Anti-Fuzzy Soft Subhemiring of a Hemiring

N.Anitha¹, J.Venkatesan *²

¹ Assistant professor, Department of Mathematics, Periyar University PG Extension centre, Dharmapuri- Tamilnadu, India.
* Assistant professor, Department of Mathematics, Sri vidya mandir arts and science college, Uthangarai Krishnagiri - Tamilnadu, India.

Abstract

In this paper, we made an endeavor to consider the logarithmic idea of an anti-fuzzy soft subhemirings of a hemiring.

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Keywords: anti-Fuzzy soft set, fuzzy soft sub hemiring, anti-fuzzy soft subhemiring, and pseudo Fuzzy soft coset.

INTRODUCTION:

There are numerous ideas of all inclusive algebras summing up a cooperative ring (R; +; .). Some of them specifically, nearrings and a few sorts of semirings have been demonstrated extremely valuable. Semirings (called likewise halfrings) are algebras (R; +; .) spread an indistinguishable properties from a ring with the excception of that (R ; + ) is thought to be a semigroup instead of a commutative group. Semirings show up in a characteristic way in a few applications to the hypothesis of automata and formal dialects. A polynomial math (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups fulfilling a. (b + c ) = a. b + a. c and (b + c) .a = b. a + c. a for every one of the a, b and c in R. A semiring R is said to be additively commutative if a + b = b + a for every a, b and c in R. A semiring R may have an identity 1, characterized by 1. a = a = a. 1 and a zero 0, characterized by 0 + a = a = a + 0 and a. 0 = 0 = 0. a for every a in R. A semiring R is said to be a hemiring in the event that it is additively commutative with zero. After the presentation of Fuzzysets by L.A. Zadeh [17], a few analysts investigated on the speculation of the idea of Fuzzysets. M. Borah, T. J. Neog and D. K. Sut,[5] were produced a few operations of Fuzzy soft sets, On operations of soft sets was created by A. Sezgin and A. O. Atan, [13] and Kumud Borgohain and Chittaranjan Gohain, [7] was produced some New operations on Fuzzy Soft Sets. In this paper, we present the idea of anti-fuzzy soft subhemirings of a hemiring.

1. PRELIMINARIES

1.1 Definition: A couple (F,E) is known as a soft set (over U) if and just if F is a mapping of E into the arrangement of all subsets of the set U.

As it were, the soft set is a parameterized group of subsets of the set U. Each set F(ε)(ε ϵ E) from this family might be considered as the arrangement of ε - components of the soft sets (F, E) or as the arrangement of ε - surmised components of the soft set.

1.2 Definition: Let (U,E) be a soft universe and A ⊆ E. Give F(U) a chance to be the arrangement of every Fuzzy subset in U. A couple (F,A) is known as a Fuzzy soft set over U, where F is a mapping given by F: A → F(U).

1.3 Definition: Let (R , + , . ) be a hemiring. An anti-Fuzzy soft normal subhemiring (AFSNSHR) of R is said to be a hemiring in the event that it is additively commutative with zero. After the presentation of Fuzzysets by L.A. Zadeh [17], a few analysts investigated on the speculation of the idea of Fuzzysets. M. Borah, T. J. Neog and D. K. Sut,[5] were produced a few operations of Fuzzy soft sets, On operations of soft sets was created by A. Sezgin and A. O. Atan, [13] and Kumud Borgohain and Chittaranjan Gohain, [7] was produced some New operations on Fuzzy Soft Sets. In this paper, we present the idea of anti-fuzzy soft subhemirings of a hemiring.
1.5 Definition: Let (F, A) and (G, B) be Fuzzy soft subsets of sets G and H, individually. The counter result of (F, A) and (G, B), signified by (F, A)\( \times \) (G, B), is characterized as

\[
(F, A)(x)(y) = \big\{ (x \in F, (x \in A, y \in G, B)) / \text{ for all } x \in F, y \in G, B \big\},
\]

where \( \mu_{F \times A}(x, y) = \max\{ \mu_F(x), \mu_A(x), \mu_G(y), \mu_B(y) \} \).

1.6 Definition: Let (F, A) be a Fuzzy soft subset in a set S, the anti-strongest Fuzzy soft connection on S, that is a Fuzzy soft relation on (F, A) is given by

\[
\mu_{F,A}(x, y) = \max\{ \mu_F(x, y), \mu_A(x, y), \mu_G(y, y) \}, \text{ for all } x \in F, y \in A, \text{ and } \mu_G(y, y) \in S.
\]

1.7 Definition: A anti Fuzzy soft subhemiring (F, A) of a hemiring R is called an anti- Fuzzy soft trademark subhemiring of R if

\[
\mu_{F,A}(x, y) = \mu_F(x, y), \text{ for all } x \in F, y \in A, \text{ and } \mu_G(y, y) \in R.
\]

1.8 Definition: Let (F, A) and (G, B) be two soft sets over a typical universe U. The union of (F, A) and (G, B) is

\[
\mu_{F,A}(x, y) = \min\{ \mu_F(x, y), \mu_A(x, y), \mu_G(y, y) \}, \text{ for all } x \in F, y \in A, \text{ and } \mu_G(y, y) \in R.
\]

1.9 Definition: Let (F, A) be an anti fuzzy soft subhemiring of a hemiring (R, +, \cdot ) and a in R. At that point (F, A) is known as a Fuzzy soft subhemiring in (R, +, \cdot ) characterized by

\[
\mu_{F,A}(x, y) = \min\{ \mu_F(x, y), \mu_A(x, y), \mu_G(y, y) \}, \text{ for all } x \in F, y \in A, \text{ and } \mu_G(y, y) \in R.
\]

1.10 Definition: Let (F, A) and (G, B) be two soft sets over a typical universe U. The union of (F, A) and (G, B) is characterized as the soft set (H, C) fulfilling the accompanying conditions:

(i) C = A \cup B
(ii) For all x \in C,

\[
H(x) = \begin{cases} 
F(x) & \text{if } x \in A - B, \\
G(x) & \text{if } x \in B - A, \\
F(x) \cup G(x) & \text{if } x \in A \cap B,
\end{cases}
\]

This is denoted by (F, A) \( \cup \) (G, B) = (H, C).

1.11 Definition: Let (F, A) be a anti-Fuzzy soft subhemiring of a hemiring R. At that point (F, A) is characterized as (F, A) is a anti-Fuzzy soft subhemiring of a hemiring R. At that point (F, A) is characterized as (F, A) is a anti-Fuzzy soft subhemiring of a hemiring R.

2. ANTI-FUZZY SOFT SUBHEMRIING OF A HEMIRING

2.1 Theorem: Union of any two anti fuzzy soft subhemiring of a hemiring R is anti-Fuzzysoft subhemiring of R.

Proof: Let (F, A) and (G, B) be any two anti-fuzzy soft subhemirings of a hemiring R and x_{F,A} and y_{G,B} in R. Let (F, A) = \{ (x_{F,A}, \mu_{F,A}(x_{F,A})) / x_{F,A} \in R \} and (G, B) = \{ (x_{G,B}, \mu_{G,B}(x_{G,B})) / x_{G,B} \in R \}.

Furthermore, let (H, C) = (F, A) \( \cup \) (G, B) = \{ (x_{H,C}, \mu_{H,C}(x_{H,C})) / x_{H,C} \in R \}, where max\{ \mu_{F,A}(x_{F,A}), \mu_{G,B}(x_{G,B}) \} = \mu_{H,C}(x_{H,C}).

It is clear that [(F, A) \( \cup \) (G, B)](x) = (F, A)(x) When \( \mu_{F,A}(x_{F,A}) \neq 0 \) and \( \mu_{G,B}(x_{G,B}) = 0 \).

Additionally, [(F, A) \( \cup \) (G, B)](x) = (G, B)(x) When \( \mu_{F,A}(x_{F,A}) = 0 \) and \( \mu_{G,B}(x_{G,B}) \neq 0 \).

It is sufficient to demonstrate that [(F, A) \( \cup \) (G, B)](x) = max\{ \mu_{F,A}(x_{F,A}), \mu_{G,B}(x_{G,B}) \}. Now, \mu_{H,C}(x_{H,C}) = max\{ \mu_{F,A}(x_{F,A}) + \mu_{G,B}(x_{G,B}) \} = max\{ \mu_{F,A}(x_{F,A}) \}, \mu_{G,B}(x_{G,B}) \} = max\{ \mu_{F,A}(x_{F,A}), \mu_{G,B}(x_{G,B}) \} = max\{ \mu_{H,C}(x_{H,C}) \}.

Therefore, \mu_{H,C}(x_{H,C}) \leq \mu_{H,C}(x_{H,C}) = max\{ \mu_{F,A}(x_{F,A}), \mu_{G,B}(x_{G,B}) \} = max\{ \mu_{H,C}(x_{H,C}), \mu_{H,C}(y_{H,C}) \}.

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\( \mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)}) \), \( \max \{ \mu_{(G,B)}(x_{(G,B)}), \mu_{(G,B)}(y_{(G,B)}) \} = \max \{ \max \{ \mu_{(F,A)}(x_{(F,A)}), \mu_{(G,B)}(x_{(G,B)}) \}, \max \{ \mu_{(F,A)}(y_{(F,A)}), \mu_{(G,B)}(y_{(G,B)}) \} \}.

In this way, \( \mu_{(H,C)}(x_{(H,C)}), \mu_{(H,C)}(y_{(H,C)}) \) for all \( x_{(F,A)} \) and \( y_{(G,B)} \) in R. Therefore, \( (H,C) \) is an anti-fuzzy soft subhemiring of a hemiring \( R \).

Hence the union of any two anti-fuzzy soft subhemirings of a hemiring \( R \) is an anti-fuzzy soft subhemiring of \( R \).

2.2 Theorem: The union of a group of anti-fuzzy soft subhemirings of hemiring \( R \) is an anti-fuzzy soft subhemiring of \( R \).

**Proof:** Let \( \{ (F_i,V_i) \}_{i=1} \) be a group of anti-fuzzy soft subhemirings of a hemiring \( R \) and let \( (F,A) = \bigcup_i V_i \). Let \( x_{(F,A)} \) and \( y_{(F,A)} \) in \( R \). Then, \( \mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) = \sup_i \mu_{F_i,V_i}(x_{(F,A)} + y_{(F,A)}) \leq \sup \max \{ \mu_{F_i,V_i}(x_{(F,A)}), \mu_{F_i,V_i}(y_{(F,A)}) \} \), \( \sup_i \mu_{F_i,V_i}(y_{(F,A)}) \) = \( \max \{ \mu_{F_i,V_i}(x_{(F,A)}), \mu_{F_i,V_i}(y_{(F,A)}) \} \) for all \( x_{(F,A)} \) and \( y_{(F,A)} \) in \( R \). And, \( \mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) = \sup_i \mu_{F_i,V_i}(x_{(F,A)} + y_{(F,A)}) \) = \( \max \{ \mu_{F_i,V_i}(x_{(F,A)}), \mu_{F_i,V_i}(y_{(F,A)}) \} \). Therefore, \( \mu_{(F,A)}(x_{(F,A)} + y_{(F,A)}) \leq \max \{ \mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(F,A)}) \} \).

2.3 Theorem: If \( (F,A) \) and \( (G,B) \) are any two anti-fuzzy soft subhemirings of hemiring \( R_1 \) and \( R_2 \) respectively. Let \( x_{(F,A)} \) and \( y_{(F,A)} \) be in \( R_1 \), \( y_{(G,B)} \) and \( y_{(G,B)} \) be in \( R_2 \). Then, \( (x_{(F,A)} + y_{(G,B)}), (x_{(F,A)} + y_{(G,B)}) \) = \( \mu_{F,A}(x_{(F,A)} + y_{(G,B)}), \mu_{G,B}(x_{(F,A)} + y_{(G,B)}) \) = \( \max \{ \mu_{F,A}(x_{(F,A)}), \mu_{G,B}(x_{(F,A)}), \mu_{G,B}(x_{(F,A)}) \} \) = \( \mu_{F,A}(x_{(F,A)}) \). Therefore, \( \mu_{(F,A)}(x_{(F,A)} + y_{(G,B)}) \leq \max \{ \mu_{(F,A)}(x_{(F,A)}), \mu_{(F,A)}(y_{(G,B)}), \mu_{(G,B)}(y_{(G,B)}) \} \).

2.4 Theorem: Let \( (F,A) \) be a Fuzzy subset of a hemiring \( R \) and \( (G,V) \) be the most grounded anti Fuzzy soft connection of \( R \). At that point \( (F,A) \) is an anti-fuzzy soft subhemiring of \( R \) if and only if \( (G,V) \) is an anti-fuzzy soft subhemiring of \( R \).

**Proof:** Suppose that \( (F,A) \) is an anti-fuzzy soft subhemiring of a hemiring \( R \). Then for any \( x = (x_{(F,A)}), y = (y_{(F,A)}) \) be in \( R \). We have, \( \mu_{G,V}(x_{(G,V)} + y_{(G,V)}) = \mu_{G,V}(x_{(G,V)}, x_{(G,V)} + y_{(G,V)}) \leq \max \{ \mu_{G,V}(x_{(G,V)}), \mu_{G,V}(y_{(G,V)}) \} \). Therefore, \( \mu_{(G,V)}(x_{(G,V)} + y_{(G,V)}) \leq \max \{ \mu_{(G,V)}(x_{(G,V)}), \mu_{(G,V)}(y_{(G,V)}) \} \).
\[(x_{G,V}) + y_{(G,V)} \leq \max\{ \mu_{G,V}(x_{G,V}), \mu_{G,V}(y_{G,V}) \} = \max\{ \mu_{G,V}(x_{G,V}), \mu_{G,V}(y_{G,V}) \} = \max\{ \mu_{F,A}(x_{F,A}), \mu_{F,A}(y_{F,A}) \}, \mu_{F,A}(x_{F,A}) \}\]

If \( \mu_{F,A}(x_{F,A}+y_{(F,A)}) \geq \mu_{F,A}(x_{F,A}+y_{F,A}), \mu_{F,A}(x_{F,A}) \geq \mu_{F,A}(x_{F,A}) \), \( \mu_{F,A}(y_{(F,A)}) \geq \mu_{F,A}(y_{F,A}) \), we get, \( \mu_{F,A}(x_{F,A}+y_{(F,A)}) \leq \max\{ \mu_{F,A}(x_{F,A}), \mu_{F,A}(y_{F,A}) \} \), for all \( x_{F,A} \) and \( y_{(F,A)} \) in \( R \). And, \( \mu_{F,A}(x_{F,A}+y_{F,A}) \) is a anti Fuzzy soft subhemiring of a hemiring \( R \).

2.5 Theorem: \((F,A)\) is a anti –fuzzy soft subhemiring of a hemiring \((R,+,\cdot)\) if and just if \( \mu_{F,A}(x_{F,A}+y_{(F,A)} \leq \mu_{F,A}(x_{F,A}) \leq \mu_{F,A}(x_{F,A}), \mu_{F,A}(y_{(F,A)}) \leq \mu_{F,A}(y_{F,A}) \), \( \mu_{F,A}(x_{F,A}) \leq \mu_{F,A}(x_{F,A}), \mu_{F,A}(y_{(F,A)}) \leq \mu_{F,A}(y_{F,A}) \), for all \( x_{F,A} \) and \( y_{(F,A)} \) in \( R \).

Proof: It is trivial.

2.6 Theorem: If \((F,A)\) is an anti-fuzzy soft subhemiring of a hemiring \((R,+,\cdot)\), then \( H = \{ x_{(F,A)} / x_{(F,A)} \in R : \mu_{F,A}(x_{F,A}) = 0 \} \) is either unfilled or is a subhemiring of \( R \).

Proof: If no component fulfills this condition, at that point \( H \) is vacant. On the off chance that \( x(F,A) \) and \( y(F,A) \) in \( H \), at that point \( \mu_{F,A}(x_{F,A}+y_{F,A}) \leq \max\{ \mu_{F,A}(x_{F,A}), \mu_{F,A}(y_{F,A}) \} = \max\{ 0, 0 \} = 0 \). Therefore, \( \mu_{F,A}(x_{F,A}+y_{F,A}) = 0 \) and \( \mu_{F,A}(y_{F,A}) \leq \max\{ \mu_{F,A}(x_{F,A}), \mu_{F,A}(y_{F,A}) \} = \max\{ 0, 0 \} = 0 \). Therefore, \( \mu_{F,A}(x_{F,A}) = 0 \). We get \( x_{(F,A)} \leq y_{(F,A)} \) in \( H \).

In this way, \( H \) is a subhemiring of \( R \). Hence \( H \) is either unfilled or is a subhemiring of \( R \).

2.7 Theorem: Let \((F,A)\) be an anti-fuzzy soft subhemiring of a hemiring \((R,+,\cdot)\). On the off chance that \( \mu_{F,A}(x_{F,A}+y_{(F,A)}) = 1 \), at that point either \( \mu_{F,A}(x_{F,A}) = 1 \) or \( \mu_{F,A}(y_{(F,A)}) = 1 \), for all \( x_{F,A} \) and \( y_{(F,A)} \) in \( R \).

Proof: Let \( x_{F,A} \) and \( y_{(F,A)} \) in \( R \). By the definition \( \mu_{F,A}(x_{F,A}+y_{(F,A)}) \leq \mu_{F,A}(x_{F,A}), \mu_{F,A}(y_{(F,A)}) \), which implies that \( 1 \leq \mu_{F,A}(x_{F,A}), \mu_{F,A}(y_{(F,A)}) \).

In this manner, either \( \mu_{F,A}(x_{F,A}) = 1 \) or \( \mu_{F,A}(y_{(F,A)}) = 1 \).

In the accompanying Theorem \( \circ \) is the structure operation of capacities:

2.8 Theorem: Let \((F,A)\) be an anti-fuzzy soft subhemiring of a hemiring \( H \) and \( f \) is an anti- isomorphism from a hemiring \( R \) onto \( H \). At that point \( (F,A)^f \) is a anti Fuzzy soft subhemiring of \( R \).

Proof: Let \( x_{(F,A)} \) and \( y_{(F,A)} \) in \( R \) and \((F,A)\) be an anti-fuzzy soft subhemiring of a hemiring \((R,+,\cdot)\). Then we have, \( \mu_{F,A}(f(x_{(F,A)}+y_{(F,A)}) = \mu_{F,A}(f(x_{(F,A)}), \mu_{F,A}(y_{(F,A)}) \), as \( f \) is an anti-isomorphism \( \leq \max\{ \mu_{F,A}(f(x_{(F,A)}), \mu_{F,A}(y_{(F,A)}) \} \), which suggests that \( (F,A)^f \) is a anti Fuzzy soft subhemiring of a hemiring \((R,+,\cdot)\).

2.9 Theorem: Let \((F,A)\) be a anti Fuzzy soft subhemiring of a hemiring \((R,+,\cdot)\), then the pseudo anti Fuzzy soft coset \((a(F,A))^p\) is an anti fuzzy soft subhemiring of a hemiring \((R,+,\cdot)\), for each \( a \) in \( R \).

Proof: Let \((F,A)\) be an anti-fuzzy soft subhemiring of a hemiring \((R,+,\cdot)\). For every \( x_{(F,A)} \) and \( y_{(F,A)} \) in \( R \), we have, \( (a(F,A))^p(x_{(F,A)}+y_{(F,A)}) = (a(F,A))^p(x_{(F,A)}, (a(F,A))^p(y_{(F,A)}) \). Therefore, \((a(F,A))^p(x_{(F,A)}+y_{(F,A)}) \leq \max\{ (a(F,A)^p(x_{(F,A)}), (a(F,A))) \} = \mu_{F,A}(x_{(F,A)}+y_{(F,A)}) \).

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2.10 Theorem: Let (R, +, . ) and (R,*, . ) be any two hemirings. The homomorphic image of an anti Fuzzy soft subhemiring of R is an anti Fuzzy soft subhemiring of R’.

Proof: Let (R, +, . ) and (R’, +, . ) be any two hemirings. Let f : R → R’ be a homomorphism. At that point, i) f(x+y) = f(x) + f(y) and ii) f(xy) = f(x)f(y), for all x and y in R. Let (G,V) = f(F,A), where (F,A) is an anti Fuzzy soft subhemiring of R. We need to demonstrate that (G,V) is an anti Fuzzy soft subhemiring of R’.

Now, for f(x(G,V)), f(y(G,V)) in R’, μ_{G,V}( f(x(G,V)) + f(y(G,V))) = μ_{G,V} ( f(x(G,V)) + f(y(G,V))), as f is a homomorphism ≤ μ_{F,A}( (F,A)(x(G,V)) + (F,A)(y(G,V)) ), which suggests that μ_{G,V}( f(x(G,V)) + f(y(G,V))) ≤ max { μ_{G,V}( f(x(G,V))), μ_{G,V}( f(y(G,V))) }, as f is a homomorphism ≤ μ_{F,A}( (F,A)(x(G,V)) ≤ max { μ_{F,A}( (F,A)(x(G,V))), μ_{F,A}( (F,A)(y(G,V))) } which infers that μ_{G,V}( f(x(G,V)) + f(y(G,V))) ≤ max { μ_{G,V}( f(x(G,V))), μ_{G,V}( f(y(G,V)))%(G,V)}(f(G,V)), Hence (G,V) is an anti-fuzzy soft subhemiring of R’.

2.11 Theorem: Let (R, +, . ) and (R’, +, . ) be any two hemirings. The homomorphic preimage of an anti-fuzzy soft subhemiring of R’ is an anti-fuzzy soft subhemiring of R.

Proof: Let (R, +, . ) and (R’, +, . ) be any two hemirings. Let f : R → R’ be a homomorphism. At that point, i) f(x+y) = f(x) + f(y) and ii) f(xy) = f(x)f(y), for all x and y in R.

Let (G,V) = f(F,A), where (G,V) is an anti-fuzzy soft subhemiring of R’. We need to demonstrate that (F,A) is an anti Fuzzy soft subhemiring of R. Let x(F,A) and y(F,A) in R, μ_{F,A}( x(F,A) + y(F,A) ) = μ_{G,V}( f(x(F,A)) + f(y(F,A))) ≤ max { μ_{G,V}( f(x(F,A))), μ_{G,V}( f(y(F,A))) }, Since μ_{G,V}( f(x(F,A)) = μ_{G,V}( f(x)), μ_{G,V}( f(y(F,A)) = μ_{G,V}( f(y)), as f is a homomorphism ≤ μ_{F,A}( x(F,A) ≤ max { μ_{F,A}( x(F,A))), μ_{F,A}( y(F,A)) }, Since μ_{G,V}( f(x(F,A)) = μ_{G,V}( f(x)) = μ_{G,V}( f(y(F,A)) = μ_{G,V}( f(y)), as f is a homomorphism ≤ μ_{F,A}( x(F,A) ≤ max { μ_{F,A}( x(F,A))), μ_{F,A}( y(F,A)) }, Again, μ_{G,V}( f(y(F,A)) = μ_{G,V}( f(x)), μ_{G,V}( f(y(F,A)) = μ_{G,V}( f(y)), as f is a homomorphism ≤ μ_{F,A}( x(F,A) ≤ max { μ_{F,A}( x(F,A))), μ_{F,A}( y(F,A)) }, Hence (F,A) is an anti-fuzzy soft subhemiring of R.

2.12 Theorem: Let (F,A) be an anti-fuzzy soft subhemiring of a hemiring R. At that point (F,A) is an anti-fuzzy soft subhemiring of a hemiring R , for all x(F,A) and y(F,A) in R.

Proof: For any x(F,A) ∈ R, we have (F,A) = (F,A) / (F,A) (0) ) / / (F,A) (0) ) , (F,A) (y(F,A)) / (F,A) (0) ) , = max{ (F,A) (x(F,A)) / (F,A) (0) ) , (F,A) (y(F,A)) / (F,A) (0) ) , (F,A) (x(F,A)) / (F,A) (0) ) , (F,A) (y(F,A)) / (F,A) (0) ) , That is (F,A) is an anti-fuzzy soft subhemiring of a hemiring R , for all x(F,A) and y(F,A) in R.

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