Viscous dissipation and ohmic heating effects on heat and mass transfer of MHD mixed convection flow over an inclined porous plate in the presence of soret effect and temperature gradient dependent heat source

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I. INTRODUCTION

The study of heat and mass transfer of fluid flow along a porous medium has attracted the attention of a good number of investigators in view of its varied applications in reciprocating engines, pulse combustors and chemical reactors etc. Flow through porous medium has many applications in the field of petroleum technology to study the movement of gas, oil and water through oil reservoirs, aero dynamics to study the rocket engine combustion chamber walls, gas turbine blades etc., at the same time convection layer flows adjacent to inclined plates or wedges have received less attention. The analysis of Sparrow et.al [1] about the fluid flow over a boundary layer with natural and forced convection. Singh [2] studied the fluid flow over an inclined plate in the presence of viscous dissipation. Due to importance of Soret (Thermo Diffusion) and Dufour effects for the fluids with very light molecular weight as well as medium molecular weight many investigators have studied and reported. Chand et.al [3] were discussed the Soret effects on Heat and Mass Transfer in MHD free convective flow through a porous medium in a vertical porous channel.

Rabin N. Barik et.al., [4] have investigated chemical reaction and Soret effect on MHD oscillatory flow through a porous medium bounded by two vertical porous plates in the presence of heat source. Balamurugan K, et.al., [5] have investigated Chemical reaction effects on Heat and Mass Transfer of Unsteady flow over an infinite vertical porous plate embedded in a porous medium with heat source. Due to importance of free convection flow over a inclined plate, Cheng P [6] have investigated that the Combined free and forced convection flow about inclined surfaces in porous media. Zyauddin, Kumar M [7] has explained that Radiation effect on Unsteady MHD Heat and mass transfer flow on a moving inclined porous heated plate in the presence of chemical reaction.

Raju M.C., et.al., [8] have analysed the effect of magneto hydrodynamics transient mixed convection and chemically reactive flow past a porous inclined plate with radiation and heat source. Due to importance of Soret effects and temperature gradient dependent heat source, extended the work of Raju M.C., et.al., by including the Soret effect on fluid flow along a porous inclined plate with temperature gradient dependent heat source.

II. FLOW DESCRIPTION AND GOVERNING EQUATIONS

We consider the flow of an electrically conducting viscous incompressible fluid flow along a semi-infinite inclined porous plate at an angle $\alpha$ in a vertical direction embedded in a porous medium, which are subjected to thermal and concentration buoyancy effects. Assume it is homogeneous chemical reaction of first order. The wall is
maintained at the constant temperature \( T_w \) and concentration \( C_w \) respectively.

Under the usual Boussinesq approximations the flow is governed by the following system of equations.

\[
\frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T)
\]

\[
\frac{\partial C}{\partial t} = \nabla \cdot (D \nabla C) - \nabla \cdot (DC) + R
\]

\[
\frac{\partial \rho}{\partial t} = \nabla \cdot (\rho \mathbf{v})
\]

\[
\frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \rho \mathbf{g} + \nabla \left( \tau - \tau' \right) + \frac{1}{
\rho}
\nabla \left( \rho D \nabla C - \nabla C \right)
\]

The boundary conditions are

\[
u' = 0, \quad T' = T_w, \quad C = C_w \quad \text{at } y = 0
\]

\[
u' = 0, \quad T' = T_w, \quad C = C_w \quad \text{at } y \to \infty
\]

\( g \) is the acceleration due to gravity, \( \beta \) and \( \beta_c \) are the coefficients of volume expansion, \( V \) is the kinematic viscosity, \( K' \) is the chemical reaction of the fluid flow, \( \rho \) is the density of the fluid, \( \sigma \) is the electrical conductivity of the fluid, \( B_0 \) is the uniform magnetic field, \( T' \) is the temperature, \( C_o \) is the specific heat at constant pressure, \( q_r \) is the radioactive heat flux, \( T' \) is the temperature of the plate, \( T_w \) is the temperature of the fluid away from the plate, being the mean free path where \( m_1 \) is the Maxwell reflection coefficient, \( C \) is the concentration, \( C_w \) is the concentration of the fluid at the wall as well as \( C_w \) is the concentration of the fluid away from the plate.

The equation of continuity (1) yields that \( V \) is a constant.

Consider the fluid which is optically thin with a relatively low flow density and radioactive heat flux is given by

\[
\frac{\partial V}{\partial y} = 4(T' - T_w) \nu, \quad \text{where} \quad I \theta_1 \text{ is the absorption coefficient at the plate.}
\]

On introducing the following dimensionless quantities

\[
y = \frac{\nu y}{\nu}, \quad u = \frac{u}{v}, \quad M = \frac{\sigma B_o v^2}{\nu}, \quad Gr = \frac{g \beta v^2(T_w - T_o)}{\nu^2} \mu, \quad Gm = \frac{g \beta v^2(C_w - C_0)}{\nu^2} \mu, \quad \epsilon = \frac{C^2}{C_p}
\]

\[
K = \frac{K' v}{\nu}, \quad T = \frac{T - T_w}{T_w}, \quad C = \frac{C - C_w}{C_w}, \quad Pr = \frac{\mu C}{\rho C_p}, \quad Ec = \frac{\nu^2}{C_p(T_w - T_o)}
\]

\[
F = \frac{4 \nu' v}{\rho C_p \nu^2} \mu, \quad H = \frac{\theta v}{\rho C_p \nu^2} \mu, \quad Sc = \frac{v}{D}
\]

Substituting (7) in equation (2), (3), and (4) we get

\[
d\theta \frac{d^2 \theta}{dy^2} + Pr \frac{d\theta}{dy} + Pr H \frac{d\theta}{dy} + Pr \frac{dC}{dy} = \frac{d\theta^2}{dy^2} - Pr F + Pr Ec \frac{M^2 u^2}{
\]

with the following boundary conditions

\[
u = 0, \quad \theta = 1, \quad C = 1, \quad \text{at } y = 0
\]

\[
u = 0, \quad \theta = 0, \quad C = 0, \quad \text{at } y \to \infty
\]

where,

\[
Gr = \frac{v^2 \beta(T_w - T_0)}{v^2} \quad \text{is the Grashof number},
\]

\[
Gm = \frac{v^2 \beta(C_w - C_0)}{v^2} \quad \text{is the modified Grashof number},
\]

\[
M = \frac{\sigma B_o v^2}{\nu^2} \mu \quad \text{is the magnetic number}, \quad K = \frac{K' \nu}{v^2} \quad \text{is the Permeability}, \quad K_r = \frac{K' \nu}{v^2} \quad \text{is the chemical reaction parameter}, \quad Sr = \frac{DK(M' - M_o)}{M(T_w - T_o)} \quad \text{is the sorot number},
\]

\[
F = \frac{4 \nu' v}{\rho C_p \nu^2} \mu \quad \text{is the Radiation Parameter}, \quad Pr = \frac{\mu C}{\rho C_p} \quad \text{is the Prandtl number}, \quad H = \frac{\theta v}{\rho C_p \nu^2} \mu \quad \text{is the heat source.}
\]

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**III. METHOD OF SOLUTION**

To solve equations (8), (9) and (10), Assuming \( \epsilon \) to be small so that one can express \( F, T' \) and \( C \) as a regular perturbation series in terms of \( \epsilon \) in the neighbourhood of the plate as

\[
u = \nu_0(y) + \epsilon \nu_1(y), \quad \theta = \theta_0(y) + \epsilon \theta_1(y), \quad C = C_0(y) + \epsilon C_1(y)
\]

Using (12) in equations (8), (9) & (10) for to reduce partial differential equations into ordinary differential equations, then equating the coefficient of \( \epsilon^0 \), \( \epsilon^1 \) neglecting \( \epsilon^2 \) terms etc., then we get the set of ordinary differential equations

\[
u_0'' + \nu_0' - \nu_0 = -G_1 \theta_0 - G_2 C_0
\]

\[
\theta_0'' + (1 + H) \theta_0' + Pr \theta_0' = 0
\]

\[
C_1 = Sc C_0 - Sc K \theta_0 = -ScSr \theta_0
\]

Equating the coefficients of first order of Eckert Number, we get

\[
u_1'' + \nu_1' - \nu_1 = -G_1 \theta_1 - G_2 C_1
\]

\[
\theta_1'' + (1 + H) \theta_1' + Pr \theta_1' + Pr u_0' + Pr M^2 u_0 = 0
\]

\[
C_1 = Sc C_1 - Sc K \theta_1 = -ScSr \theta_1
\]

Where...
\[ p = M^2 + \frac{1}{R} \quad G_1 = Gr \cos \alpha, \quad G_2 = Gr \cos \alpha \]

The corresponding boundary conditions are

\[ u_0 = 0, u_1 = \theta_0, \theta_0 = 0, \theta_1 = 1, C_1 = 0, C_1 = 0 \quad \text{at} \quad y = 0 \]
\[ u_1 = 0, u_2 = 0, \theta_0 = 1, C_0 = 0, C_1 = 0 \quad \text{at} \quad y = \infty \quad (19) \]

On solving the above differential equations with following boundary conditions, we get the set of solutions as below

\[ U_0 = C_6 e^{M_y} + C_8 e^{M_y} + C_9 e^{M_y} + C_{10} e^{M_y} \]
\[ U_1 = A_{11} e^{M_y} + B_{11} e^{M_y} + B_{12} e^{M_y} + B_{13} e^{M_y} + B_{14} e^{M_y} + B_{15} e^{M_y} + B_{16} e^{M_y} + B_{17} e^{M_y} + B_{18} e^{M_y} + B_{19} e^{M_y} + B_{20} e^{M_y} \]
\[ \theta_0 = e^{M_y} \]
\[ \theta_1 = C_8 e^{M_y} + B_1 e^{M_y} + B_2 e^{M_y} + B_3 e^{M_y} + B_4 e^{M_y} + B_5 e^{M_y} + B_6 e^{M_y} + B_7 e^{M_y} + B_8 e^{M_y} + B_9 e^{M_y} + B_{10} e^{M_y} + B_{11} e^{M_y} + B_{12} e^{M_y} + B_{13} e^{M_y} + B_{14} e^{M_y} + B_{15} e^{M_y} + B_{16} e^{M_y} + B_{17} e^{M_y} + B_{18} e^{M_y} + B_{19} e^{M_y} + B_{20} e^{M_y} \]
\[ C_0 = C_4 e^{M_y} + C_5 e^{M_y} \]
\[ C_1 = A_{10} e^{M_y} + D_{11} e^{M_y} + D_{12} e^{M_y} + D_{13} e^{M_y} + D_{14} e^{M_y} + D_{15} e^{M_y} + D_{16} e^{M_y} + D_{17} e^{M_y} + D_{18} e^{M_y} + D_{19} e^{M_y} + D_{20} e^{M_y} + D_{21} e^{M_y} \]

A. Skin friction

The skin friction for the velocity u is given by

\[ \left( \frac{du}{dy} \right)_{y=0} = -(C_6 \mu_6 + C_8 \mu_8 + C_9 \mu_4 + C_{10} \mu_2) \]

\[ + E_{4c} (A_{11} \mu_6 + B_{11} \mu_2 + E_2 (2 \mu_6) + E_4 (2 \mu_4)) \]
\[ + E_4 (2 \mu_2) + E_5 (2 \mu_2) + E_6 (2 \mu_6) \]
\[ + E_7 (2 \mu_4) + E_8 (2 \mu_4) + E_9 (2 \mu_6) \]
\[ + E_{10} (2 \mu_2) + E_{11} (2 \mu_2) + E_{12} (2 \mu_4) \]
\[ + E_{13} (2 \mu_2) + E_{14} (2 \mu_4) + E_{15} (2 \mu_2) \]
\[ + E_{16} (2 \mu_2) + E_{17} (2 \mu_2) + E_{18} (2 \mu_2) + E_{19} (2 \mu_4) \]
\[ + E_{20} (2 \mu_2) + E_{21} (2 \mu_2) + E_{22} (2 \mu_4) \]
\[ + E_{23} \mu_2 + E_{24} \mu_2 + E_{25} \mu_2 + E_{26} \mu_4 \]
\[ + E_{27} (2 \mu_2) + E_{28} (2 \mu_2) + E_{29} (2 \mu_4) \]
\[ + E_{30} (2 \mu_4) + E_{31} (2 \mu_4) + E_{32} (2 \mu_2) + E_{33} (2 \mu_4) \]
\[ + E_{34} (2 \mu_4) + E_{35} (2 \mu_4) + E_{36} (2 \mu_4) + E_{37} (2 \mu_4) + E_{38} (2 \mu_4) \]
\[ + E_{39} (2 \mu_4) + E_{40} (2 \mu_4) + E_{41} (2 \mu_4) \]
\[ + E_{42} (2 \mu_4) + E_{43} (2 \mu_4) \]

B. Heat Flux

The rate of heat transfer at the plate of non-dimensional Nusselt number is given by

\[ \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = -(M_2 + E_{4c} (C_8 \mu_2 + 2 B_1 \mu_6) \]
\[ + 2 B_2 \mu_2 + 2 B_3 \mu_4 + 2 B_4 \mu_2 + 3 B_5 \mu_2 + 4 + B_6 (\mu_2 + \mu_4) + B_7 (\mu_2 + \mu_4) + B_8 (\mu_2 + \mu_4) \]
\[ + 2 B_9 \mu_2 + B_{10} (\mu_2 + \mu_2 + 2 \mu_4) + 2 B_{11} \mu_2 + 2 B_{12} \mu_2 \]
\[ + 2 B_{13} \mu_2 + 2 B_{14} \mu_2 + B_{15} (\mu_2 + \mu_4) + B_{16} (\mu_2 + \mu_4) + B_{17} \mu_2 + B_{18} (\mu_2 + \mu_4) + B_{19} \mu_2 + B_{20} (\mu_2 + \mu_4) \]

C. Mass Flux

The rate of mass transfer at the plate of non-dimensional Sherwood number is given by

\[ \left( \frac{\partial C}{\partial y} \right)_{y=0} = -(C_4 \mu_4 + C_5 \mu_5 + E_{4c} (A_{10} \mu_4) \]
\[ + D_1 \mu_2 + D_2 \mu_6 + D_3 \mu_2 + D_4 \mu_4 + D_5 \mu_2 + D_6 \mu_2 + D_7 (\mu_2 + \mu_4) + D_8 (\mu_2 + \mu_4) + D_9 (\mu_2 + \mu_4) + D_{10} \mu_4 \]
\[ + D_{11} \mu_2 + D_{12} \mu_4 + D_{13} \mu_2 + D_{14} \mu_4 + D_{15} \mu_2 + D_{16} \mu_4 \]
\[ + D_{17} \mu_2 + D_{18} \mu_2 + D_{19} \mu_2 + D_{20} \mu_2 + D_{21} (\mu_2 + \mu_4) \]

IV. RESULTS AND DISCUSSIONS

Here, some of the results of physical interest on the velocity, temperature, concentration distribution and also on the wall shear stress and the rate of heat transfer, rate of mass transfer at the wall were discussed. Further, the result is also in good agreement with the result of Raju M.C., et. al. If omitted, sorbent effect and temperature dependent heat source.
We have studied the main flow velocity, temperature and concentration by including various parameters like Prandtl Number, Schmidt number, Thermal Grashof number and mass Grashof number. The effect of flow parameters on velocity field, Temperature field, Concentration field, skin friction, heat flux and mass flux have been analysed numerically and discussed with the help of numerical values.

Figures 1 and 2 depict the temperature of the fluid decreases if increase of Radiation and Heat source. Figure 3 depict that increase of Prandtl number retards the temperature profile of the fluid flow because decreases of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of prandtl numbers are equivalent to increase the thermal conductivity of the fluid. Figure 4 illustrates Concentration of fluid flow increase if increase of Soret number. Figure 5 illustrates the increase of Schmidt number retards the concentration of the fluid flow. From Figure 6 concentration of the fluid flow retards while increase of chemical reaction. Figures 7 and 8 indicates that the increase of Grashof number and modified Grashof number increases the velocity of the fluid flow. This implies that the present study is a buoyancy assisting flow with thermal buoyancy and mass buoyancy. Figure 9 shows that the increase of velocity of the fluid flow while increases of soret number. Influence of magnetic parameter on figure 10 shows velocity of the fluid goes on decrease due to resistance force which is oppose to the velocity of the fluid flow.

Figure 11 depicts velocity of the fluid flow goes on increase while increase of permeability parameter. Variation of angle of plate retards the velocity of the fluid flow as shown in the figure 12. Similar effects shows in the figure 13 while increase of prandtl number.

In figure 14 shows that the increase effects of skin friction while increase of permeability parameter with function of Grashof number. It is concluded that the thermal buoyancy accelerates the skin friction. Mass flux shows the increase effect if increase of Schmidt number. Heat flux shows increase effects while increase of prandtl number.
Fig 5: Concentration of fluid flow for various value of $Sc$
$Pr=0.71; H=2; H=1.0; H=1.0; Sc=0.25; K=1.0; M=2.0; Gr=6.0; Gm=1.0; Ec=0.01;$
$= \cos[30]; G1=Gr \alpha; G2=Gm \alpha;$

Fig 6: Concentration of fluid flow for various value of $Kr$
$Pr=0.71; H=2; H=1.0; Sc=0.25; K=1.0; M=2.0; Gr=6.0; Gm=1.0; Ec=0.01;$
$= \cos[30]; G1=Gr \alpha; G2=Gm \alpha;$

Fig 7: Velocity of fluid flow for various value of $Gr$
$Pr=0.71; H=2; H=1.0; Sc=0.25; K=1.0; M=2.0; Gr=6.0; Gm=1.0; Ec=0.01;$
$= \cos[30]; G1=Gr \alpha; G2=Gm \alpha;$

Fig 8: Velocity of fluid flow for various value of $Gm$
$Pr=0.71; H=2; H=1.0; Sc=0.25; K=1.0; M=2.0; Gr=6.0; Kr=1.0; Sr=1.0; Gm=1.0; Ec=0.01;$
$= \cos[30]; G1=Gr \alpha; G2=Gm \alpha;$

Fig 9: Velocity of fluid flow for various value of $Sr$
$Pr=0.71; H=2; H=1.0; Sc=0.25; K=1.0; M=2.0; Gr=6.0; Kr=1.0; Sr=1.0; Gm=1.0; Ec=0.01;$
$= \cos[30]; G1=Gr \alpha; G2=Gm \alpha;$

Fig 10: Velocity of fluid flow for various value of $M$
$Pr=0.71; H=2; H=1.0; Sc=0.25; K=1.0; M=2.0; Gr=6.0; Kr=1.0; Sr=1.0; Gm=1.0; Ec=0.01;$
$= \cos[30]; G1=Gr \alpha; G2=Gm \alpha;
Fig 11: Velocity of fluid flow for various value of K
Pr=0.71; H=2; H=1.0; Sc=0.25; Sr=2.0; Ec=0.01; Sr=2.0; G1=Gr α; G2=Gm α; Gr=6.0; K=1.0; Sr=1.0;

V. CONCLUSION
In this work, we have studied the soret and heat dependent source parameter on magneto hydrodynamics past a porous inclined plate with viscous and ohmic effects. The governing equations are solved by using perturbation techniques. An asymptotic solution of the resulting differential equations under the prescribed boundary conditions is obtained. Numerical results are discussed with...
help of graphs. The conclusions of the study are as follows:

Velocity increase while increase of Grashof numbers. Increase of Soret effect enhances the velocity of the fluid flow. Angle of inclined plate retards the velocity of the fluid. Increase of Magnetic parameter retards velocity of the fluid flow. Temperature profile shows decrease effects while increase of both radiation and Prandtl number, but the reverse process exists if increase of Soret effect in the concentration field. Here, consider the skin friction as function of Grashof number enhances the skin friction profile while increase of permeability.

REFERENCES


