A Generalized Fuzzy Soft Set Based
Recruiting Method

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Abstract: Decision making is very important to deal with many complex problems in various sectors involving imprecise data and uncertainties. Generalized fuzzy soft matrix theory is a powerful tool for decision making. In this paper we defined some new types of generalized fuzzy soft matrices and gave a decision making method based on generalized fuzzy soft matrices.

Keywords — Decision making, Recruiting, Generalized Fuzzy Soft Matrix, Generalized Membership Value Matrix, Score Matrix.

1. INTRODUCTION

Many theories have been developed to deal with uncertainty in the real life problems. Some of these theories are fuzzy set [1], intuitionistic fuzzy sets [2], rough set [3], vague sets [4], interval mathematics [5] etc. These theories work to some extent but in some cases these theories do not give satisfactory results due to the insufficient parameterization tools. Zadeh [1] introduced fuzzy sets to deal with such type of problems. Molodtsov [6] pointed out the problem of inadequacy of the fuzzy set theory and gave soft set theory to overcome this problem. Maji [7] developed fuzzy soft set theory by combining soft sets and fuzzy sets. Naim Cagman introduced soft matrices and fuzzy soft matrices [8], [9]. Generalized fuzzy soft set theory was introduced by Pinaki Majumdar [10]. He applied it to a decision making problem and medical diagnosis problem. Hai Long YANG [11] pointed out some mistakes and corrected them. Pinaki Majumdar applied generalized fuzzy soft set to a student ranking problem [12]. B.K. Saikia et al. [13] defined generalized fuzzy soft matrix and studied some properties of it. They gave a decision making method based on generalized fuzzy soft matrices.

The rest of the paper is designed as follows: section 2 contains some basic definitions. In section 3 a selection method based on generalized fuzzy soft matrices is presented and an algorithm of the method is given in section 4. Section 5 gives a case study and section 6 concludes the paper.

2. SOME BASIC DEFINITIONS

In this section, we recall some basic definitions of fuzzy soft set theory which would be helpful for our discussion.


Let U be a universal set, C be set of parameters and A ⊆ C. A pair (F, A) is a soft set over U, where F: A → U.I n fact a soft set is a parameterized family of subsets of subsets over the universe U. Every set F(c), c ∈ C represents the set of elements of the soft set (F, A).


Let U be a universal set, C be set of parameters and I1 be the set of all fuzzy subsets of U. Let A ⊆ C. A pair (F, A) is a fuzzy soft set over U, where F: A → I1.

2.3. Fuzzy Soft Matrix [14]

Let U be the universal set, C be the set of parameters. Let A ⊆ C and (F, A) be a fuzzy soft set. Then the matrix form of the fuzzy soft set (F, A) is given as

\[ A = \{ a_{ij} \}, \quad 0 \leq a_{ij} \leq 1 \]

where \( a_{ij} = \mu(c_i, c_j) \) and \( c_j \in A \), \( c_j \notin A \) \nexists j, i.

Here \( \mu(c_i, c_j) \) denotes the membership of \( c_j \) in the fuzzy soft set \( F(c_j) \).

2.4. Generalized Fuzzy Soft Set [10]

Let U be a universal set, C be the set of parameters, \( F \) be a mapping of C to \( \Gamma \), where \( \Gamma \) is the collection of all fuzzy subsets of U and \( \Lambda \) be a fuzzy subset of C, i.e.

\[ \Lambda: C \to \Gamma = [0, 1]. \]

Let \( F_2: C \to \Gamma \times I \) be a function, such that \( F_2(c) = (F(c), \Lambda(c)) \), \( F(c) \in \Gamma \). Then \( F_2 \) is said to be a generalized fuzzy soft set (GFSS in short) over (U, C).

2.5. Generalized Fuzzy Soft Subset [10]

For two generalized fuzzy soft sets \( F_2 \) and \( G_2 \) over (U, C), \( F_2 \) is called generalised fuzzy soft subset of \( G_2 \), denoted by

\[ F_2 \subseteq G_2 \] if \( \mu \subseteq \Lambda \) and \( F(c) \subseteq G(c), \forall c \in C \).


Let U be the universal set, C be the set of parameters and \( A \subseteq C \). Suppose that \( (F_A, C) \) be a GFSS over (U, C). A uniquely defined subset of \( U \times C \), \( R_A = \{(u, c), c \in C, u \in F_A(c)\} \) is a relation form of \( (F_A, C) \). The membership function \( \mu_{R_A} \) and the function \( \Lambda_{R_A} \) are written as

\[ \mu_{R_A}: U \times C \to [0, 1] \] and \( \Lambda_{R_A}: U \times C \to [0, 1] \),

where \( \mu_{R_A}: \{(u, c) \in [0, 1], \forall u \in U, c \in C \} \).
2.7. Generalized Fuzzy Soft Sub Matrix
Let \( [a_{ij}, \lambda_{ij}]_{m \times n} \) be a generalized fuzzy soft matrix. Then \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called generalized fuzzy soft sub matrix of \( [b_{ij}, \psi_{ij}]_{m \times n} \) if \( a_{ij} \leq b_{ij} \) and \( \lambda_{ij} \leq \psi_{ij} \) for all \( i, j \).

2.8. Proper Generalized Fuzzy Soft Sub Matrix
Let \( [a_{ij}, \lambda_{ij}]_{m \times n} \) and \( [b_{ij}, \psi_{ij}]_{m \times n} \) be two generalized fuzzy soft matrices. Then \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called proper generalized fuzzy soft sub matrix of \( [b_{ij}, \psi_{ij}]_{m \times n} \) if \( a_{ij} \leq b_{ij} \) and \( \lambda_{ij} \leq \psi_{ij} \) for at least one term.

2.9. Strictly Proper Generalized Fuzzy Soft Sub Matrix
Let \( [a_{ij}, \lambda_{ij}]_{m \times n} \) and \( [b_{ij}, \psi_{ij}]_{m \times n} \) be two generalized fuzzy soft matrices. Then \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called strictly proper generalized fuzzy soft sub matrix of \( [b_{ij}, \psi_{ij}]_{m \times n} \) if \( a_{ij} < b_{ij} \) and \( \lambda_{ij} < \psi_{ij} \) for all \( i, j \).

2.10. Generalized Fuzzy Soft Rectangular Matrix
A generalized fuzzy soft \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called generalized fuzzy soft rectangular matrix, if \( [a_{ij}, \lambda_{ij}]_{m \times n} = \mu_R A_{ij} ((u_i, c_i), \lambda (u_i, c_i)) \) and \( m \neq n, \forall i, j \).

2.11. Generalized Fuzzy Soft Square Matrix
A generalized fuzzy soft \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called generalized fuzzy soft square matrix, if \( [a_{ij}, \lambda_{ij}]_{m \times n} = \mu_R A_{ij} ((u_i, c_i), \lambda (u_i, c_i)) \) and \( m = n, \forall i, j \).

A generalized fuzzy soft \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called generalized fuzzy soft diagonal matrix, if \( [a_{ij}, \lambda_{ij}]_{m \times n} = \mu_R A_{ij} ((u_i, c_i), \lambda (u_i, c_i)) \) and \( \mu_{ij} = (0, 0), \forall i \neq j \).

A generalized fuzzy soft \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called generalized fuzzy soft scalar matrix, if \( [a_{ij}, \lambda_{ij}]_{m \times n} = \mu_R A_{ij} ((u_i, c_i), \lambda (u_i, c_i)) \) and \( (\mu_{ij}, \lambda_{ij}) = (k, \lambda), \forall i = j \).

A generalized fuzzy soft \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called generalized fuzzy soft row matrix, if \( [a_{ij}, \lambda_{ij}]_{m \times n} = \mu_R A_{ij} ((u_i, c_i), \lambda (u_i, c_i)) \) and \( m = 1, \forall i, j \).

2.15. Generalized Fuzzy Soft Column Matrix
A generalized fuzzy soft \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called generalized fuzzy soft column matrix, if \( [a_{ij}, \lambda_{ij}]_{m \times n} = \mu_R A_{ij} ((u_i, c_i), \lambda (u_i, c_i)) \) and \( n = 1, \forall i, j \).

2.16. Generalized Fuzzy Soft Upper Triangular Matrix
A generalized fuzzy soft \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called generalized fuzzy soft upper triangular matrix, if \( \lambda_{ij} \leq \lambda_{kl} \) for \( i < j \).

2.17. Generalized Fuzzy Soft Lower Triangular Matrix
A generalized fuzzy soft \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called generalized fuzzy soft lower triangular matrix, if \( \lambda_{ij} \leq \lambda_{kl} \) for \( i > j \).

2.18. Generalized Fuzzy Soft Triangular Matrix
A generalized fuzzy soft matrix \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is called generalized fuzzy soft triangular matrix, if it is either generalized fuzzy soft upper triangular matrix or generalized fuzzy soft lower triangular matrix.

2.19. Scalar Multiple of Generalized Fuzzy Soft Matrix
Let \( [a_{ij}, \lambda_{ij}]_{m \times n} \) be a generalized fuzzy soft matrix. Then scalar multiple of \( [a_{ij}, \lambda_{ij}]_{m \times n} \) by a scalar \( k \) is defined as \( k \cdot [a_{ij}, \lambda_{ij}]_{m \times n} = [ka_{ij}, k\lambda_{ij}]_{m \times n} \), where \( 0 \leq k \leq 1 \).

2.20. Addition of Generalized Fuzzy Soft Matrices
Let \( [a_{ij}, \lambda_{ij}]_{m \times n} \) and \( [b_{ij}, \psi_{ij}]_{m \times n} \) be two generalized fuzzy soft matrices. Then addition of \( [a_{ij}, \lambda_{ij}]_{m \times n} \) and \( [b_{ij}, \psi_{ij}]_{m \times n} \) is defined as \( [a_{ij}, \lambda_{ij}]_{m \times n} + [b_{ij}, \psi_{ij}]_{m \times n} = [a_{ij} + b_{ij}, \lambda_{ij} + \psi_{ij}]_{m \times n} \), where \( \lambda_{ij} = \max \{ a_{ij}, b_{ij} \} \) and \( \psi_{ij} = \max \{ \lambda_{ij}, \psi_{ij} \}, \forall i, j \).

2.21. Complement of Generalized Fuzzy Soft Matrix
Let \( [a_{ij}, \lambda_{ij}]_{m \times n} \) be a generalized fuzzy soft matrix. Then complement of \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is denoted by \( [\bar{a}_{ij}, \bar{\lambda}_{ij}]_{m \times n} \) and is defined as \( [\bar{a}_{ij}, \bar{\lambda}_{ij}]_{m \times n} = [c_{ij}, \mu_{ij}]_{m \times n} \), where \( c_{ij} = 1 - a_{ij} \) and \( \mu_{ij} = 1 - \lambda_{ij}, \forall i, j \).

2.22. Generalized Membership Value Matrix
The membership value matrix corresponding to the generalized fuzzy soft matrix \( [a_{ij}, \lambda_{ij}]_{m \times n} \) is
MV $[a_{ij}, \lambda_{ij}]_{m\times n}$ = $[a_{ij}]_{m\times n}$, $\forall$ i, j.
Where $a_{ij}$ represents the fuzzy membership function in the fuzzy soft matrix.

### 2.2.3. Generalized Score Matrix
Let $[a_{ij}, \lambda_{ij}]_{m\times n}$ and $[b_{ij}, \psi_{ij}]_{m\times n}$ be two generalized fuzzy soft matrices and their corresponding generalized membership value matrices are

MV $[a_{ij}, \lambda_{ij}]_{m\times n}$ = $[c_{ij}]_{m\times n}$ and
MV $[b_{ij}, \psi_{ij}]_{m\times n}$ = $[d_{ij}]_{m\times n}$.

Then the score matrix $S[a_{ij}, \lambda_{ij}, b_{ij}, \psi_{ij}]$ is defined as $S[a_{ij}, \lambda_{ij}]_{m\times n}; [b_{ij}, \psi_{ij}]_{m\times n} = [s_{ij}]_{m\times n}$, where $[s_{ij}]_{m\times n} = [c_{ij}]_{m\times n} - [d_{ij}]_{m\times n}$, $\forall$ i, j.

### 3. Mathematical Modeling of the Problem
In this section we represent a selection method to select a best possible candidate from the set of short listed candidates under consideration of the authority of an organization. The authority of the organization has to conduct interviews of the short listed candidates. Suppose that there are k number of candidates and r members in the expert committee of the organization. They are conducting interviews on the basis of some selection criteria say $S = \{s_1, s_2, ..., s_n\}$.

The observations of the authority are expressed as generalized fuzzy soft matrices $[\mu_{ij}]^p$, $[\lambda_{ij}]^p$ over S, where $p = 1, 2, ..., r$.

We calculate sum of the generalized fuzzy soft matrices $[\mu_{ij}]^p$, $[\lambda_{ij}]^p = [a_{ij}]_{m\times n}$, where $a_{ij} = \sum_{p=1}^{r} \mu_{ij}^p$, and $\psi_{ij} = \sum_{p=1}^{r} \lambda_{ij}^p$.

We obtain generalized membership value matrix $[a_{ij}]_{m\times n}$ corresponding to $[a_{ij}, \psi_{ij}]_{m\times n}$.

We find compliment of the generalized fuzzy soft matrices $[\mu_{ij}]^p$, $[\lambda_{ij}]^p = [a_{ij}]_{m\times n}$, and find their sum as $\sum_{p=1}^{r} [\mu_{ij}^p - \lambda_{ij}^p]_{m\times n} = [b_{ij}]_{m\times n}$.

Again we obtain generalized membership value matrix $[b_{ij}]_{m\times n}$ corresponding to $[b_{ij}, \Phi_{ij}]_{m\times n}$.

Now we find score general membership matrix as $S[a_{ij}, \lambda_{ij}]_{m\times n} = [a_{ij}]_{m\times n} - [b_{ij}]_{m\times n}$.

Finally we find maximum score.

### 4. Algorithm

**Step 1:** Input the generalized fuzzy soft matrices $[x_{ij}, \lambda_{ij}]$ and $[y_{ij}, \mu_{ij}]$.

**Step 2:** Calculate $[x_{ij}, \lambda_{ij}]$ and $[y_{ij}, \mu_{ij}]$.

**Step 3:** Find $[x_{ij}, \lambda_{ij}] + [y_{ij}, \mu_{ij}]$ and $[x_{ij}, \lambda_{ij}] + [y_{ij}, \mu_{ij}]$.

**Step 4:** Find membership value of $[x_{ij}, \lambda_{ij}] + [y_{ij}, \mu_{ij}]$ and $[x_{ij}, \lambda_{ij}] + [y_{ij}, \mu_{ij}]$.

**Step 5:** Calculate score matrix.

**Step 6:** Find maximum score.

### 5. Case Study
Suppose that an organization wants to recruit an Eligible Candidate to fill a vacant position. There are four short listed candidates for this post i.e. C = {c1, c2, c3, c4}.

Two experts Mr. X and Mr. Y are conducting their interviews on the basis of $S = \{s_1, s_2, s_3, s_4, s_5\}$, where $s_1, s_2, s_3, s_4$ and $s_5$ stands for confidence, knowledge, personality, communication skills and ability of team work respectively.

To get the observations of the experts, we construct generalized fuzzy soft matrices $[x_{ij}, \lambda_{ij}]_{m\times n}$ and $[y_{ij}, \lambda_{ij}]_{m\times n}$ as

$[x_{ij}, \lambda_{ij}]_{m\times n} =$

\[
\begin{bmatrix}
(0.7, 0.6) & (0.6, 0.8) & (0.5, 0.7) & (0.6, 0.9) & (0.7, 0.5) \\
(0.5, 0.6) & (0.5, 0.8) & (0.6, 0.7) & (0.7, 0.9) & (0.4, 0.5) \\
(0.8, 0.6) & (0.7, 0.8) & (0.8, 0.7) & (0.4, 0.9) & (0.6, 0.5) \\
(0.5, 0.6) & (0.8, 0.8) & (0.4, 0.7) & (0.8, 0.9) & (0.8, 0.5)
\end{bmatrix}
\]

And

$[y_{ij}, \lambda_{ij}]_{m\times n} =$

\[
\begin{bmatrix}
(0.8, 0.6) & (0.4, 0.8) & (0.6, 0.7) & (0.8, 0.9) & (0.6, 0.5) \\
(0.5, 0.6) & (0.6, 0.8) & (0.5, 0.7) & (0.6, 0.9) & (0.7, 0.5) \\
(0.7, 0.6) & (0.7, 0.8) & (0.8, 0.7) & (0.5, 0.9) & (0.8, 0.5) \\
(0.9, 0.6) & (0.5, 0.8) & (0.4, 0.7) & (0.7, 0.9) & (0.4, 0.5)
\end{bmatrix}
\]

We find the sum of $[x_{ij}, \lambda_{ij}]$ and $[y_{ij}, \lambda_{ij}]$ as

$[x_{ij}, \lambda_{ij}] + [y_{ij}, \lambda_{ij}] =$

\[
\begin{bmatrix}
(0.8, 0.6) & (0.6, 0.8) & (0.6, 0.7) & (0.8, 0.9) & (0.7, 0.5) \\
(0.5, 0.6) & (0.6, 0.8) & (0.6, 0.7) & (0.7, 0.9) & (0.7, 0.5) \\
(0.8, 0.6) & (0.7, 0.8) & (0.8, 0.7) & (0.5, 0.9) & (0.8, 0.5) \\
(0.9, 0.6) & (0.8, 0.8) & (0.4, 0.7) & (0.8, 0.9) & (0.8, 0.5)
\end{bmatrix}
\]

We obtain generalized membership value matrix $[c_{ij}]$ corresponding to $[x_{ij}, \lambda_{ij}] + [y_{ij}, \lambda_{ij}]$ as

$[c_{ij}] =$

\[
\begin{bmatrix}
0.8 & 0.6 & 0.6 & 0.8 & 0.7 \\
0.6 & 0.6 & 0.7 & 0.7 & 0.7 \\
0.8 & 0.7 & 0.8 & 0.5 & 0.8 \\
0.9 & 0.8 & 0.4 & 0.8 & 0.8
\end{bmatrix}
\]

We calculate $[x_{ij}, \lambda_{ij}]$ and $[y_{ij}, \lambda_{ij}]$ as

$[x_{ij}, \lambda_{ij}] =$

\[
\begin{bmatrix}
(0.3, 0.4) & (0.4, 0.2) & (0.5, 0.3) & (0.4, 0.1) & (0.3, 0.5) \\
(0.4, 0.4) & (0.5, 0.2) & (0.4, 0.3) & (0.3, 0.1) & (0.6, 0.5) \\
(0.2, 0.4) & (0.3, 0.2) & (0.2, 0.3) & (0.6, 0.1) & (0.4, 0.5) \\
(0.5, 0.4) & (0.2, 0.2) & (0.6, 0.3) & (0.2, 0.1) & (0.2, 0.5)
\end{bmatrix}
\]

and

$[y_{ij}, \lambda_{ij}] =$
\[
(0.2,0.4) \quad (0.6,0.2) \quad (0.4,0.3) \quad (0.2,0.1) \quad (0.4,0.5) \\
(0.5,0.4) \quad (0.6,0.2) \quad (0.5,0.3) \quad (0.4,0.1) \quad (0.3,0.5) \\
(0.3,0.4) \quad (0.3,0.2) \quad (0.2,0.3) \quad (0.5,0.1) \quad (0.2,0.5) \\
(0.1,0.4) \quad (0.5,0.2) \quad (0.6,0.3) \quad (0.3,0.1) \quad (0.6,0.5)
\]

We obtain the sum of \([x_{ij}, \lambda_j]\) and \([y_{ij}, \lambda_j]\) as

\[
[x_{ij}, \lambda_j] + [y_{ij}, \lambda_j] =
\]

\[
(0.3,0.4) \quad (0.6,0.2) \quad (0.5,0.3) \quad (0.4,0.1) \quad (0.4,0.5) \\
(0.5,0.4) \quad (0.6,0.2) \quad (0.3,0.3) \quad (0.4,0.1) \quad (0.6,0.5) \\
(0.3,0.4) \quad (0.3,0.2) \quad (0.2,0.3) \quad (0.6,0.1) \quad (0.4,0.5) \\
(0.5,0.4) \quad (0.5,0.2) \quad (0.6,0.3) \quad (0.3,0.1) \quad (0.6,0.5)
\]

We get generalized membership value matrix \([d_{ij}]\) corresponding to \([x_{ij}, \lambda_j]\) + \([y_{ij}, \lambda_j]\) as

\[
[d_{ij}] =
\]

\[
0.3 \quad 0.6 \quad 0.5 \quad 0.4 \quad 0.4 \\
0.5 \quad 0.5 \quad 0.3 \quad 0.4 \quad 0.6 \\
0.3 \quad 0.3 \quad 0.2 \quad 0.6 \quad 0.4 \\
0.5 \quad 0.5 \quad 0.6 \quad 0.3 \quad 0.6
\]

Finally we find generalized score matrix and maximum scores respectively as

\[
S[e_{ij},t_{ij}] =
\]

\[
\begin{bmatrix}
0.5 & 0.0 & 0.1 & 0.4 & 0.3 \\
0.1 & 0.1 & 0.3 & 0.3 & 0.1 \\
0.5 & 0.4 & 0.6 & -0.1 & 0.4 \\
0.4 & 0.3 & -0.2 & 0.5 & 0.2 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0.13 \\
0.9 \\
0.18 \\
0.12
\end{bmatrix}
\]

Which shows that \(c_3\) is the best candidate to be recruited.

**6. Conclusion**

In this paper we defined some new types of generalized fuzzy soft matrices and studied some new operations of it. We gave a selection method based on generalized fuzzy soft matrices and applied it to recruiting a best possible eligible candidate. The method may also be applied to other similar problems.

**REFERENCES**


