pathan and P.H.Bhathawala ([9]) developed a new model of unemployment with four dynamic variables (i) Number of unemployed persons \( U(t) \), (ii) Number of new migrant workers \( M(t) \), (iii) Number of Employed persons \( E(t) \) and (iv) Number of newly created vacancies by government and private sector\( V(t) \). Using this concept we developed a new model with this four variables and analyzed the result with delay and without delay. We assumed that native unemployed and new migrant workers can apply for available vacancies and get chances equally. Therefore new migrant attracts to the territory and government realized more burden of native unemployed workers and migrant workers. So, government tried to take a step of creating new vacancies with the help of private sector. We consider the situation that native unemployed and migrant both try for their independent work and taking a step of self-employment to survive.

The paper is organized as follows: Section 2 describes Model for unemployment, Section 3 describes an equilibrium analysis, Section 4 describes the stability of equilibrium point, Numerical simulation describes in section 5 and Conclusion is given in section 6.

II. MATHEMATICAL MODEL

In the process of making a model we assume that all entrants of the category unemployment are fully qualified to do any job at any time \( t \). Number of unemployed persons \( U \), increases with constant rate \( a_1 \). The rate of movement from unemployed class to employed class is jointly proportional to \( U \) and \( (P+V-E) \). Where present jobs in the market provided by government and private sector is constant denoted by \( P \), Government and private sector try to create new vacancies denoted by \( V \) and number of employed persons denoted by \( E \). So, total available vacancies in the market are \( P+V-E \).

We assumed that job search is open for native unemployed as well as new migrant. So, new migrant also become part of the labor workforce of the territory denoted by \( M \). Number of migrant increases with constant rate \( m_1 \). The rate of movement of migrant workers in employment is jointly proportional to \( M \) and \( (P+V-E) \). Native unemployed and migrant both try for self-employment to survive which is proportional to number of unemployed and migrant with the rate \( a_5 \) and \( a_7 \) respectively. Employed persons joint unemployed class with rate \( a_3 \) because offered from the job or leave the job. The death and retirement rate of employed person is \( a_8 \). The death rate of
unemployed and migrant are \( a_3 \) and \( m_3 \) respectively. The rate of migrant who registered as unemployed or fired from their job is \( a_6 \).

\[
\frac{dU}{dt} = a_1 - a_1 U(P + V - E) - a_4 U + a_4 E - a_8 U + a_8 M \quad (1)
\]

\[
\frac{dM}{dt} = m_1 - m_2 M(P + V - E) - a_6 M - m_3 M - a_7 M \quad (2)
\]

\[
\frac{dE}{dt} = a_1 U(P + V - E) + m_2 M(P + V - E) - a_4 E
+ a_4 U + a_5 M - a_8 E \quad (3)
\]

\[
\frac{dV}{dt} = aU(t - \tau) + \beta M(t - \tau) - \delta V \quad (4)
\]

Here, \( \alpha \) and \( \beta \) is the rate of newly created vacancies by government and Private sector respectively and \( \delta \) is the diminution rate of newly created vacancies.

**Lemma:**

The set \( \Omega = \{(U, M, E, V) : 0 \leq U + M + E \leq \frac{a_1 + m_1}{\gamma} , 0 \leq \frac{(\alpha + \beta)(a_1 + m_1)}{\gamma \delta} \} \),

where \( \gamma = \min(a_1, m_1, a_8) \) is a region of attraction for the system \((1) - (4)\) and it attracts all solutions initiating in the interior of the positive octant.

**Proof:**

From equation \((1) - (3)\) we get,

\[
\frac{d}{dt}(U(t) + M(t) + E(t)) = a_1 + m_1 - a_4 U(t) - m_3 M(t) - a_8 E(t)
\]

Which gives

\[
\frac{d}{dt}(U(t) + M(t) + E(t)) \leq a_1 + m_1 - \gamma(U(t) + M(t) + E(t))
\]

Where \( \gamma = \min(a_1, m_1, a_8) \).

By taking limit supremum

\[
\lim_{t \to \infty} \sup(U(t) + M(t) + E(t)) \leq \frac{a_1 + m_1}{\gamma}
\]

from \((4)\) we have

\[
\frac{dV}{dt} = aU(t) + \beta M(t) - \delta V(t)
\]

\[
\therefore \frac{dV}{dt} \leq (\alpha + \beta)U(t) - \delta V(t)
\]

By taking limit supremum which leads to,

\[
\lim_{t \to \infty} \sup V(t) \leq \frac{(\alpha + \beta)(a_1 + m_1)}{\delta \gamma}
\]

This proves the lemma.

**III. EQUILIBRIUM ANALYSIS**

The model system \((1) - (4)\) has only one non-negative equilibrium point \( E_0(U^*, M^*, E^*, V^*) \) which obtained by solving the following set of algebraic equations.

\[
a_1 - a_2 U(P + V - E) - a_4 U + a_4 E - a_8 U + a_8 M = 0 \quad (5)
\]

\[
m_1 - m_2 M(P + V - E) - a_6 M - m_3 M - a_7 M = 0 \quad (6)
\]

\[
a_2 U(P + V - E) + m_2 M(P + V - E) - a_4 E + a_4 U + a_5 M - a_8 E = 0 \quad (7)
\]

\[
\alpha U + \beta M - \delta V = 0 \quad (8)
\]

Taking addition of equation \((5)\), \((6)\) and \((7)\)

\[
a_1 + m_1 - a_4 U - m_3 M - a_8 E = 0
\]

\[
\therefore E = \frac{a_1 + m_1 - a_4 U - m_3 M}{a_8}
\]

From equation \((8)\)

\[
V = \frac{aU + \beta M}{\delta} \quad (9)
\]

\[
\therefore P + V = \frac{aa_4 U + ba_8 M - (a_1 + m_3 - Pa_8)}{a_8} \quad (10)
\]

Where \( a = \frac{\alpha}{\delta} + \frac{a_3}{a_8} \), \( b = \frac{\beta}{\delta} + \frac{m_3}{a_8} \)

Put values of equations \((9)\) and \((11)\) in \((5)\) and \((6)\) we get,

\[
A_0 U^2 + A_1 UM - A_2 U - A_3 M - A_4 = 0 \quad (12)
\]

\[
B_0 M^2 + B_1 UM - B_2 M - B_3 = 0 \quad (13)
\]

Where,
\( A_0 = a a_1 a_8, \quad A_i = a_i a_8, \quad B_2 = [a_2 (a_1 m_1 - Pa_8) - a_3 (a_4 + a_8) - a_s a_8] \)

\( A_i = [a_i a_s - m_i a_4], \)

\( A_3 = [a_i a_8 + a_s (a_1 + m_1)], \)

\( B_0 = b a_2 m_2, \quad B_1 = a a_2 m_2, \)

\( B_2 = [m_2 (a_1 + m_1 - Pa_8) - a_8 (a_6 + m_3 + a_1)], \)

\( B_3 = m_1 a_8. \)

equation (12) and (13) represent the equation of hyperbolas.

from equation (12),

\[
M = \frac{A_3 + A_4 U - A_0 U^2}{A_0 U - A_1} \tag{14}
\]

put value of equation (14) in (13) we get,

\[
H_0 U^3 + H_1 U^2 - H_2 U - H_3 = 0 \tag{15}
\]

\[
1 + + - \quad \therefore
\]

Where

\[
H_0 = A_2 A_4 B_1 - A_0 (A_2 B_1 + A_4 B_2),
\]

\[
H_1 = A_2 B_2 A_4 - B_0 A_2^2 + A_4 (A_2 B_1 + A_4 B_2) + A_2 A_4 B_2 + B_3 A_1^2,
\]

\[
H_2 = 2 A_2 A_4 B_0 - A_4 (A_2 B_1 + A_4 B_2) + A_2 A_4 B_2 + 2 A_1 A_5 B_3,
\]

\[
H_3 = B_0 A_4 A_1 - A_1 A_2 B_2 - A_2 A_1^2 B_3.
\]

Since \( H_i, \quad i=1,2,3,4 \) all are positive and number of changes in signs of equation (15) is only one. By Descart's rule equation (15) has only one positive solution say \( U^* \). So, we get the non-negative equilibrium point of model with coordinates:

\[
M^* = \frac{A_3 + A_4 U^* - A_0 (U^*)^2}{A_0 U^* - A_1},
\]

\[
E^* = \frac{a_1 + m_1 - a_s U^* - m_s M^*}{a_8},
\]

\[
V^* = \frac{\alpha U^* + \beta M^*}{\delta}
\]

So, \( E_0 (U^*, M^*, E^*, V^*) \) is required non negative solution of the Model.

IV. STABILITY ANALYSIS

Stability of equilibrium point without any delay:

To check the local stability for \( \tau = 0 \) at equilibrium point \( E_0 (U^*, M^*, E^*, V^*) \), we calculate the variational matrix \( T \) of the model system (1) – (4), corresponding to \( E_0 (U^*, M^*, E^*, V^*) \):

\[
T = \begin{bmatrix}
C_{11} & a_6 & C_{13} & -p_3 \\
0 & C_{23} & p_4 & -p_4 \\
C_{31} & C_{32} & C_{33} & C_{34} \\
\alpha & \beta & 0 & -\delta
\end{bmatrix}
\]

Where

\[
p_1 = a_1 (P + V - E), \quad p_2 = m_2 (P + V - E),
\]

\[
p_3 = a_2 U, \quad p_4 = m_2 M,
\]

\[
C_{11} = -p_1 - a_1 - a_s, \quad C_{13} = p_1 + a_4,
\]

\[
C_{23} = -p_2 - a_6 - m_3 - a_7,
\]

\[
C_{31} = p_1 + a_5, \quad C_{32} = p_2 + a_5,
\]

\[
C_{33} = -p_3 - p_4 - a_4 - a_8,
\]

\[
C_{34} = p_3 + p_4.
\]

The characteristic equation of above matrix is

\[
\lambda^4 + d_1 \lambda^3 + d_2 \lambda^2 + d_3 \lambda + d_4 = 0
\]

\[\tag{16}\]

Where

\[
d_1 = \delta - C_{33} - C_{23} - C_{11},
\]

\[
d_2 = C_{11} (C_{23} + C_{33}) - \delta (C_{11} + C_{33} + C_{23}) + C_{23} C_{33} + p_4 (\beta - C_{32}) - C_{13} C_{31} + \alpha p_3,
\]

\[
d_3 = C_{33} \delta (C_{11} + C_{23}) + C_{23} C_{11} (\delta - C_{33}) - \beta p_4 (C_{11} + C_{34} + C_{34}) + C_{32} p_4 (C_{11} - \delta) + C_{13} C_{31} (C_{23} - \delta) + p_4 a_6 (\alpha - C_{31}) - \alpha (C_{13} C_{34} + p_3 C_{33} + p_3 C_{23})
\]

\[
d_4 = (C_{34} + C_{33}) (\beta p_4 C_{11} - \alpha a_6 p_4) + (p_3 - C_{13}) (\beta p_4 C_{31} - \alpha p_4 C_{32}) + C_{11} \delta (C_{32} p_4 - C_{23} C_{33}) + C_{31} \delta (C_{13} C_{34} - p_3 a_6) + \alpha C_{23} (C_{13} C_{34} + p_3 C_{33})
\]

Since, \( d_1, d_2, d_3, d_4 \) are positive then all coefficients of equation (16) are positive and some algebraic manipulation convey that \( d_1 d_2 > d_3 \) and \( d_1 d_2 d_3 > d_1^2 d_4 \). So, by Routh Hurwitz criteria all roots of equation (16) are negative or having a negative real part. Therefore equilibrium point...
\[ E_0 = (U^*, M^*, E^*, V^*) \]
is locally asymptotically stable.

**Stability of equilibrium point with delay:**

To check the local stability for \( \tau \neq 0 \) at
equilibrium point \[ E_0 = (U^*, M^*, E^*, V^*) \]
we calculate the variational matrix \( T_1 \) and \( T_2 \) of
the model system (1) - (4) corresponding to
\[ E_0 = (U^*, M^*, E^*, V^*). \]

\[
\frac{dx}{dt} = T_1 x(t) + T_2 x(t - \tau) \quad (17)
\]

Where \( x(t) = [u(t) \quad m(t) \quad e(t) \quad v(t)]^T \)

\( u(t), m(t), e(t) \) and \( v(t) \) are small perturbations
around the equilibrium point \( E_0. \)

\[ T_1 = \begin{bmatrix}
C_{11} & a_6 & C_{13} & -p_3 \\
0 & C_{23} & p_4 & -p_4 \\
C_{31} & C_{32} & C_{33} & C_{34} \\
0 & 0 & 0 & -\delta
\end{bmatrix}
\]

\[ T_2 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\alpha & \beta & 0 & 0
\end{bmatrix}
\]

Where \( p_1 = a_5 (P + V - E), \)
\( p_2 = m_2 (P + V - E), \)
\( p_3 = a_2 U, p_4 = m_2 M, \)
\( C_{11} = -p_1 - a_3 - a_5 \cdot C_{13} = p_3 + a_4, \)
\( C_{23} = -p_2 - a_6 - m_3 - a_7, \)
\( C_{31} = p_1 + a_5, C_{32} = p_2 + a_7, \)
\( C_{33} = -p_3 - p_4 - a_4 - a_8, \)
\( C_{34} = p_3 + p_4. \)

The characteristic equation of system (17) is
\[ \psi^4 + j_1 \psi^3 + j_2 \psi^2 + j_4 \psi + j_4 + (k_1 \psi^2 + k_2 \phi + k_3)e^{-i\tau} \psi = 0 \quad (18) \]

Where
\[ j_1 = \delta - C_{33} - C_{35} - C_{11}, \]
\[ j_2 = C_{11} (C_{33} + C_{23}) - \delta (C_{11} + C_{33} + C_{23}) + C_{23} C_{33} - p_2 C_{32} - C_{13} C_{31}, \]
\[ j_3 = C_{33} \delta (C_{11} + C_{23}) + C_{23} C_{11} (\delta - C_{33}) \]
\[ + C_{32} p_4 (C_{11} - \delta) + C_{13} C_{33} (C_{23} - \delta) - p_4 a_6 C_{31}, \]
\[ j_4 = C_1 \delta (C_{32} p_4 - C_{23} C_{33}) + C_1 \delta (C_{13} C_{23} - p_4 a_6), \]
\[ k_1 = p_4 \beta + p_3 \alpha, \]
\[ k_2 = \alpha (p_4 a_6 - (C_{13} C_{34} + p_3 C_{33} + p_3 C_{23})) - \beta p_4 (C_{11} + C_{34} + C_{33}), \]
\[ k_3 = (C_{34} + C_{33}) (p_4 \beta C_{11} - \alpha a_6 p_4) + (p_3 - C_{13}) (\beta p_4 C_{34} - \alpha p_2 C_{32}) + \alpha C_{23} (C_{11} C_{34} + p_3 C_{33}) \]

Now to check the stability of Eq. (18) we
should not directly use Routh-Hurwitz criterion.
We check that Hopf bifurcation occurs for that we have
to show that Eq. (18) has a pair of purely imaginary roots.

For this we substitute \( \psi = e^{i\omega t} \)
in Eq. (18) and we get
\[ \omega^4 - j_1 \psi^3 - j_2 \psi^2 + j_4 \psi + j_4 + (-k_1 \psi^2 + ik_2 \phi + k_3) e^{-i\tau} \psi = 0 \quad (19) \]
\[ .\quad \omega^4 - j_1 \psi^3 i - j_2 \psi^2 + i j_4 \psi + j_4 + (-k_1 \psi^2 + ik_2 \phi + k_3) (\cos \omega \tau - i \sin \omega \tau) = 0 \]
\[ .\quad \omega^4 - j_1 \psi^3 + j_4 = (k_1 \psi^2 - k_3) \cos \omega \tau - k_2 \cos \omega \tau \]
\[ .\quad i(-j_1 \psi^3 + j_4 w = (-k_1 \psi^2 + k_3) \sin \omega \tau + k_2 \cos \omega \tau) = 0 \quad (20) \]

Separating real and imaginary part of Eq. (20) we
get
\[ \omega^4 - j_2 \psi^2 + j_4 = (k_1 \psi^2 - k_3) \cos \omega \tau - k_2 \cos \omega \tau \]
\[ (21) \]
\[ j_1 \omega^3 - j_3 \psi = (k_1 \psi^2 - k_3) \sin \omega \tau + k_2 \psi \cos \omega \tau \]
\[ (22) \]

By squaring and adding Eq. (21) and Eq. (22)
\[ (\omega^4 - j_2 \psi^2 + j_4)^2 + (j_1 \omega^3 - j_3 \psi)^2 = (k_1 \psi^2 - k_3)^2 + k_2^2 \psi^2 \]
\[ (23) \]

By taking expansion of this
\[ \omega^8 + r_1 \omega^6 + r_2 \omega^4 + r_2 \omega^2 + r_4 = 0 \]
\[ (24) \]
\[ r_i = (j_1^2 - 2j_2), \]
\[ r_2 = (j_2^2 + 2j_3 - 2j_1j_3 - k_1^2), \]
\[ r_3 = (j_3^2 - 2j_1j_2 + 2k_1k_3 - k_2^2), \]
\[ r_4 = (j_4^2 - k_3^2) \]

Substituting \( \omega^2 = \sigma \) in above Eq. then we have

\[ f(\sigma) = \sigma^4 + e_1\sigma^3 + e_2\sigma^2 + e_3\sigma + e_4 = 0 \]

(25)

Where

\[ e_1 = (j_1^2 - 2j_2), \]
\[ e_2 = (j_2^2 + 2j_3 - 2j_1j_3 - k_1^2), \]
\[ e_3 = (j_3^2 - 2j_1j_2 + 2k_1k_3 - k_2^2), \]
\[ e_4 = (j_4^2 - k_3^2) \]

If all \( e_i > 0 \) \((i=1,2,3,4)\) and satisfies

Routh-Hurwitz criterion then there is no positive root of Eq. (25) i.e., all roots of Eq. (24) are negative or having a negative real part. So, by Routh-Hurwitz criterion equilibrium \( E_0 \) is asymptotically stable for all delay \( \tau > 0 \).

Contrary to all \( e_i \) does not satisfy the Routh-Hurwitz criterion then there is at least one positive root \( \omega_0 \) exist of Eq. (24) for \( e_4 < 0 \).

From this we get that \((j_4^2 - k_3^2) < 0\) since \( j_4 + k_3 > 0 \) so, \( j_4 - k_3 < 0 \). Which gives the condition for the existence of pair of purely imaginary roots \((\pm i\omega)\) of Eq. (19).

\[ j_4 - k_3 < 0 \]

i.e.

\[ \delta((p_2 + a_6 + m_1 + a_7)(p_1 + a_7) + a_7(p_7 + a_7 + a_7)) + \delta(p_7(m_7(p_7 + a_7) + a_7m_7 + m_7 + a_7)) - \delta(p_7a_7 + a_7m_7) - \alpha(p_7a_7 + a_7m_7 + a_7a_7) < 0 \]

(26)

From Eq. (21) and Eq. (22) we get

\[ \tan \omega \sigma = \frac{(k_1, \omega^2 - k_2)(j_1, \omega^3 - j_1, \omega)(k_3, \omega^4 - j_2, \omega^2 + j_4)}{k_2, \omega^2(\omega^2 - j_1, \omega)(\omega^3 - j_1, \omega)(\omega^4 - j_2, \omega^2 + j_4)} \]

For positive \( \omega_0 \) we have corresponding \( \omega_0 \) is given by

\[ \tau_n = n\pi + \frac{1}{\omega_0} \tan^{-1} \frac{(k_4, \omega^2 - k_5)(j_5, \omega^3 - j_5, \omega) - k_4, \omega(\omega^4 - j_5, \omega^2 + j_5)}{k_5, \omega^2(\omega^2 - j_5, \omega)(\omega^3 - j_5, \omega)(\omega^4 - j_5, \omega^2 + j_5)} \]

(27)

\[ n=0, 1, 2, 3, \ldots \]

By Butler’s lemma we can say that equilibrium \( E_0 \) remains stable for \( \tau < \tau_0 \).

Now to check that Hopf-bifurcation occurs at \( \tau_0 \) we have to check that \( \tau_0 \) satisfies the transversality condition.

**Lemma 2:** Transversality condition is

\[ \text{sgn} \left[ \frac{d(\text{Re}(\psi))}{d\tau} \right]_{\tau=\tau_0} > 0 \]

Proof: By differentiating Eq. (18) with respect to \( \tau \), we have

\[ \frac{d\psi}{d\tau} = 4\psi^3 + 3j_2\psi^2 + 2j_3\psi + j_4 + (2k_1\psi + k_2)\psi - \frac{\tau}{\psi(\psi^2 - k_1\psi^2 + k_2\psi + k_3)} \]

Now,

\[ \text{sgn} \left[ \frac{d(\text{Re}(\psi))}{d\tau} \right]_{\tau=\tau_0} = \text{sgn} \left[ \text{Re} \left( \frac{d\psi}{d\tau} \right) \right]_{\tau=\tau_0} \]

\[ = \text{sgn} \left[ 4\psi_0^3 + 3m_1\psi_0^2 + 2m_2\psi_0 + m_3 \right] \]

Here \( m_1 = (j_2^2 - 2j_2), \)
\[ m_2 = (2j_2 + j_3^2 - 2j_1j_3), \]
\[ m_3 = (j_3^2 + k_3^2 - 2j_1j_2 - 2k_1k_3) \]

Since condition (26) is satisfied then we have positive \( \omega_0 \) and for that Transversality condition is satisfies.

This shows that if condition (26) satisfies then equilibrium \( E_0 \) is asymptotically stable for \( \tau < \tau_0 \) (i.e. \( \tau \in (0, \tau_0) \)) and unstable for \( \tau > \tau_0 \). The condition of Hopf-bifurcation is satisfied so, periodic solution occurs when \( \tau \) passes to the \( \tau_0 \) for equilibrium \( E_0 \).

**V. NUMERICAL SIMULATION**

For the Numerical simulation using MATLAB 7.6.0 we consider the following data,
\[ a_1 = 5000, a_2 = 0.00004, a_3 = 0.04, a_4 = 0.004, a_5 = 0.03, a_6 = 0.1, \]
\[ a_7 = 0.01, a_8 = 0.07, m_1 = 3000, m_2 = 0.00002, m_3 = 0.05, \alpha = 0.2, \]
\[ \beta = 0.001, \delta = 0.08, P = 10000 \]

The equilibrium values of the model are:

\[ U^* = 31978, M^* = 13101, E^* = 86654, V^* = 80108 \]

The eigenvalues of the variational matrix corresponding to the equilibrium point \( E_0 = (U^*, M^*, E^*, V^*) \) of model system (1) - (4) are:

\[ -1.6004, -0.2318 + 0.0302i, -0.2318 - 0.0302i, -0.0683 \]

All eigenvalues are negative or having negative real part. So, equilibrium \( E_0 = (U^*, M^*, E^*, V^*) \) is locally asymptotically stable.

Using above parameter and equation (24) we get \( \omega = 0.1357 \). From equation (27) we get \( \tau = 13.66 \)

**Figure-1:**

**Figure-2:**

**Figure-3:**

**VI. CONCLUSION**

The paper presented a nonlinear mathematical model for unemployment using four dynamic variables: Number of unemployed persons, number of migrant workers, number of employed persons and number of newly created vacancies by government and private sector. It shows the theoretical calculation and compare it with numerical simulation using MATLAB 7.6.0.

Fig.1 shows that unemployment is lower for higher rate of persons who joined employed class. Similarly From Fig.2 it can be observe that unemployment is lower for higher self-employment rate. That is to control unemployment more and more people have to join employed class which is possible with efforts of government and private sector by creating new vacancies and also efforts of unemployed by create chances for self-employment. From Fig.3 we can observe that rate of unemployment of migrant workers goes lower as they joined employed class. That is government and private sector should create new vacancies for both native unemployed and migrant workers to control unemployment.

We observed that if territory allow new migrant workers then it should be create new vacancies proportional to native unemployed as well as migrant workers. We get the equilibrium point without any condition in absence of delay. In presence of delay equilibrium point is stable if it satisfies the eq. 26. Equilibrium point is unstable if it cross the critical value of delay (\( \tau \)) given by eq. 27.

**REFERENCES**


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