Unpreserved Inventory Models with Inflation Induced Demand under Progressive Credit Limit

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ABSTRACT
Inventory control has one of the most important tasks faced by modern manager. The investment in inventories for most form their assets committed to inventories. Further inventories one often the least stable and difficult to manage type of assist. Rapid change in level of business activities effect on inventories. In recent year, change in interest rate effect the inventories. Employ and customer theft has also led to increased cost of maintaining inventories. But carrying inventory is a costly thing as the storage cost, stock out cost, capacity related cost, item cost, ordering cost, deterioration and expiration of the product etc. must be taken in to account. Some policies, procedures and techniques employed in maintaining the optimum number of amount of each inventory item is the inventory management. While inventory is an asset, it is a non productive asset since it earns no interest but costs an organization in handling insurance, taxes, shrinkage and space. Careful inventory management can make a huge difference in the profitability of a firm.

Keywords: Inventory, modern manager, policies

I. INTRODUCTION
The effect of deterioration of physical goods can not be disregarded in many inventory systems. Deterioration is defined as decay, damage or spoilage. Food items, photographic films, drugs, chemicals, electronic components and radioactive substances are some examples of items in which sufficient deterioration may occur during the normal storage period of the units and consequently this loss must be taken into account while analyzing the inventory system. An order level inventory model for deteriorating items with inflation induced demand and shortage has been developed. Since most decision makers think that inflation does not have significant influence on the inventory policy, the effects of inflation are not considered in some inventory models. However, from a financial point of view, an inventory represents a capital investment and must compete with other assets for a firm’s limited capital funds. Thus, it is necessary to consider the effects of inflation on the inventory system, as many countries experience high annual inflation rate. The whole environment of business dealing has been assumed to be progressive credit period. Further, we use a numerical example to illustrate the model and sensitivity analysis on some parameters is made.

II. ASSUMPTION AND NOTATIONS
The following assumptions are used to develop a foresaid model:
• Shortages are allowed
• If the retailer pays by M, then the supplier does not charge to the retailer. If the retailer pays after M and before N (N>M), he can keep the difference in the unit sale price and unit purchase price in an interest bearing account at the rate of IC/Unit/Year. During [M,N], the supplier charges the retailer an interest rate of IC₁/Unit/Year on unpaid balance. If the retailer pays after N, then supplier charges the retailer an interest rate of IC₂/Unit/Year (IC₁>IC₂) on unpaid balance.

The notations are as follows
• \( s \) = selling price /unit
• \( C₀ \) = the unit purchase cost with \( C₀<s \)
• \( M \) = the first offered credit period in selling the account without any charges.
• \( N \) = the second permissible credit period in settling the account with interest charge \( IC₂ \) on unpaid balance and \( N>M \)
• \( IC₁ \) = the interest charged per $ in stock per year by the supplier when retailer pays during [M,N]
• \( IC₂ \) = the interest charged per $ in stock per year by the supplier when retailer pays during [N,T](IC₁>IC₂)
• \( IE \) = the interest earned / $ / year
• \( r \) = discount rate \( r > \alpha \)
• \( IE \) = the interest earned / time unit
• \( IC \) = the interest charged /time unit
• \( T \) = length of replenishment cycle.
• The demand rate is exponentially increasing and \( D(t)=λ₀e^{at} \) where \( 0 ≤ \alpha ≤ 1 \) a
constant inflation rate is and $\lambda_0$ is initial demand rate.
- $A_0$ = ordering cost / order
- $C_{10}$ = carrying cost / unit time
- $C_{20}$ = shortage cost / unit time
- $\theta t$ = variable deterioration rate
- A discounted cash flow (DCF) approach is used to consider the various costs at various times ($r \geq \alpha$) is discount rate.
- L is the length of finite planning horizon.

### III. MATHEMATICAL FORMULATION

Assuming continuous compounding of inflation, the ordering cost, unit cost of the item, out of pocket inventory carrying cost and storage cost at any time $t$ are

$$A(t) = A_0 e^{\alpha t}$$

$$C(t) = C_{10} e^{\alpha t}$$

and

$$C_2(t) = C_{20} e^{\alpha t}$$

the planning horizon $L$ has been discarded into $n$ equal cycles of length $T$ (i.e. $T = \frac{L}{n}$) let us consider the $i$th cycle i.e. $t_{i-1} \leq t \leq t_i$ where $t_0 = 0$, $t_n = L$, $t_i - t_{i-1} = T$ and $t_0(t) = (i = 1, 2, ..., n)$. At the beginning of each cycle a batch of $q_i$ units enters the inventory system from $S_i$ units which are delivered towards backorders leaving a balance of $I_{0i}$ units as the initial inventory level of the cycle $q_i = I_{0i} + S_i$, after as time passes, the inventory level gradually decreasing mainly due to demand and partially due to deterioration and reaches zero at time $t_{i1}$ (Fig.1) further demands during the remaining period of the cycle from $t_{i1}$ to $t_i$ are backlogged and are of fulfilled by a new procurement.

Now

$$t_{i1} = t_i - kT = (i - k) \frac{L}{n} (0 \leq k \leq 1)$$

where $kT$ is the fraction of the cycle having shortages. Let $I(t)$ be the inventory level of the $i$th cycle at time $t$ ($t_{i-1} \leq t \leq t_i$, $i = 1, 2, ..., n$). Now at the beginning of each cycle there will be cash out flow of ordering cost and purchase cost. Further since the inventory carrying cost is proportional to the value of the inventory, the out of pocket (physical storage) inventory carrying cost per unit time at time $t$ is $I(t)C(t)$. Similarly the shortage cost can also be obtained. The inventory level is represented by the following differential equations:

$$\frac{dI(t)}{dt} + \theta t I(t) = -\lambda_0 e^{\alpha t} \quad t_{i-1} \leq t \leq t_i, \quad i = 1, 2, ..., n \quad \ldots (1.1)$$

$$\frac{dI(t)}{dt} = -\lambda_0 e^{\alpha t} \quad t_{i1} \leq t \leq t_i, \quad i = 1, 2, ..., n$$

The solution of the above differential equation along with the boundary condition $I(t_{i-1}) = I_{0i}$ and $I_i(t_{i1}) = 0$ is

$$I(t_i) = I_{0i} e^{\lambda_0 t_i} \quad t_{i1} \leq t \leq t_i, \quad i = 1, 2, ..., n$$

Since $I_1(t_{i1}) = 0$ and $I_i(t_i) = -S_i$

Now put $I_i(t_{i1}) = 0$ in (3a) then

$$B_i = -\frac{\lambda_0}{\alpha} (e^{\alpha t_i} - e^{\alpha t_{i1}}) \quad i = 1, 2, ..., n$$

Now we put the value of $I_{0i}$ in (3a) then

$$B_i = -\lambda_0 \left[\frac{(t_{i1} - t_i) + \frac{\alpha}{2} (t_{i1} - t_i)^2 + \frac{(\alpha + \theta) a}{6} (t_{i1} - t_i)^3}{(t_i - t_{i1}) + \frac{\alpha}{2} (t_i - t_{i1})^2 + \frac{(\alpha + \theta) a}{6} (t_i - t_{i1})^3}\right] e^{\alpha t_i} \quad i = 1, 2, ..., n \quad \ldots (1.4)$$

Now put $I_i(t_i) = -S_i$ in (3b) then

$$S_i = \frac{\lambda_0}{\alpha} (e^{\alpha t_i} - e^{\alpha t_{i1}}) \quad i = 1, 2, ..., n$$

Now we put the value of $I_{0i}$ in (3a) then

$$I_i(t) = I_{0i} e^{\alpha t} \left[\frac{(t_{i1} - t_i) + \frac{\alpha}{2} (t_{i1} - t_i)^2 + \frac{(\alpha + \theta) a}{6} (t_{i1} - t_i)^3}{(t_i - t_{i1}) + \frac{\alpha}{2} (t_i - t_{i1})^2 + \frac{(\alpha + \theta) a}{6} (t_i - t_{i1})^3}\right] e^{\alpha t_i} \quad i = 1, 2, ..., n \quad \ldots (1.6)$$

Further batch size $q_i$ for the $i$th cycles is :

$$q_i = q_i(t_{i1} + S_i)$$

$$q = \lambda_0 \left[\frac{(t_{i1} - t_i) + \frac{\alpha}{2} (t_{i1} - t_i)^2 + \frac{(\alpha + \theta) a}{6} (t_{i1} - t_i)^3}{(t_i - t_{i1}) + \frac{\alpha}{2} (t_i - t_{i1})^2 + \frac{(\alpha + \theta) a}{6} (t_i - t_{i1})^3}\right] e^{\alpha t_i} \quad i = 1, 2, ..., n$$

(1) Present worth of ordering cost for the $i$th cycle $A_i$ is

$$A_i = A(t_{i1}) e^{-\alpha t_{i1}} = A_0 e^{(\alpha - \alpha t_{i1})} \quad i = 1, 2, ..., n$$

(2) Present worth of the purchase cost for the $i$th cycle $P_i$ is

$$P_i = q_i C(t_{i1}) e^{-\alpha t_{i1}} = q_i C_0 e^{(\alpha - \alpha t_{i1})} \quad i = 1, 2, ..., n$$

(3) Present worth of the inventory carrying cost for the $i$th cycle $H_i$ is
\[ H_i = C_i (t_{i-1})e^{-\eta_i} \int_{t_{i-1}}^{t_i} I_i(t)e^{-\alpha t}dt \]

\[ H_i = C_i \alpha e^{(e^{-\eta_i})} \int_{t_{i-1}}^{t_i} \left( \frac{\theta}{2} + \frac{\alpha}{2} - t_i \right) e^{\alpha t}dt \]

\[ \ldots (1.10) \]

(4) Present worth of the shortage cost for the \( i \)th cycle is

\[ \pi_i = C_i (t_{i-1})e^{-\eta_i} \int_{t_{i-1}}^{t_i} I_i(t)e^{-\alpha t}dt \]

\[ = C_i \alpha e^{(e^{-\eta_i})} \int_{t_{i-1}}^{t_i} \left( \frac{\theta}{2} + \frac{\alpha}{2} - t_i \right) e^{\alpha t}dt \]

\[ = \alpha e^{(e^{-\eta_i})} \left( \frac{\theta}{2} + \frac{\alpha}{2} - t_i \right) \int_{t_{i-1}}^{t_i} e^{\alpha t}dt \]

\[ \ldots (1.19) \]

Therefore the present worth of the total variable cost for the \( i \)th cycle \( PW_i \) is the sum of the ordering cost \( A_i \) purchase cost \( P_1 \), inventory carrying cost \( (H_i) \) and shortage cost \( (\pi_i) \) i.e.

\[ PW_i = A_i + P_1 + H_i + \pi_i \]

The present worth of the total variable cost of the system during the entire time horizon \( L \) is given by

\[ \text{Case } I \ M \geq t_{i1} \]

\[ \text{Inventory level Fig. 2 } M \geq t_{i1} \]

In the first case, retailer does not pay any interest to the supplier. Here retailer sells \( I_i \) units during \((0, t_{i1})\) time internal and paying for \( C_i \) units in full to the supplier at time \( M_t \), so interest charges are zero i.e.

\[ IC_{11} = 0 \]

Retailers deposits the revenue in an interest bearing account at the rate of \( Ie/\text{unit/year} \).

Therefore, interest earned \( IE_1 \), per year is

\[ IE_1 = \frac{s L \sum \text{PW}_i}{t_{i1}} \left[ \int_0^{t_{i1}} D(t)I_i(t)dt + (M - t_{i1}) \int_0^{t_{i1}} D(t)I_i(t)dt \right] \]

Total cost per unit time of an inventory system is

\[ T[PW_1(k,n)] = \sum_{i=1}^{n} PW_i + IC_1 - IE_1 \]

\[ = \sum_{i=1}^{n} (A_i + P_i + H_i + \pi_i) + IC_1 - IE_1 \]

\[ \text{Case II} \ - \ M < t_{i1} \leq N \]

In the second case, supplier charges interest at the rate \( IC_1 \) on unpaid balance – Interest earned, \( IE_2 \) during \([0,M]\) is

\[ IE_2 = s \int_0^M D(t)I(t)dt \]

\[ \ldots \]

Retailer pays \( Is \) units purchased at time \( t = 0 \) at the rate of \( C_i/\text{unit} \) to the supplier during \([0,M]\). The retailer sells \( D(M)_i \) units at selling price \( s_i \) unit. So, he has generated revenue of \( s D(M)_i + IE_2 \) in the second case.

\[ \text{Sub Case 2.1} \ - \]

Let \( SD \) \( M_i + IE_2 \geq CI_i \) retailer has enough money to settle, his account for all \( IS \) units procured at time \( t = 0 \) then interest charge will be

\[ IC_{21} = 0 \]

And interest earned

\[ IE_{21} = \frac{IE_2}{T_2} \]

So the total cost \( T_{21} [PW_1(k,n)] \) per unit time of inventory system is

\[ \text{Sub Case 2.2} \ - \]

Let \( SD \) \( M_i + IE_2 < CI_i \) here retailer will have to pay interest on unpaid balance \( U_1 = CI_i - (sD(M)_i + IE_2) \) at the rate of \( IC_1 \) at time \( M \) to the supplier. Then interest paid per unit time is given by

\[ IC_{22} = \frac{U_1IC}{T_2} \int_0^M I_i(t)dt \]

and interest earned

\[ IE_{22} = \frac{IE_2}{T_2} \]

So the total cost \( T_{22}[PW_1(k,n)] \) per unit time of inventory system is

\[ T_{22}[PW_1(k,n)] = \sum_{i=1}^{n} (A_i + P_i + H_i + \pi_i) + IC_{21} - IE_{21} \]

\[ \text{Case III} \ t_{i1} > N \]

\[ \text{Inventory level Fig. 3 } t_{i1} > N \]

In the final case, retailer pays interest at the rate of \( IC_2 \) to the supplier. Based on the total purchased cost
Cl's, the total money \( sD(M)M + IE_2 \) in account at M and total money \( SD(N)N + IE_2 \) at N, there are three sub cases may arise. This case is same as sub case 2.1 have 3.1 designate decision variables and objective function. 

Sub case 3.2 – Let \( sD(M)M + IE_2 < CI \) and

\[
sD(N-M)(N-M) + sI \int_0^N D(t)dt \geq CI_1(sD(M)M + IE_2)
\]

Here, retailer does not have enough money to pay off total purchase dost at N. He will not pay money \( sD(M)M + IE_2 \) at M and \( sD(N-M)(N-M) + sI \int_0^N D(t)dt \) at N. That’s why we has to pay interest on unpaid balance \( U_1 = CI_1 - (sD(M)M + IE_2) \) with IC1 interest rate during \( (M, N) \) and \( U_2 = U_1 - sD(N-M)(N-M) + sI \int_0^N D(t)dt \) with interest rate IC2 during \( (N, t_i) \). Therefore, total interest charged on retailer IC3,3 per unit time is –

\[
IC_{3,3} = \frac{U_2}{T_2} + \frac{U_1}{T_2} \sum_{i=1}^{K} I_i \int_{N}^{t_i} D(t)dt
\]

And interest earned per unit time is

\[
IE_{3,3} = \frac{IE_2}{T_2}
\]

So total cost \( T_{3,3}[PW_i(k,n)] \) per unit time of inventory system is

\[
T_{3,3}[PW_i(k,n)] = \sum_{i=1}^{K} (A_i + P_i + H_i + P_i) + IC_{3,3} - IE_{3,3}
\]

### IV. NUMERICAL ILLUSTRATIONS

The preceding theory can be illustrated by the following numerical example where the parameters are given as follows:

- Demand parameters, \( \lambda_0 = 420 \), \( a = 0.02 \)
- Selling price, \( s = 23 \)
- Buyer’s purchased cost, \( C_s = 27 \)
- Buyer’s percentage holding cost per year per dollar, \( C_{bh} = 0.03 \)
- Buyer’s ordering cost per order, \( C_{bo} = 410 \)
- Buyer’s shortage cost, \( C_s = 10 \)
- Vendor’s unit cost, \( C_v = 18 \)
- Vendor’s percentage holding cost per year per dollar, \( C_{vh} = 0.2 \)
- Vendor’s setup cost per order, \( C_{sy} = 550 \)
- Vendor’s production rate per year, \( K = 2 \)
- Deterioration rate, \( \theta = 0.02 \)
- First delay period, \( M = 0.06 \)
- Second delay period, \( N = 0.2 \)
- The interest earned, \( Ie = 0.04 \)
- The interest charged, \( Ic_1 = 0.20 \)
- The interest charged, \( Ic_2 = 0.16 \left( Ic_1 > Ic_2 \right) \)

| Table 1.1: Retailer does not pay any interest to the Supplier |
| --- | --- | --- | --- |
| \( N \) | \( T_2 \) | \( t_1 \) | \( TC \) |
| 1 | 0.754546 | 0.116863 | 1284.561 |
| 2 | 0.786092 | 0.118116 | 1289.345 |
| 3 | 0.806756 | 0.118522 | 1292.717 |
| 4 | 0.836491 | 0.118723 | 1331.382 |
| 5 | 0.865356 | 0.118844 | 1348.485 |

| Table 1.2: Supplier charges interest but Retailer has enough money to settle his account |
| --- | --- | --- | --- |
| \( N \) | \( T_2 \) | \( t_1 \) | \( TC \) |
| 1 | 0.764371 | 0.124818 | 1615.17 |
| 2 | 0.787892 | 0.125711 | 1531.16 |
| 3 | 0.815642 | 0.125993 | 1518.25 |
| 4 | 0.826030 | 0.126018 | 1487.83 |
| 5 | 0.857485 | 0.126614 | 1411.19 |

| Table 1.3: Retailer will have to pay interest on unpaid balance at the rate of interest \( Ic_1 \), Retailer does not have enough money to pay off at M |
| --- | --- | --- | --- |
| \( N \) | \( T_2 \) | \( t_1 \) | \( TC \) |
| 1 | 0.764371 | 0.124818 | 1513.88 |
| 2 | 0.787892 | 0.125711 | 1478.79 |
| 3 | 0.815642 | 0.125993 | 1445.51 |
| 4 | 0.826030 | 0.126018 | 1431.04 |
| 5 | 0.857485 | 0.126614 | 1401.45 |

| Table 1.4: Retailer pays interest at the rate of \( Ic_2 \) to the Supplier; Retailer does not have enough money to pay off at N |
| --- | --- | --- | --- |
| \( N \) | \( T_2 \) | \( t_1 \) | \( TC \) |
| 1 | 1.218450 | 0.896290 | 1553.54 |
| 2 | 1.297826 | 0.758734 | 1497.97 |
| 3 | 1.433762 | 0.689980 | 1411.22 |
4 | 1.563398 | 0.549718 | 1351.21
5 | 1.784529 | 0.427600 | 1314.65

V. CONCLUSION

The goal of this work is to develop an inventory model with shortages, in which units are deteriorate with time dependent rates and the demand rate is increasing exponentially due to inflation under trade credit. Most products experience a period of rapid demand increase during the introduction phase of product life cycle, level off in demand after reaching their maturity period, and will enter a period of sales decline due to new competing products or changes in consumer preference. An inventory control is an intriguing yet practicable issue of decision science when inflation induced demand is involved. The effect of inflation on an inventory system has been taken into consideration. Cost minimization technique is used to get the expressions for total cost and other parameters. A numerical assessment of the theoretical model has been done to illustrate the theory. The whole combination of the setup is very unique and more practical.

VI. REFERENCES


