Contra Harmonic Mean Labeling of Some Disconnected Graphs

S.S. Sandhya, S.Somasundaram, J. Rajeshni Golda

Assistant Professor in Mathematics, Sree Ayyappa College for Women, Chunnakadai, Kanyakumari District, Tamilnadu, India.
Professor in Mathematics, Manonmaniam Sundaranar University, Tirunelveli, Tamilnadu, India
Assistant Professor in Mathematics, Women’s Christian College, Nagercoil, Kanyakumari District, Tamilnadu, India.

Abstract:

A graph G(V,E) is called a Contra Harmonic mean graph with p vertices and q edges, if it is possible to label the vertices x ∈ V with distinct elements f(x) from 0, 1, …, q in such a way that when each edge e = uv is labeled with distinct edge labels. Here f is called a Contra Harmonic mean labeling of G.

Key words: Graph, Contra Harmonic mean labeling, Contra Harmonic mean graphs, Path, Cycle, Comb.

1. INTRODUCTION

All graph in this paper are simple, finite, undirected. Let G be a graph with p vertices and q edges. For a detail survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notation we follow Harary [2]. S. Somasundaram and R.Ponraj introduced mean labeling for some standard graphs in 2013 [3]. S. Somasundaram and S.S. Sandhya introduced Harmonic mean labeling of graph [4]. We have introduced Contra Harmonic mean labeling in [5].The Contra Harmonic mean labeling of disconnected graphs are introduced in [6]. In this paper we investigate the Contra Harmonic mean labeling behaviour of some more disconnected graphs. The following definition are useful for our present study.

Definition 1.1

A graph G (V,E) is called a Contra Harmonic mean graph with p vertices and q edges, if it is possible to label the vertices x ∈ V with distinct elements f(x) from 0, 1, …, q in such a way that when each edge e = uv is labeled with

\[ f(e = uv) = \left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor \text{ or } \left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil \]

with distinct edge labels. Here f is called a Contra Harmonic mean labeling of G.

Definition 1.2: The union of two graphs G_1 = (V_1,E_1) and G_2 = (V_2, E_2) is a graph G = G_1 \cup G_2 with vertex set V = V_1 \cup V_2 and edge set E = E_1 \cup E_2.

Definition 1.3: The corona of two graphs G_1 and G_2 is the graph G = G_1 \circ G_2 formed by taking one copy of G_1 and |V(G_1)| copies of G_2 where the i-th vertex of G_1 is adjacent to every vertex in the i-th copy of G_2.

Definition 1.4: A graph G is called a (nxm)-flower graph if it has n vertices which form a n-cycle and n sets of m-2 vertices which form m-cycles around the n-cycle on a single edge. This graph is denoted by f_{nxm}.

Theorem 1.5: Any Path is a Contra Harmonic mean graph.

Theorem 1.6: Any Cycle is a Contra Harmonic mean graph.

Theorem 1.7: Any Comb is a Contra Harmonic mean graph.

Theorem 1.8: Any Crown is a Contra Harmonic mean graph.

2. Main Results

Theorem 2.1: (C_m \circ K_3) \cup P_n is a Contra Harmonic mean graph for m ≥ 3.
**Proof:** Let \( u_1u_2\ldots u_m \) be the cycle \( C_m \). Let \( v_i,w_i \) be the vertices of \( K_3 \) that are joined to the vertex \( u_i \) of \( C_m \), \( 1 \leq i \leq m \).

Let \( P_n \) be a path with vertices \( z_1,\ldots, z_n \).

Let \( G = (C_m \odot K_3) \cup P_n \).

Define \( f: V(G) \to \{0,1,\ldots,q\} \) by

\[ f(u_i) = 4i-3, \ 1 \leq i \leq m-1, \quad f(u_m) = 4m-2 \]

\[ f(v_i) = 2, \ f(v_i) = 4i-4, \ 2 \leq i \leq m \]

\[ f(w_i) = 4i-1, \ 1 \leq i \leq m-1, \ f(w_m) = 4m \]

\[ f(z_i) = 0, \ f(z_i) = 4m+i-1, \ 2 \leq i \leq n \]

Then the distinct edge labels are

\[ f(u_{i+1}) = 4i, \ 1 \leq i \leq m-1, \ f(u_mu_1) = 4m-2 \]

\[ f(u_v_i) = 4i-3, \ 1 \leq i \leq m \]

\[ f(u_{w_i}) = 4i-1, \ 1 \leq i \leq m-1, \ f(u_mw_m) = 4m \]

\[ f(v_{w_i}) = 4i-2, \ 1 \leq i \leq m-1, \ f(v_{w_m}) = 4m-1 \]

\[ f(z_{z+1}) = 4m+i, \ 1 \leq i \leq n-1 \]

Clearly, \( (C_m \odot K_3) \cup P_n \) is a Contra Harmonic mean graph.

**The Contra Harmonic mean labeling of \( (C_3 \odot K_3) \cup P_5 \) is**

![Figure: 1](image)

**Theorem 2.2**

\( (C_m \odot K_3) \cup (P_n \odot K_3) \) is a Contra Harmonic mean graph, for \( m \geq 3 \).

**Proof:** Let \( u_1,\ldots,u_m \) be the cycle \( C_m \). Let \( v_i,w_i \) be the vertices of \( K_3 \) that are joined to the vertex \( u_i \) of \( C_m \). The resultant graph is \( (C_m \odot K_3) \).

Let \( P_n \) be a path with vertices \( t_1,\ldots,t_n \) and let \( s_i \) be the vertex that are joined to the vertex \( t_i, 1 \leq i \leq n \) of \( P_n \). The resultant graph is \( (P_n \odot K_3) \).

Let \( G = (C_m \odot K_3) \cup (P_n \odot K_3) \).

Define \( f: V(G) \to \{0,1,\ldots,q\} \) by

\[ f(u_i) = 4i-3, \ 1 \leq i \leq m-1, \ f(u_m) = 4m-2 \]

\[ f(v_i) = 2, \ f(v_i) = 4i-4, \ 2 \leq i \leq m \]

\[ f(w_i) = 4i-1, \ 1 \leq i \leq m-1, \ f(w_m) = 4m \]

\[ f(t_i) = 0, \ f(t_i) = 4m+2i-2, \ 2 \leq i \leq n \]

\[ f(s_i) = 4m+2i-1, \ 1 \leq i \leq n \]

Then the distinct edge labels are

\[ f(u_{i+1}) = 4i, \ 1 \leq i \leq m \]

\[ f(v_{w_i}) = 4i-2, \ 1 \leq i \leq m-1 \]

\[ f(v_{w_m}) = 4m-1 \]

\[ f(u_{w_i}) = 4i-1, \ 1 \leq i \leq m-1, \ f(u_mw_m) = 4m \]

\[ f(u_{u_{i+1}}) = 4i, \ 1 \leq i \leq m-1, \ f(u_mu_1) = 4m-2 \]

\[ f(t_{t+i}) = 4m+2i, \ 1 \leq i \leq n-1 \]

\[ f(t_{s_i}) = 4m+2i-1, \ 1 \leq i \leq n \]

Clearly, \( f \) is a Contra Harmonic mean graph of \( G \).
The Contra Harmonic mean labeling of

\((C_5 \odot K_3) \cup (P_6 \odot K_3)\)

Define a function \(f : V(G) \rightarrow \{0, 1, \ldots, q\}\) by

\[f(u_1) = 0, f(u_i) = 4i-3, 2 \leq i \leq m-1, f(u_m) = 4m-2\]
\[f(v_1) = 1, f(v_i) = 4i-4, 2 \leq i \leq m\]
\[f(w_i) = 4i-1, 1 \leq i \leq m-1, f(w_m) = 4m\]
\[f(t_1) = 4m+4i-3, 1 \leq i \leq n\]
\[f(s_i) = 4m+4i-2, 1 \leq i \leq n\]
\[f(x_i) = 4m+4i-1, 1 \leq i \leq n\]

Then the distinct edge labels are

\[f(u_1v_1) = 1, f(u_2v_2) = 4, f(u_iv_i) = 4i-3, 3 \leq i \leq m\]
\[f(v_1w_i) = 4i-2, 1 \leq i \leq m-1 \text{ and } f(v_mw_m) = 4m-1,\]
\[f(u_1w_i) = 4i-1, 1 \leq i \leq m-1, f(u_m, w_m) = 4m\]
\[f(u_1u_2) = 5 , f(u_1u_{i+1})=4i, 2 \leq i \leq m-1,\]
\[f(u_mu_1) = 4m-2\]
\[f(t_1t_{i+1}) = 4m+4i, 1 \leq i \leq n-1\]
\[f(t_1s_i) = 4m+4i-3, 1 \leq i \leq n\]
\[f(t_1x_i) = 4m+4i-1, 1 \leq i \leq n\]
\[f(s_ix_i) = 4m+4i-2, 1 \leq i \leq n\]

Clearly, \(f\) is a Contra Harmonic mean graph of \(G\).

The Contra Harmonic mean labeling of

\((C_m \odot K_3) \cup (P_n \odot K_3)\) is

\[f(u_1) = 0, f(u_i) = 4i-3, 2 \leq i \leq m-1, f(u_m) = 4m-2\]
\[f(v_1) = 1, f(v_i) = 4i-4, 2 \leq i \leq m\]
\[f(w_i) = 4i-1, 1 \leq i \leq m-1, f(w_m) = 4m\]
\[f(t_1) = 4m+4i-3, 1 \leq i \leq n\]
\[f(s_i) = 4m+4i-2, 1 \leq i \leq n\]
\[f(x_i) = 4m+4i-1, 1 \leq i \leq n\]

The resultant graph is \((P_n \odot K_3)\).

Let \(G = (C_m \odot K_3) \cup (P_n \odot K_3)\)
Theorem 2.4: \((C_m \odot K_1) \odot K_3 \cup P_n\) is a Contra Harmonic mean graph, for \(m \geq 3\).

Proof:

Let \(u_1u_2...u_m\) be the cycle \(C_m\). Let \(v_i\) be the vertex which is adjacent to \(u_i\), \(1 \leq i \leq m\). The resultant graph is \(C_m \odot K_1\). Let \(x_i\), \(y_i\) be the vertices of \(K_3\) which are attached to each of the vertex \(v_i\). The resultant graph is \((C_m \odot K_1) \odot K_3\).

Let \(P_n\) path with vertices \(t_1...t_n\).

Let \(G = (C_m \odot K_1) \odot K_3 \cup P_n\)

Define \(f: V(G) \rightarrow \{0, 1,..., q\}\) by

\[
\begin{align*}
f(u_1) &= 1, f(u_i) = 5i-1, 2 \leq i \leq m-1, f(u_m) = 5m \\
f(v_1) &= 2, f(v_i) = 5i-2, 2 \leq i \leq m \\
f(x_i) &= 3, f(x_i) = 5i-5, 2 \leq i \leq m \\
f(y_i) &= 4, f(y_i) = 5i-4, 2 \leq i \leq m \\
f(t_1) &= 0, f(t_i) = 5m+i-1, 2 \leq i \leq n 
\end{align*}
\]

Then the distinct edge labels are

\[
\begin{align*}
f(u_1u_2) &= 9, f(u_2u_{i+1}) = 5i+2, 2 \leq i \leq m-2 \\
f(u_{m-2}u_{m-1}) &= 5(m-1)+3, \\
f(u_{m-1}u_m) &= 5m-1 \\
f(u_1v_1) &= 1, f(v_2v_3) = 8, f(u_1v_1) = 5i-1, 3 \leq i \leq m-1, \\
f(u_mv_m) &= 5m \\
f(v_1x_i) &= 2, f(v_1x_i) = 5i-4, 2 \leq i \leq m. 
\end{align*}
\]

Clearly, \(f\) is a Contra Harmonic mean graph of \(G\).

The Contra Harmonic mean labeling of

\((C_4 \odot K_1) \odot K_3 \cup P_7\) is

![Figure 3](image.png)

![Figure 4](image.png)

Theorem 2.5: \(((C_m \odot K_1) \odot K_3) \cup (P_n \odot K_1)\) is a Contra Harmonic mean graph, for \(m \geq 3\).

Proof:

Let \(u_1u_2...u_m\) be the cycle \(C_m\). Let \(v_i\) be the vertex which is adjacent to \(u_i\), \(1 \leq i \leq m\). The resultant graph is \(C_m \odot K_1\). Let \(x_i\), \(y_i\) be the vertices of \(K_3\) which are attached to each of the vertex \(v_i\). The resultant graph is \(((C_m \odot K_1) \odot K_3)\).

Let \(t_1...t_n\) be a path \(P_n\).
Let \( s_i \) be the vertex that is joined to \( t_i \) of the path \( P_n \).

\( 1 \leq i \leq n \). The resultant graph is \( (P_n \circ K_1) \).

Let \( G = ((C_m \circ K_1) \circ K_3) \cup (P_n \circ K_1) \)

Define \( f: V(G) \rightarrow \{0,1,\ldots,q\} \) by

\[
\begin{align*}
    f(u_1) &= 1, f(u_i) = 5i-1, 2 \leq i \leq m-1, f(u_m) = 5m \\
    f(v_1) &= 2, f(v_i) = 5i-2, 2 \leq i \leq m. \\
    f(x_1) &= 3, f(x_i) = 5i-5, 2 \leq i \leq m \\
    f(y_1) &= 4, f(y_i) = 5i-4, 2 \leq i \leq m \\
    f(t_1) &= 0, f(t_i) = 5m+2i-2, 2 \leq i \leq n \\
    f(s_i) &= 5m+2i-1, 1 \leq i \leq n
\end{align*}
\]

Then the distinct edge labels are

\[
\begin{align*}
    f(u_iu_{i+1}) &= 9, f(u_iu_{i+1}) = 5i+2, 2 \leq i \leq m-2 \\
    f(u_{i+1}u_{i+2}) &= 5(m-1)+3, \\
    f(u_{i}u_{i+1}) &= 5m-1 \\
    f(u_iu_{i+1}) &= 8, f(u_iu_{i+1}) = 5i-1, 3 \leq i \leq m-1 \\
    f(u_{i+1}u_m) &= 5m \\
    f(v_{i+1}x_i) &= 2, f(v_{i+1}x_i) = 5i-4, 2 \leq i \leq m \\
    f(v_iy_i) &= 4, f(v_iy_i) = 5i-2, 3 \leq i \leq m-1 \\
    f(v_{i+1}y_m) &= 5m-3, \\
    f(x_iy_{i}) &= 3, f(x_iy_{i}) = 5i-5, 2 \leq i \leq m \\
    f(t_{i+1}t_i) &= 5m+2i-1, 1 \leq i < n-1 \\
    f(t_sn) &= 5m+2i-1, 1 \leq i \leq n
\end{align*}
\]

Clearly, \( f \) is a Contra Harmonic mean labeling of \( G \).

**The Contra Harmonic mean labeling of**

\(( (C_3 \circ K_1) \circ K_3 ) \cup (P_7 \circ K_1) \) is

\[
\begin{align*}
    f(u_1) &= 1, f(u_i) = 5i-1, 2 \leq i \leq m-1, f(u_m) = 5m \\
    f(v_1) &= 2, f(v_i) = 5i-2, 2 \leq i \leq m \\
    f(x_1) &= 3, f(x_i) = 5i-5, 2 \leq i \leq m \\
    f(y_1) &= 4, f(y_i) = 5i-4, 2 \leq i \leq m \\
    f(t_1) &= 0, f(t_i) = 5m+i-1, 2 \leq i \leq n-1 , f(t_n) = 5m+n
\end{align*}
\]
Then the distinct edge labels are
\[ f(u_1u_2) = 9, f(u_iu_{i+1}) = 5i+2, 2 \leq i \leq m-2 \]
\[ f(u_{m-2}u_{m-1}) = 5(m-1)+3, \]
\[ f(u_{m-1}u_m) = 5m-1 \]
\[ f(u_1v_1) = 1, f(u_2v_2) = 8, f(u_iv_i) = 5i-1, 3 \leq i \leq m-1 \]
\[ f(u_mv_m) = 5m \]
\[ f(v_1x_1) = 2, f(v_ix_i) = 5i-4, 2 \leq i \leq m \]
\[ f(v_1y_1) = 4, f(v_2y_2) = 7, f(v_ix_i) = 5i-2, 3 \leq i \leq m-1 \]
\[ f(v_my_m) = 5m-3, \]
\[ f(x_1y_1) = 3, f(x_2y_2) = 5i-5, 2 \leq i \leq m \]
\[ f(t_1t_{i+1}) = 5m+i, 1 \leq i \leq n-1 \]
\[ f(t_nt_1) = 5m+n \]

Clearly, \( f \) is a Contra Harmonic mean labeling of \( G \).

**The Contra Harmonic mean labeling of**

\( ((C_5 \odot K_3) \odot K_3) \cup C_6 \) is

Then the distinct edge labels are
\[ f(u_1u_2) = 9, f(u_iu_{i+1}) = 5i+2, 2 \leq i \leq m-2 \]
\[ f(u_{m-2}u_{m-1}) = 5(m-1)+3, \]
\[ f(u_{m-1}u_m) = 5m-1 \]
\[ f(u_1v_1) = 1, f(u_2v_2) = 8, f(u_iv_i) = 5i-1, 3 \leq i \leq m-1 \]
\[ f(u_mv_m) = 5m \]
\[ f(v_1x_1) = 2, f(v_ix_i) = 5i-4, 2 \leq i \leq m \]
\[ f(v_1y_1) = 4, f(v_2y_2) = 7, f(v_ix_i) = 5i-2, 3 \leq i \leq m-1 \]
\[ f(v_my_m) = 5m-3, \]
\[ f(x_1y_1) = 3, f(x_2y_2) = 5i-5, 2 \leq i \leq m \]
\[ f(t_1t_{i+1}) = 5m+i, 1 \leq i \leq n-1 \]
\[ f(t_nt_1) = 5m+n \]

**Theorem 2.7:** \( ((C_m \odot K_1) \odot K_3) \cup (P_n \odot K_3) \) is a Contra Harmonic mean graph for \( m \geq 3 \).

**Proof:** Let \( u_1u_2 \ldots u_m \) be the cycle \( C_m \). Let \( v_i \) be the vertex which is adjacent to \( u_i \), \( 1 \leq i \leq m \). The resultant graph is \( C_m \odot K_1 \). Let \( x_i, y_i \) be the vertices of \( K_3 \) which are attached to each of the vertex \( v_i \). The resultant graph is \( (C_m \odot K_1) \odot K_3 \).

Let \( t_1 \ldots \ldots t_n \) be the path \( P_n \). Let \( s_i, w_i \) be the vertex of \( K_3 \) that are joined with the vertices \( t_i \) of \( P_n \), \( 1 \leq i \leq n \).

The resultant graph is \( P_n \odot K_3 \).

Let \( G = ((C_m \odot K_1) \odot K_3) \cup (P_n \odot K_3) \)

Define \( f: V(G) \to \{0,1,\ldots,q\} \) by
\[ f(u_1) = 0, f(u_i) = 5i-1, 2 \leq i \leq m-1, f(u_m) = 5m \]
\[ f(v_1) = 1, f(v_i) = 5i-2, 2 \leq i \leq m \]
\[ f(x_1) = 2, f(x_i) = 5i-5, 2 \leq i \leq m \]
\[ f(y_1) = 4, f(y_i) = 5i-4, 2 \leq i \leq m \]
\[ f(t_1) = 5m+4i-3, 1 \leq i \leq n \]
\[ f(s_1) = 5m+4i-2, 1 \leq i \leq n \]
\[ f(w_1) = 5m+4i-1, 1 \leq i \leq n \]

Then the distinct edge labels are
f(u_1u_2) = 9, f(u_{m+1}) = 5(m-1)+3,
f(u_{m-1} u_m) = 5m
f(u_1 u_2) = 5i+2, 2\leq i \leq m-2
f(u_{m-1} u_m) = 5(m-1)+3,

\textbf{Theorem 2.8:} \ f_{m\times 3}\cup P_n \ is a \ Contra \ Harmonic \ mean \ graph \ for \ m \geq 3.

\textbf{Proof:}
Let \ f_{m\times 3} \ be \ a \ flower \ graph. \ Let \ P_n \ be \ the \ path \ with \ vertices \ t_1 \ldots t_n.
Let \ G = f_{m\times 3}\cup P_n
Define \ f: V(G) \rightarrow \{0,1,\ldots, q\} \ by

\begin{align*}
  f(v_i) &= 2i, \text{ for } i = 1,2, 3 \leq i \leq m-1, \\
  f(v_{m}) &= 3m-3, \\
  f(x_1x_2) &= 3, f(x_iy_i) = 5i-5, 2 \leq i \leq m \\
  f(t_1t_2) &= 5m+4i, 1 \leq i \leq n-1 \\
  f(t_3) &= 5m+4i-3, 1 \leq i \leq n \\
  f(t_{i}w_{i}) &= 5m+4i-1, 1 \leq i \leq n \\
  f(s_{i}w_{i}) &= 5m+4i-2, 1 \leq i \leq n \\
\end{align*}

Then distinct edge labels are
\begin{align*}
  f(u_1u_2) &= 3, f(u_{i+1}u_{i+1}) = 3i-1, 2 \leq i \leq m-1, \\
  f(u_{m+1}) &= 3m-2 \\
  f(u_1v_1) &= 3i+1, 1 \leq i \leq m-1, f(u_{m}v_{m}) = 3m-1 \\
  f(u_{i}v_{2}) &= 3i, 2 \leq i \leq m \\
  f(t_{i+1}) &= 3m+i, 1 \leq i \leq n-1 \\
\end{align*}

Clearly, \ f \ is \ a \ Contra \ Harmonic \ mean \ graph.

\textbf{The Contra Harmonic mean labeling of } \ f_{5\times 3}\cup P_7 \ is

\textbf{Figure 7}
Theorem 2.9: $f_{mx3} \cup (P_n \odot K_1)$ is a Contra Harmonic mean graph, for $m \geq 3$.

Proof:

Let $f_{mx3}$ be a flower graph. Let $t_1, \ldots, t_n$ be the path $P_n$. Let $s_i$ be the vertex that is joined to $t_i$ of the path $P_n$ for $1 \leq i \leq n$.

Let $G = f_{mx3} \cup P_n \odot K_1$

Define $f: V(G) \rightarrow \{0, 1, \ldots, q\}$ by

$f(v_i) = 2i$, for $i = 1, 2$
$f(v_i) = 3i-2$, $3 \leq i \leq m-1$, $f(v_m) = 3m$
$f(u_1) = 1$, $f(u_i) = 3i-3$, $2 \leq i \leq m-1$, $f(u_m) = 3m-2$
$f(t_1) = 0$, $f(t_i) = 3m+2i-2$, $2 \leq i \leq n$
$f(s_i) = 3m+2i-1$, $1 \leq i \leq n$

Then the distinct edge labels are

$f(u_1u_2) = 3$, $f(u_iu_{i+1}) = 3i-1$, $2 \leq i \leq m-1$
$f(u_mu_1) = 3m-2$
$f(u_1v_1) = 3i-2$, $1 \leq i \leq m-1$, $f(u_mv_m) = 3m-1$
$f(u_1v_2) = 2$, $f(u_iu_{i+1}) = 3i$, $2 \leq i \leq m$
$f(t_{i+1}) = 3m+2i$, $1 \leq i \leq n-1$
$f(t_1s_1) = 3m+2i-1$, $1 \leq i \leq n$

Clearly, $f$ is a Contra Harmonic mean graph of $G$.

The Contra Harmonic mean labeling of $f_{5x3} \cup (P_6 \odot K_1)$ is

Figure 8

Theorem 2.10: $f_{mx3} \cup C_n$ Contra Harmonic mean graph for $m \geq 3$, $n \geq 3$.

Proof: Let $f_{mx3}$ be a flower graph.

Let $t_1, \ldots, t_n$ be a cycle $C_n$.

Let $G = f_{mx3} \cup C_n$

Define $f: V(G) \rightarrow \{0, 1, \ldots, q\}$ by

$f(v_i) = 2i$, for $i = 1, 2$
$f(v_i) = 3i-2$, $3 \leq i \leq m-1$, $f(v_m) = 3m$
$f(u_1) = 1$, $f(u_i) = 3i-3$, $2 \leq i \leq m-1$, $f(u_m) = 3m-2$
$f(t_i) = 0$, $f(t_i) = 3m+i-1$, $2 \leq i \leq n$, $f(t_n) = 3m+n$

Then the distinct edge labels are

$f(u_1u_2) = 3$, $f(u_iu_{i+1}) = 3i-1$, $2 \leq i \leq m-1$
$f(u_mu_1) = 3m-2$
$f(u_1v_1) = 3i-2$, $1 \leq i \leq m-1$
$f(u_mv_m) = 3m-1$
\[ f(u_1v_2) = 2, \ f(u_iu_{i+1}) = 3i, \ 2 \leq i \leq m \]
\[ f(t_iu_i) = 3m+i, \ 1 \leq i \leq n-1 \]
\[ f(t_0t_1) = 3m+n \]

Clearly, \( f \) is a Contra Harmonic mean graph of \( G \).

**The Contra Harmonic mean labeling of \( f_{S_5 \cup C_6} \)**

\[ \text{Figure 10} \]

**REFERENCES**