Symmetric Skew Reverse n-Derivations on Prime Rings and Semiprime rings

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Abstract: Let n ≥ 2 be any fixed positive integer and δ denote the trace of symmetric skew reverse n-derivation Δ: R^n → R, associated with an antiautomorphism α. Let I be any Ideal of R. (1) If R is non commutative prime ring such that [δ(x), α(x)] = 0, for all x ∈ I then Δ = 0 in R. (2) Let R be non commutative semiprime ring such that δ is commuting on I and [δ(x), α(x)] ∈ Z(R), for all x ∈ I then [δ(x), α(x)] = 0 for all x ∈ I.

Key words: Prime ring, semiprime ring, commuting mapping, centralizing mapping, derivation, skew derivation, reverse derivation, skew reverse derivation, automorphism, antiautomorphism.

1. INTRODUCTION

The concept of reverse derivations of prime rings was introduced by Bresar and Vukman [2]. Relations between derivations and reverse derivations with examples were given by Samman and Alyamani [17]. Recently there has been a great deal of work done by many authors on commutativity and centralizing mappings on prime rings and semi prime rings in connection with derivations, skew derivations, reverse derivations, skew reverse derivations [1, 4-6, 7-12, 15-17, 19-22]. Vukman [19-22], Mohammad Ashraf [1], Jung and Park [7] have studied the concepts of symmetric biderivations, 3-derivations, 4-derivations and n-derivations. Ajda Fosner [5], Faiza Shujat and Abuzaid Ansari [17], Jayasubba Reddy et al. [8] and Basudeb Dhar and Faiza Shujat [4], Yadav and Sharma [22] have extended and studied the concepts of symmetric skew 3-derivations, 4-derivations and n-derivations. Recently Jayasubba Reddy et al., [9-12] have studied the concepts of symmetric skew reverse derivations. Inspired by these works we have made an attempt to introduce the notion of symmetric skew reverse n- derivations and prove some properties, which may be a contribution to the theory of commuting and centralizing mappings of prime rings and semi prime rings.

2. Preliminaries

Throughout this paper R will represent a ring with center Z (R). Let n be any integer. A ring R is said to be n-torsion free, if for all x, y ∈ R, n x = 0 ⇒ x = 0. Recall that a ring R is said to be prime, if a R b = 0, ∀ a, b ∈ R ⇒ either a=0 (or) b=0 and a ring R is said to be semiprime, if a R a = 0, ∀ a ∈ R ⇒ a=0. Let us denote the commutator xy - yx by [x, y]. In this paper we extensively make use of the commutator [x, y z] and [x, y z] = [x, y] z + y [x, z]. Let I be any non-empty two sided ideal of R. Then the mapping δ: R → R is said to be commuting on I, if [δ(x), x] = 0, ∀ x ∈ I and centralizing on I if, [δ(x), x] ∈ Z(R), ∀ x ∈ I. An additive mapping D: R → R is called a derivation if, D(xy) = D(x) y + x D(y) holds ∀ x, y ∈ R and is called a reverse derivation, if D(xy) = D(y) x + y D(x) holds ∀ x, y ∈ R. An additive mapping D: R → R is called a skew derivation (α-derivation) associated with an automorphism α if, D(xy) = D(x) y + α(x) D(y) holds ∀ x, y ∈ R and is called symmetric skew reverse derivation (α'- derivation) associated with an antiautomorphism α' if, D(xy) = D(y) x + α'(y) D(x) holds ∀ x, y ∈ R.

Definition 2.1. Let n ≥ 2 be any fixed positive integer and a mapping Δ: R^n → R is said to be symmetric (permuting), if the equation

Δ(x_1, x_2, x_3, ..., x_n) = Δ(x_π(1), x_π(2), ..., x_π(n)) holds ∀ x_i ∈ R and for every permutation π ∈ S_n, the Symmetric permutation group of order n.

Definition 2.2. Let n ≥ 2 be any fixed positive integer and a mapping Δ: R^n → R is said to be symmetric if, D(xy) = D(x) y + α(x) D(y) holds ∀ x, y ∈ R and is called symmetric skew reverse derivation (α'- derivation) associated with an antiautomorphism α' if, D(xy) = D(y) x + α'(y) D(x) holds ∀ x, y ∈ R.

Definition 2.3. An n – additive mapping Δ: R^n → R is said to be symmetric skew n – derivations, associated with an automorphism α, if for every i = 1, 2, 3..., n and for all x_1, x_2, ..., x_i-1, x_i+1, ..., x_n ∈ R, the mapping

x → Δ(x_1, x_2, ..., x_i-1, x, x_i+1, ..., x_n) is a skew derivation of R associated with the automorphism α. In particular, for all x, y, x_1, x_2, ..., x_i-1, x_i+1, ..., x_n ∈ R,
\( \Delta(x_1, x_2, \ldots, y) = \Delta(x_1, x_2, \ldots, y) + \alpha(x) \Delta(x_1, x_2, \ldots, y) \)

**Definition 2.4.** An \( n \)-additive mapping \( \Delta: R \rightarrow R \) is said to be symmetric skew reverse \( n \)-derivation, associated with the antiautomorphism \( \alpha \), if for every \( i=1,2,3,\ldots,n \) and for all \( x, y \in R \), the mapping \( x \mapsto \Delta(x_1, x_2, \ldots, x_{i-1}, x, x_{i+1}, \ldots, x_n) \) is a skew reverse derivation of \( R \) associated with the antiautomorphism \( \alpha \). In particular, for all \( x, y, x_1, x_2, \ldots, x_{i-1}, x, x_{i+1}, \ldots, x_n \in R \),

\[
\Delta(x_1, x_2, \ldots, y) = \Delta(x_1, x_2, \ldots, y) + \alpha(x) \Delta(x_1, x_2, \ldots, y),
\]

Further,
\[
\Delta(0, x_2, \ldots, y) = \Delta(0, x_2, \ldots, y).
\]

\( \Delta(0, x_2, \ldots, y) + \Delta(0, x_2, \ldots, y) = 0 \Rightarrow \Delta(x_1, x_2, \ldots, y) + \Delta(x_1, x_2, \ldots, y) = 0. \]

Clearly, \( \Delta \) is an odd function if \( n \) is odd and is an even function, if \( n \) is even. If \( \Delta: R \rightarrow R \) is a symmetric map which is \( n \)-additive then the trace \( \delta \) of \( \Delta \) satisfy the relation:

\[
\delta(x + y) = C_0 \Delta(x, x, \ldots, x) + C_1 \Delta(x, x, \ldots, x) + C_2 \Delta(x, x, \ldots, x) + \ldots + C_n \Delta(x, x, \ldots, x) + \ldots + \Delta(x, x, x, \ldots, x, x, \ldots, x), \forall x, y \in R.
\]

For symmetric biderivations \( \delta \) satisfies the relation:

\[
\delta(x + y) = \delta(x) + 2\Delta(x, x) + \delta(y), \forall x, y \in R.
\]

For symmetric 3-derivations \( \delta \) satisfies the relation:

\[
\delta(x + y) = \delta(x) + 3\Delta(x, x, x) + 3\Delta(x, y) + \delta(y), \forall x, y \in R.
\]

For symmetric 4-derivations \( \delta \) satisfies the relation:

\[
\delta(x + y) + \delta(x) + 6\Delta(x, x, y) + 6\Delta(x, y, x) + \delta(y), \forall x, y \in R.
\]

For convenience let us define \( \delta(x + y) \) as follows:

\[
\delta(x + y) = \sum_{r=0}^{n} c_r f_r(x, y).
\]

For symmetric biderivations \( \alpha^* \) satisfies the relation:

\[
\alpha^*(x) = \alpha^*(x) = 0, \forall x, y \in R.
\]

\( \Delta(0, x_2, \ldots, y) = \Delta(0, x_2, \ldots, y). \)

\( \Delta(0, x_2, \ldots, y) + \Delta(0, x_2, \ldots, y) = 0. \)

\[
\delta(x + y) = \alpha^*(x) \Delta(x, x, x) + \alpha^*(x) \Delta(x, x, x) + \alpha^*(x) \Delta(x, x, x) + \ldots + \alpha^*(x) \Delta(x, x, x) + \ldots + \alpha^*(x) \Delta(x, x, x).
\]

\[
\delta(x + y) = \alpha^*(x) \Delta(x, x, x) + \alpha^*(x) \Delta(x, x, x) + \alpha^*(x) \Delta(x, x, x) + \ldots + \alpha^*(x) \Delta(x, x, x) + \ldots + \alpha^*(x) \Delta(x, x, x).
\]

\[
\Delta(A_1, A_2, \ldots, A_n) = \alpha^*(A_1) \Delta(A_1, A_2, \ldots, A_n) = \alpha^*(A_1) \Delta(A_1, A_2, \ldots, A_n) = \ldots = \alpha^*(A_1) \Delta(A_1, A_2, \ldots, A_n).
\]

\[
\alpha^*(A_1) \Delta(A_1, A_2, \ldots, A_n) = \alpha^*(A_1) \Delta(A_1, A_2, \ldots, A_n) = \ldots = \alpha^*(A_1) \Delta(A_1, A_2, \ldots, A_n).
\]

\[
\Delta(A_1, A_2, \ldots, A_n) = \alpha^*(A_1) \Delta(A_1, A_2, \ldots, A_n).
\]

Hence from the above it is clear that the \( n \)-additive map \( \Delta \) is a symmetric skew reverse \( n \)-derivation associated with an antiautomorphism \( \alpha^* \). Now let us quote some important lemmas, which are useful in proving the main results.

**Lemma 2.6.** [3] Let \( n \) be a fixed positive integer and \( R \) be an \( n! \)-torsion free ring. Suppose that \( y_1, y_2, y_3, \ldots, y_n \in R \), which satisfy the relation:
Let \( R \) be a prime ring. Let \( a \in R \). If \( a(0) = 0 \) or \( a(1) = 0 \), then we have either \( a = 0 \) or \( b = 0 \).

**Lemma 2.8.** Let \( R \) be a prime ring. If \( a(0) = 0 \) or \( a(1) = 0 \), then either \( a = 0 \) or \( b = 0 \). Since \( R \) is prime then either \( a = 0 \) or \( b = 0 \).

**Proof.** Replacing \( x \) by \( xy \) in the relation \( a(0) = 0 \), we have \( \alpha(a(0)) = 0 \), for all \( x \in I \).

\[ \Delta(x, x, x, \ldots, x) = 0, \quad x \in R \]

**3. The main results:**

**Theorem 3.1.** Let \( n \geq 2 \) be any fixed positive integer. Let \( R \) be a non-commutative \( n \)-derivation associated with an antiautomorphism \( \alpha^* \). Let \( \delta \) denote the trace of \( \Delta \) such that \( \delta(x, \alpha(x)) = 0 \), for all \( x \in I \) then \( \Delta(x_1, x_2, x_3, \ldots, x_n) = 0 \), \( x_i \in R \).

**Proof.** Our supposition is that there exist a symmetric skew reverse \( n \)- derivation \( \Delta : R \to R \) associated with an antiautomorphism \( \alpha^* \). Let \( \delta \) denote the trace of \( \Delta \) such that \( \delta(x, \alpha(x)) = 0 \), for all \( x \in I \).

Substituting \( x \) by \( x + \mu y \) (\( 1 \leq \mu \leq n \)), in (1), we have

\[ \delta(x + \mu y), \alpha^*(x + \mu y) = 0 \quad \forall \ x, y \in I \]

\[ \Rightarrow \sum_{r=0}^{n-1} c_r f_r(x, \mu y), \alpha^*(x + \mu y) = 0 \]

\[ \Rightarrow \delta(x) + \delta(\mu y) + \sum_{r=0}^{n-1} c_r f_r(x, \mu y), \alpha^*(x) + \mu \alpha^*(y) = 0 \]

\[ \Rightarrow \delta(x) + \delta(\mu y) + \sum_{r=0}^{n-1} c_r f_r(x, \mu y), \alpha^*(x) + \mu \alpha^*(y) = 0 \]

Applying Lemma 2.6 to equation (2),

\[ [c_1 f_1(x, y), \alpha^*(x)] + [\delta(x), \alpha^*(y)] = 0. \]

Now replacing \( y \) by \( xy \) in equation (3),

\[ [c_1 f_1(x, y) + \alpha^*(x)] f_1(x, y), \alpha^*(x)] + [\delta(x), \alpha^*(y)] = 0 \]

**Proof.** Replacing \( x \) by \( xy \) in the relation \( a(0) = 0 \), we have \( \alpha(a(0)) = 0 \), for all \( x \in I \).

\[ \Delta(x, x, x, \ldots, x) = 0, \quad x \in R \]

Applying Lemma 2.6 to equation (2),

\[ [c_1 f_1(x, y), \alpha^*(x)] + [\delta(x), \alpha^*(y)] = 0. \]

Now replacing \( y \) by \( xy \) in equation (3),

\[ [c_1 f_1(x, y) + \alpha^*(x)] f_1(x, y), \alpha^*(x)] + [\delta(x), \alpha^*(y)] = 0 \]

**Proof.** Replacing \( x \) by \( xy \) in the relation \( a(0) = 0 \), we have \( \alpha(a(0)) = 0 \), for all \( x \in I \).

\[ \Delta(x, x, x, \ldots, x) = 0, \quad x \in R \]

Applying Lemma 2.6 to equation (2),

\[ [c_1 f_1(x, y), \alpha^*(x)] + [\delta(x), \alpha^*(y)] = 0. \]

Now replacing \( y \) by \( xy \) in equation (3),

\[ [c_1 f_1(x, y) + \alpha^*(x)] f_1(x, y), \alpha^*(x)] + [\delta(x), \alpha^*(y)] = 0 \]
Hence from the above we have,
\[ \delta(x) = 0 \quad \forall \ x \in I \cap Z \] (8)

Now for each value of \( l=1, 2, \ldots, n \) let us denote
\[ T_l(x) = \Delta(x, x, x, \ldots, x, y_l+1, y_l+2, y_l+3, \ldots, y_n), \]
where \( x, y_l \in I, \ l = l+1,l+2,\ldots,n \)
\[ \Rightarrow T_n(x) = \delta(x) = 0 \quad \forall \ x \in I \] (9)

Let \( \eta (1 \leq \eta \leq n) \) be any positive integer. Then from (9),
\[ T_n(\eta x + x_n) = 0 \]
\[ \Rightarrow T_n(x_n) + T_n(\eta x) + \sum_{l=1}^{n-1} \eta^l c_l T_l(x) = 0 \]
\[ \Rightarrow \delta(x_n) + \eta^n \delta(x) + \sum_{l=1}^{n-1} \eta^l c_l T_l(x) = 0 \]
\[ \Rightarrow \sum_{l=1}^{n-1} \eta^l c_l T_l(x) = 0 \quad \forall \ x, x_n \in I \]
\[ \Rightarrow \eta^1 c_1 T_1(x) + \eta^2 c_2 T_2(x) + \eta^3 c_3 T_3(x) \]
\[ + \ldots + \eta^{n-1} c_{n-1} T_{n-1}(x) = 0. \] (10)

Again applying Lemma 2.8 to equation (10),
\[ c_1 T_1(x) = 0 \Rightarrow T_1(x) = 0 \Rightarrow \Delta(x, x_2, x_3, \ldots, x_n) = 0 \]
\[ c_2 T_2(x) = 0 \Rightarrow T_2(x) = 0 \Rightarrow \Delta(x, x, x_2, x_3, \ldots, x_n) = 0 \ldots \]
\[ c_{n-1} T_{n-1}(x) = 0 \Rightarrow T_{n-1}(x) = 0 \Rightarrow \Delta(x, x, x, \ldots, x, x_n) = 0 \]

Hence from the above, we have \( T_n(x) = 0, \forall x \in I \) (11)
Again let \( \tau (1 \leq \tau \leq n) \) be any positive integer. Then from (11)
\[ T_{n-1}(rx + x_{n-1}) = 0, \forall x, x_{n-1} \in I \]
\[ \Rightarrow T_{n-1}(rx) + T_{n-1}(x_{n-1}) + \sum_{l=1}^{n-2} \eta^l c_l T_l(x) = 0 \]
\[ \Rightarrow r^1 c_1 T_1(x) + r^2 c_2 T_2(x) + r^3 c_3 T_3(x) \]
\[ + \ldots + r^{n-2} c_{n-2} T_{n-2}(x) = 0. \quad \forall x \in I \] (12)

Again applying Lemma (1) to result (12) we have, \( \Rightarrow \Delta(x, x, x, \ldots, x, x_{n-1}, x_n) = 0 \)
\[ \Rightarrow T_{n-2}(x) = 0, \forall x \in I \] (13)

Continuing the above process, finally we obtain \( T_1(x) = 0, \forall x \in I \).
\[ \Rightarrow \Delta(x_1, x_2, x_3, \ldots, x_{n-1}, x_n) = 0, \forall x \in I. \] (14)

Now replacing \( x_1 \) by \( x_1 p_l \), where \( p_l \in R \) in (14),
\[ \Delta(x_1 p_1, x_2, x_3, \ldots, x_{n-1}, x_n) = 0 \]
\[ \Rightarrow \Delta(p_1, x_2, x_3, \ldots, x_{n-1}, x_n) = 0 \]
\[ + \alpha(p_1) \Delta(x_1, x_2, x_3, \ldots, x_{n-1}, x_n) = 0 \]
\[ \Rightarrow \Delta(p_1, x_2, x_3, \ldots, x_{n-1}, x_n), x_1 = 0. \] (15)

Now applying Lemma 2.8 to equation (15),
\[ \Delta(p_1, x_2, \ldots, x_{n-1}, x_n) = 0, p_1 \in R, \forall x \in I \]

Now replacing \( x_2 \) by \( x_1 p_2 \), where \( p_2 \in R \) in (15),
\[ \Delta(p_1, p_2, x_3, x_4, \ldots, x_{n-1}, x_n) = 0, \forall p_1, p_2 \in R, \forall x \in I \]
Repeating the above process we finally obtain, \( \Delta(p_1, p_2, p_3, \ldots, p_{n-1}, p_n) = 0, p_i \in R \)
Hence the proof of the theorem is completed. \( \square \)

**Theorem 3.2.** Let \( n \geq 2 \) be any fixed positive integer. Let \( R \) be non commutative ! - torsion free semiprime ring and \( I \) be any non – zero two sided ideal of \( R \). Suppose that there exist a symmetric skew reverse derivation \( n \)-derivation \( R^a \rightarrow R \) associated with an antiautomorphism \( \alpha^* \). Let \( \delta \) denote the trace of \( A \) such that \( \delta(x), x \) = 0, for all \( x \in I \) and \( \{\delta(x), \alpha^*(x)\} \in Z(R), \forall x \in I \).

**Proof.** Our supposition is that there exist a symmetric skew reverse \( n \) – derivation \( A : R^a \rightarrow R \) associated with an antiautomorphism \( \alpha^* \) such that \( \delta(x), x \) = 0, for all \( x \in I \) and \( \{\delta(x), \alpha^*(x)\} \in Z(R), \forall x \in I \).

Let \( \mu (1 \leq \mu \leq n) \) be any positive integer. Substituting \( x \) by \( x+\mu y \) in (1) we have,
\[ \{\delta(x+\mu y), \alpha^*(x+\mu y)\} \in Z, \forall x, y \in I \]
\[ \Rightarrow [\sum_{r=0}^{\mu} c_r f_r(x, \mu y), \alpha^*(x) + \alpha^*(\mu y)] \in Z \]
\[ \Rightarrow \mu \left[ \{\delta(x, \alpha^*(y)) + [c_1 f_1(x, y), \alpha^*(x)] \right] \]
\[ + \mu^2 \left[ \{c_2 f_2(x, y), \alpha^*(x)] + [c_1 f_1(x, y), \alpha^*(y)] \right] \]
\[ + \mu^3 \left[ \{c_3 f_3(x, y), \alpha^*(y)] + \{\delta(y), \alpha^*(x) \} \right] \]
\[ + \mu^{n+1} \{\delta(y), \alpha^*(x) \} + [\delta(x), \alpha^*(x)] \in Z, \forall x, y \in I \] (17)

Now commuting (17) with \( \delta(x) \),
\[ \mu \left[ \{\delta(x), \alpha^*(y)] + [c_1 f_1(x, y), \alpha^*(x)] \right] \]
\[ + \mu^2 \left[ \{c_2 f_2(x, y), \alpha^*(x)] + [c_1 f_1(x, y), \alpha^*(y)] \right] \]
\[ + \mu^3 \left[ \{c_3 f_3(x, y), \alpha^*(y)] + [\delta(y), \alpha^*(x)] \right] \]
\[ + \mu^{n+1} \{\delta(y), \alpha^*(x) \}, \delta(x) \]
\[ + [\delta(x), \alpha^*(x)], \delta(x) \] = 0, \( \forall x, y \in I \) (18)

Applying Lemma 2.6 to equation (18), we have
\[ \left[ \{c_1 f_1(x, x^2), \alpha^*(x)] + [\delta(x), \alpha^*(x)] \right], \delta(x) \] = 0 (19)
\[ (c_1 + 1)\alpha^*(x)[\delta(x),\alpha^*(x)], \delta(x) \]
\[ + \left[ \delta(x), \alpha^*(x) \right] \alpha^*(x), \delta(x) \]
\[ + c_1[\delta(x), \alpha^*(x)]x, \delta(x) \]
\[ + c_1\delta(x)[x, \alpha^*(x)], \delta(x) = 0 \]
\[ \Rightarrow (c_1 + 1)[\alpha^*(x), \delta(x)][\delta(x), \alpha^*(x)] \]
\[ + (c_1 + 1)\alpha^*(x)[\delta(x), \alpha^*(x)], \delta(x) \]
\[ + [\delta(x), \alpha^*(x)][\alpha^*(x), \delta(x)] + \left[ [\delta(x), \alpha^*(x)], \delta(x) \right] \alpha^*(x) \]
\[ + c_1[\delta(x), \alpha^*(x)][x, \delta(x)] + c_1[\delta(x), \alpha^*(x)], \delta(x) \]
\[ + c_1\delta(x)[x, \alpha^*(x)], \delta(x)] + c_1[\delta(x), \alpha^*(x)][x, \alpha^*(x)] = 0 \]
\[ \Rightarrow (c_1 + 1)[\delta(x), \alpha^*(x)]^2 + [\delta(x), \alpha^*(x)]^2 \]
\[ + c_1\delta(x)[x, \alpha^*(x) - \alpha^*(x)x] = 0 \]
\[ \Rightarrow (c_1 + 2)[\delta(x), \alpha^*(x)]^2 + c_1\delta(x)[x, \delta(x), \alpha^*(x)] = 0 \]
\[ \Rightarrow (c_1 + 2)[\delta(x), \alpha^*(x)]^2 = 0, \forall x \in 1 \] (20)

Again commuting equation (17) with \( \alpha^*(x) \) and using Lemma.2.6,
\[ \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(y)], \alpha^*(x) = 0 \] (21)

Now replacing \( y \) by \( yx \) in the above equation (21),
\[ \left[ c_1 f_1(x, yx), \alpha^*(x) \right] + [\delta(x), \alpha^*(yx)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \right] \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)]^2, \alpha^*(x) \]
\[ \Rightarrow \left[ \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \right] \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow c_1 f_1(x, y), \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
\[ \Rightarrow \left[ c_1 f_1(x, y), \alpha^*(x) \right] + [\delta(x), \alpha^*(x)], \alpha^*(x) \]
\[ + [\delta(x), \alpha^*(x)][\delta(x), \alpha^*(x)], \alpha^*(x) = 0 \]
Now combining (20) and (24), \( [\delta(x), \alpha^*(x)]^2 = 0, \forall x \in I \).

As the center of the semiprime ring contains no non-zero nilpotent elements we have \( [\delta(x), \alpha(x)] = 0, \forall x \in I \).

Hence the proof of the theorem is completed. □

**Theorem 3.3.** Let \( n \geq 2 \) be any fixed positive integer. Let \( R \) be an \( n! \)-torsion free prime ring and \( I \) be any non zero two sided ideal of \( R \). Suppose that there exist a non zero symmetric skew reverse \( n \)-derivation \( \delta^* \) associated with an antiautomorphism \( \alpha^* \). Let \( \delta \) denote the trace of \( \Delta \) such that \( \delta \) is commuting on \( I \) and \( [\delta(x), \alpha(x)] \in Z(R) \), for all \( x \in I \) then \( R \) must be commutative.

**Proof.** On contrary suppose that \( R \) is a non commutative prime ring. Then from Theorem 3.2, we have \( [\delta(x), \alpha(x)] = 0, \forall x \in I \).

Now from Theorem 3.1, we have \( \Delta = 0, \forall x \in R \), which is a contradiction. Hence \( R \) must be commutative prime ring. □

**Conflict of Interests:**

The authors declare that there is no conflict of interests.

**References:**