On $\text{rgw}_\alpha$-Continuous and $\text{rgw}_\alpha$-Irresolute Maps in Topological Spaces

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Abstract: The aim of this paper is to introduce a new type of functions called the $\text{rgw}_\alpha$- continuous map, $\text{rgw}_\alpha$- irresolute maps, strongly $\text{rgw}_\alpha$-continuous maps, perfectly $\text{rgw}_\alpha$-continuous maps and study some of these properties.

Keywords: $\text{rgw}_\alpha$-open sets, $\text{rgw}_\alpha$-closed sets, $\text{rgw}_\alpha$-continuous map, $\text{rgw}_\alpha$-irresolute maps, strongly $\text{rgw}_\alpha$-continuous maps, perfectly $\text{rgw}_\alpha$-continuous.

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I. Introduction:

The continuous functions play a very important role in Topology. Balachandran et.al [5], Levine [18], Mashhour et.al [16], Gnanmbal et.al [11], S. P. Arya and Gupta. R [24] have introduced g-continuity, Semi-continuity, pre-continuity, g-continuity, regular and completely-continuous respectively. In 1972 Crossley and Hildebrand [6] introduced the notion of irresoluteness. In 1981, Munshi and Bassan [17] introduced the notion of generalized continuous (briefly g - continuous) functions which are called in [5] as g- irresolute functions. Furthermore, the notion of gs- irresolute [10] (resp.ggp-irresolute [2], ag-irresolute [9], gb- irresolute [3], gsp-irresolute [32]) functions is introduced. Also, the concept of $w_\alpha$-continuous functions was introduced by S S Benchalli et al [30]. Recently R S Wali and Vijayalaxmi R.Patil [23] introduced and studied the properties of $\text{rgw}_\alpha$-closed set. The purpose of this paper is to introduce a new class of functions, namely, $\text{rgw}_\alpha$-continuous functions and $\text{rgw}_\alpha$-irresolute functions strongly $\text{rgw}_\alpha$-continuous maps, perfectly $\text{rgw}_\alpha$-continuous maps. Also, we study some of the characterization and basic properties of $\text{rgw}_\alpha$-continuous functions.

II. Preliminaries:

Throughout this paper, $(X, \tau)$ and $(Y, \sigma)$ (or simply $X$ and $Y$) represent a topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset $A$ of a space $X$, $\text{cl}(A)$ and $\text{int}(A)$ denote the closure of $A$ and the interior of $A$ respectively. $X\setminus A$ or $A'$ denotes the complement of $A$ in $X$. We recall the following definitions and results.

Definition 2.1: A subset $A$ of a topological space $(X, \tau)$ is called.

1. semi-open set [19] if $A \subseteq \text{cl}(\text{int}(A))$ and semi-closed set if $\text{int}(\text{cl}(A)) \subseteq A$.
2. pre-open set [1] if $A \subseteq \text{int}(\text{cl}(A))$ and pre-closed set if $\text{cl}(\text{int}(A)) \subseteq A$.
3. $\alpha$-open set [21] if $A \subseteq \text{int}(\text{cl}(A))$ and $\alpha$ -closed set if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.
4. semi-pre open set [7] ($\beta$-open) if $A \subseteq \text{cl}(\text{cl}(\text{cl}(A)))$ and a semi-pre closed set ($\beta$-closed ) if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.
5. regular open set [15] if $A = \text{int}(\text{cl}(A))$ and a regular closed set if $A = \text{cl}(\text{int}(A))$.
6. Regular semi open set [11] if there is a regular open set $U$ such that $U \subseteq A \subseteq \text{cl}(U)$.
7. Regular $\alpha$-open set [8] (briefly, $r\alpha$-open) if there is a regular open set $U$ s.t $U \subseteq A \subseteq \text{acl}(U)$.

Definition 2.2: A subset $A$ of a topological space $(X, \tau)$ is called

1. $w$-closed set [23] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $X$.
2. $w_\alpha$- closed set [30] if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $w$-open in $X$.
3. generalized closed set(briefly g-closed) [18] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
4. generalized semi-closed set(briefly gs-closed) [27] if $\text{sc}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
5. generalized pre regular closed set(briefly gpr-closed) [33]if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$.
6. regular generalized $\alpha$-closed set (briefly, rga-closed) [2] if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular $\alpha$-open in $X$.
7. $\alpha$-generalized closed set (briefly ag -closed) [13] if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$. 

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(8) generalized α-closed set (briefly gα-closed) 
(20) if acl(A) ⊆ U whenever A⊆U and U is α-open in X.
(9) weakly generalized closed set (briefly, wg-closed)[20] if cl(int(A))⊆U whenever A⊆U and U is open in X.
(10) regular weakly generalized closed set (briefly, rwg-closed) [20] if cl(int(A)) ⊆ U whenever A⊆U and U is regular open in X.
(11) generalized pre closed (briefly gp-closed) set [12] if pcl(A) ⊆ U whenever A⊆U and U is open in X.
(12) regular w-closed (briefly rw -closed) set [31] if cl(A) ⊆ U whenever A⊆U and U is regular semi-open in X.
(13) generalized regular closed (briefly gr-closed) set [24] if rcl(A)⊆ U whenever A⊆ U and U is open in X.
(14) regular generalized weak (briefly rgw-closed) set [25] if cl(int(A)) ⊆ U whenever A⊆ U and U is regular semi open in X.
(15) generalized weak α-closed (briefly gwa-closed) set [29] if acl(A) ⊆ U whenever A⊆U & U is wa-open in X.
(16) generalized star weakly α-closed set (briefly g*wa-closed) [28] if cl(A) ⊆ U whenever A⊆U & U is wa-open in X.

The compliment of the above mentioned closed sets are their open sets respectively.

Definition 2.3: A map f: (X, τ) → (Y, σ) is said to be
(i) regular-continuous(r-continuous) [24] if f(V) is r-closed in X for every closed subset V of Y.
(ii) completely-continuous [24] if f(V) is regular closed in X for every closed subset V of Y.
(iii) strongly-continuous [15] if f(V) is clopen (both open and closed) in X for every closed subset V of Y.
(iv) g-continuous [30] if f(V) is g-closed in X for every closed subset V of Y.
(v) w-continuous [23] if f(V) is w-closed in X for every closed subset V of Y.
(vi) α-continuous [21] if f(V) is α-closed in X for every closed subset V of Y.
(vii) wa-continuous [30] if f(V) is wa-closed in X for every closed subset V of Y.
(viii) strongly α-continuous [34] if f(V) is α-closed in X for every semi-closed subset V of Y.
(ix) ag-continuous [13] if f(V) is ag-closed in X for every closed subset V of Y.
(x) wg-continuous [20] if f(V) is wg-closed in X for every closed subset V of Y.
(xi) rwg-continuous [20] if f(V) is rwg-closed in X for every closed subset V of Y.
(xii) gs-continuous [18] if f(V) is gs-closed in X for every closed subset V of Y.
(xiii) gpr-continuous [33] if f(V) is gpr-closed in X for every closed subset V of Y.
(xiv) gα-continuous [2] if f(V) is gα-closed in X for every closed subset V of Y.
(xv) gr-continuous [24] if f(V) is gr-closed in X for every closed subset V of Y.
(xvi) rw-continuous [31] if f(V) is rw-closed in X for every closed subset V of Y.
(xvii) gwa-continuous [25] if f(V) is gwa-closed in X for every closed subset V of Y.

Definition 2.4: A map f: (X, τ) → (Y, σ) is said to be
(i) irresolute [30] if f(V) is semi-closed in X for every semi-closed subset V of Y.
(ii) α-irresolute [21] if f(V) is α-closed in X for every α-closed subset V of Y.
(iii) contra irresolute [21] if f(V) is semi-open in X for every semi-open subset V of Y.
(iv) contra w-irresolute [23] if f(V) is w-open in X for every w-closed subset V of Y.
(v) contra r-irresolute [24] if f(V) is regular-open in X for every regular-closed subset V of Y.
(vi) contra continuous [4] if f(V) is open in X for every closed subset V of Y.
(vii) rw*-open(resp rw*-closed) [31] map if f(U) is rw-open (resp. rw-closed) in Y for every rw-open (resp rw-closed) subset U of X.

Lemma 2.5: see [23]
1) Every closed (resp. regular-closed, w-closed, α-closed and β-closed) set is rgwα-closed set in X.
2) Every rw-closed (resp. rs-closed, ra-closed, wa-closed, ga-closed, rgα-closed, gwa-closed and g*wa-closed) set is rgwα-closed set in X.
3) Every rgwα-closed set is gβ-closed set
4) The set g-closed (resp. wg-closed, rg-closed, gr-closed, gpr-closed, rgw-closed, rwg-closed and ag closed) set is independent with rgwα-closed set.

Lemma 2.6: see [23] If a subset A of a topological space X, and
1) If A is weak-open and rgwα-closed then A is α-closed set in X.
2) If A is both weak α-open and rgwα-closed then it is ra-closed set in X
3) If A is weak-open and ra-closed then A is rgwα-closed set in X
4) If A is both open and g-closed then A is rgwα-closed set in X.

Definition 2.7: A topological space (X, τ) is called
(1) an $\alpha$-space if every $\alpha$-closed subset of $X$ is closed in $X$.

3. rgw$\alpha$- Continuous Functions.

**Definition 3.1**: A function $f$ from a topological space $X$ into a topological space $Y$ is called regular generalized weakly $\alpha$- continuous (briefly rgw$\alpha$- continuous) if $f^{-1}(V)$ is rgw$\alpha$- closed in $X$ for every closed set $V$ in $Y$.

**Theorem 3.2**: If a map $f$ is continuous, then it is rgw$\alpha$-continuous but not conversely.

**Proof**: Let $f: X \to Y$ be continuous. Let $F$ be any closed set in $Y$. Then the inverse image $f^{-1}(F)$ is closed set in $X$. Since every closed set is rgw$\alpha$- closed, by Lemma 2.5, $f^{-1}(F)$ is rgw$\alpha$- closed in $X$. Therefore $f$ is rgw$\alpha$-continuous.

**Theorem 3.3**: If a map $f: X \to Y$ is $\alpha$-continuous, then it is rgw$\alpha$-continuous but not conversely.

**Proof**: Let $f: X \to Y$ be $\alpha$-continuous. Let $F$ be any closed set in $Y$. Then the inverse image $f^{-1}(F)$ is $\alpha$-closed set in $X$. Since every $\alpha$-closed set is rgw$\alpha$- closed by Lemma 2.5, $f^{-1}(F)$ is rgw$\alpha$-closed in $X$. Therefore $f$ is rgw$\alpha$-continuous.

The converse need not be true as seen from the following example.

**Example 3.4**: Let $X = Y = \{a,b,c,d,e\}$, $\tau = \{X, \phi, \{a\}, \{d\}, \{e\}, \{a,d\}, \{a,e\}, \{d,e\}, \{a,d,e\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{d\}, \{e\}, \{a,d\}, \{a,e\}, \{d,e\}, \{a,d,e\}\}$. Let map $f: X \to Y$ defined by $f(a)=c$ , $f(b)=a$ , $f(c)=b$ , $f(d)=d$ , $f(e)=e$, then $f$ is rgw$\alpha$- continuous but not $\alpha$-continuous and not $\alpha$-continuous as closed set $F=\{a,e\}$ in $Y$, then $f^{-1}(F)$ is not $\alpha$-closed set in $X$.

**Theorem 3.5**: If a map $f: X \to Y$ is continuous, then the following holds.

(i) $f$ is rgw$\alpha$-continuous.

(ii) $f$ is w-continuous, $\beta$- continuous, rw- continuous, rs- continuous, ra- continuous, wa- continuous, ga- continuous, rga- continuous, gw\alpha- continuous, g*wa- continuous, then $f$ is rgw$\alpha$-continuous.

**Proof**: (i) Let $F$ be a closed set in $Y$. Since $F$ is $\alpha$-continuous, then $f^{-1}(F)$ is $\alpha$-closed in $X$. Since every $\alpha$-closed set is rgw$\alpha$-closed by Lemma 2.5, then $f^{-1}(F)$ is rgw$\alpha$-closed in $X$. Hence $f$ is rgw$\alpha$-continuous.

Similarly we can prove (ii).

The converse need not be true as seen from the following example.

**Example 3.6**: Let $X=Y=\{a,b,c,d,e\}$, $\tau = \{X, \phi, \{a\}, \{d\}, \{e\}, \{a,d\}, \{a,e\}, \{d,e\}, \{a,d,e\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{d\}, \{e\}, \{a,d\}, \{a,e\}, \{d,e\}, \{a,d,e\}\}$. Let map $f: X \to Y$ defined by $f(a)=c$ , $f(b)=a$ , $f(c)=b$ , $f(d)=d$ , $f(e)=e$, then $f$ is rgw$\alpha$- continuous but not rw-continuous, w-continuous, $\beta$-continuous, rs-continuous and ra-continuous, rw-continuous, wa-continuous, ga-continuous, rga- continuous, gw\alpha- continuous, g*wa-continuous as closed set $F=\{b,c,d,e\}$ in $Y$, then $f^{-1}(F)$ is not $\beta$-closed but not $\alpha$-closed, w-closed, $\beta$-closed, rs-closed and ra-closed set in $X$. and closed set $F=\{b,c,d\}$ in $Y$ if $f(F) = \{a,b,d\}$ which is not rw-closed, wa-closed, $\alpha$-closed, rga- closed, gw\alpha- closed, g*wa- closed.

**Theorem 3.7**: If a map $f: X \to Y$ is rgw$\alpha$-continuous, then it is g$\beta$- continuous but not conversely.

**Proof**: Let $f: X \to Y$ be rgw$\alpha$- continuous. Let $F$ be any closed set in $Y$. Then the inverse image $f^{-1}(F)$ is rgw$\alpha$-closed set in $X$. Since every rgw$\alpha$-closed set is g$\beta$-closed by Lemma 2.5, $f^{-1}(F)$ is g$\beta$-closed set in $X$. Therefore $f$ is g$\beta$- continuous.

The converse need not be true as seen from the following example.

**Example 3.8**: Let $X=Y=\{a,b,c\}$, $\tau = \{X, \phi, \{a\}, \{b,c\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Let map $f: X \to Y$ defined by $f(a)=b$ , $f(b)=a$ , $f(c)=c$, then $f$ is g$\beta$- continuous but not rgw$\alpha$-continuous as a closed set $F=\{b,c\}$ in $Y$, $f^{-1}(F)=\{a\}$ which is not rgw$\alpha$-closed set.

**Remark 3.9**: The following examples show that rgw$\alpha$-continuous maps are independent of g-continuous, wg-continuous, r-continuous, gpr-continuous, gw-continuous, rwg-continuous and $\alpha$ g continuous maps.

**Example 3.9**: Let $X=Y=\{a,b,c\}$, $\tau = \{X, \phi, \{a\}, \{b,c\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Let map $f: X \to Y$ defined by $f(a)=b$ , $f(b)=a$ , $f(c)=c$, then $f$ is g-continuous, wg-continuous, rg-continuous, gr-continuous, gpr-continuous, rwg-continuous and $\alpha g$ continuous maps.
continuous, rwg-continuous and wg continuous but not rgwa-continuous function, as a closed set \( F = \{b, c\} \) in \( Y \)
\( f^{-1}(F) = f^{-1}\{b, c\} = \{a, c\} \) is not rgwa-closed set.

**Example 3.10:** Let \( X = \{a, b, c, d\} \) and \( Y = \{a, b, c\} \), \( \tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \) and \( \sigma = \{Y, \emptyset, \{a\}, \{b, c\}\} \). Let map \( f: X \rightarrow Y \) defined by \( f(a) = b, f(b) = a, f(c) = a, f(d) = c \) then \( f \) is rgwa-continuous but not g-continuous, wg-continuous, rg-continuous, gr-continuous, gpr-continuous, rgw-continuous, rwg-continuous and ag continuous, as a closed set \( F = \{a\} \) closed set in \( Y \)
\( f^{-1}(F) = \{b\} \) which is not g-closed, wg-closed, rg-closed, gr-closed, gpr-closed, rgw-closed, rwg-closed and ag closed set.

**Remark 3.11:** From the above discussion and known results we have the following implications

By \( A \rightarrow B \) we mean \( A \) implies \( B \) but not conversely and \( A \leftrightarrow B \) means \( A \) and \( B \) are independent of each other.

**Theorem 3.12:** Let \( f: X \rightarrow Y \) be a map. Then the following statements are equivalent:
(i) \( f \) is rgwa-continuous.
(ii) the inverse image of each open set in \( Y \) is rgwa-open in \( X \).

**Proof:** Assume that \( f: X \rightarrow Y \) is rgwa-continuous. Let \( G \) be open in \( Y \). The \( G \) is closed in \( Y \). Since \( f \) is rgwa-continuous, \( f^{-1}(G) \) is rgwa-closed in \( X \). But \( f^{-1}(G) = X - f^{-1}(G) \). Thus \( f^{-1}(G) \) is rgwa-open in \( X \).

Conversely, Assume that the inverse image of each open set in \( Y \) is rgwa-open in \( X \). Let \( F \) be any closed set in \( Y \). By assumption \( f^{-1}(F) \) is rgwa-open in \( X \). But \( f^{-1}(F) = X - f^{-1}(F) \). Thus \( X - f^{-1}(F) \) is rgwa-open in \( X \) and so \( f^{-1}(F) \) is rgwa-closed in \( X \). Therefore \( f \) is rgwa-continuous. Hence (i) and (ii) are equivalent.

**Theorem 3.13:** If \( f: (X, \tau) \rightarrow (Y, \sigma) \) is a map. Then the following holds.
1) \( f \) is rgwa-continuous and contra w-irresolute map then \( f \) is \( \alpha \)-continuous
2) \( f \) is rgwa-continuous and contra \( w \)- irresolute map then \( f \) is \( \alpha \)-continuous.
3) \( f \) is \( \alpha \)-continuous and contra \( w \)-irresolute map then \( f \) is rgwa-continuous
4) \( f \) is g-continuous and contra irresolute map then \( f \) is rgwa-continuous.

**Proof:**
1) Let \( V \) be w-closed set of \( Y \). As every w-closed set is closed, \( V \) is closed set in \( Y \). Since \( f \) is rgwa-continuous and contra \( w \)-irresolute map, \( f^{-1}(V) \) is rgwa-closed and \( w \)-open in \( X \). Now by Lemma 2.6, \( f^{-1}(V) \) is \( \alpha \)-closed in \( X \). Thus \( f \) is \( \alpha \)-continuous.
2) Similarly using Lemma 2.6 we can prove this.
3) Let V be closed set of Y. Since f is $\alpha$-continuous and contra w-irresolute map, $f'(V)$ is $\alpha$-closed and w-open in X, Now by Lemma 2.6, $f^{-1}(V)$ is $\alpha$rgw-closed in X. Thus f is $\alpha$rgw-continuous.
4) Similarly using Lemma 2.6 we can prove this.

**Theorem 3.14:** Let A be a subset of a topological space X. Then $x \in \alpha\text{rgwcl}(A)$ if and only if for any $\alpha\text{rgw}$-open set U containing x, $A \cap U \neq \emptyset$.

**Proof:** Let $x \in \alpha\text{rgwcl}(A)$ and suppose that there is a $\alpha\text{rgw}$-open set U in X such that $x \in U$ and $A \cap U = \emptyset$. This implies that $x \notin \alpha\text{rgwcl}(A)$, which is a contradiction.

Conversely, Suppose that, for any $\alpha\text{rgw}$-open set U containing x, $A \cap U \neq \emptyset$. To prove that $x \in \alpha\text{rgwcl}(A)$. Suppose that $x \notin \alpha\text{rgwcl}(A)$, then there is a $\alpha\text{rgw}$-closed set F in X such that $x \notin F$ and $A \subseteq F$. Since $x \notin F$ implies that $F \notin \alpha\text{rgw}$-open in X. Since $A \subseteq F$ implies that $A \cap F = \emptyset$, this is a contradiction.

Thus $x \in \alpha\text{rgwcl}(A)$.

**Theorem 3.15:** Let f: $X \rightarrow Y$ be a function from a topological space X into a topological space Y. If f: $X \rightarrow Y$ is $\alpha\text{rgw}$-continuous, then $f(\alpha\text{rgwcl}(A)) \subseteq \text{cl}(f(A))$ for every subset A of X.

**Proof:** Since $f(A) \subseteq \text{cl}(f(A))$ implies that $A \subseteq f^{-1}(\text{cl}(f(A)))$. Since $\text{cl}(f(A))$ is a closed set in Y and f is $\alpha\text{rgw}$-continuous, then by definition $f(\text{cl}(f(A)))$ is a $\alpha\text{rgw}$-closed set in X containing A. Hence $\alpha\text{rgwcl}(A) \subseteq f^{-1}(\text{cl}(f(A)))$. Therefore $f(\alpha\text{rgwcl}(A)) \subseteq \text{cl}(f(A))$.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.16:** Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c\}\}$, $\sigma = \{\emptyset, \phi, \{a\}, \{b, c\}\}$. Let map $f: X \rightarrow Y$ defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. For every subset of $X$, $f(\alpha\text{rgwcl}(A)) \subseteq \text{cl}(f(A))$ holds. But f is not $\alpha\text{rgw}$-continuous since $\alpha\text{rgw}$-closed set $\emptyset = \{b, c\}$ in Y, $f^{-1}(\emptyset) = \{a\}$ which is not $\alpha\text{rgw}$-closed set in X.

**Theorem 3.17:** Let f: $X \rightarrow Y$ be a function from a topological space X into a topological space Y. Then the following statements are equivalent:

(i) For each point $x \in X$ and each open set $V \in Y$ with $f(x) \in V$, there is a $\alpha$rgw-open set U in X such that $x \in U$ and $f(U) \subseteq V$.

(ii) For each subset A of X, $f(\alpha\text{rgwcl}(A)) \subseteq \text{cl}(f(A))$.

(iii) For each subset B of Y, $\alpha\text{rgwcl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$.

(iv) For each subset B of Y, $f^{-1}(\text{int}(B)) \subseteq \alpha\text{rgwint}(f^{-1}(B))$.

**Proof:** (i) $\rightarrow$ (ii) Suppose that (i) holds and let $y \in f(\alpha\text{rgwcl}(A))$ and let $V$ be any open set of Y. Since $y \in f(\alpha\text{rgwcl}(A))$ implies that there exists $x \in \alpha\text{rgwcl}(A)$ such that $f(x) = y$. Since $f(x) \in V$, then by (i) there exists a $\alpha$rgw-open set U in X such that $x \in U$ and $f(U) \subseteq V$. Since $x \in f(\alpha\text{rgwcl}(A))$, then by theorem 3.14 $U \cap A \neq \emptyset$. This implies that $\alpha\text{rgwcl}(A) \subseteq \text{cl}(f(A))$ and hence $f(U) \subseteq \text{cl}(f(A))$.

(ii) $\rightarrow$ (i) Let if (ii) holds and let $x \in X$ and $V$ be any open set in Y containing $f(x)$. Let $A = f^{-1}(V)$ this implies that $x \notin A$. Since $f(\alpha\text{rgwcl}(A)) \subseteq \text{cl}(f(A)) \subseteq V$, this implies that $\alpha\text{rgwcl}(A) \subseteq f^{-1}(V) = A$. Since $x \notin A$ implies that $x \notin \alpha\text{rgwcl}(A)$ and by theorem 3.14 there exists a $\alpha$rgw-open set U containing x such that $U \cap A = \emptyset$, then $U \in \alpha$rgwcl(A) and hence $f(U) \subseteq f(A)$.

(iii) $\rightarrow$ (ii) Suppose that (ii) holds and let B be any subset of Y. Replacing A by $f^{-1}(B)$ we get from (i) $f(\alpha\text{rgwcl}(f^{-1}(B))) \subseteq \text{cl}(f(f^{-1}(B))) \subseteq \text{cl}(B)$. Hence $\alpha\text{rgwcl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$.

(iv) $\rightarrow$ (iii) Suppose that (iv) holds. Let $B \subseteq Y$, then $Y \subseteq B \subseteq Y$. By (iv), $f(\text{cl}(Y - B)) \subseteq f^{-1}(\text{cl}(Y - B))$ this implies X $\alpha\text{rgw}(f^{-1}(B)) \subseteq X - f^{-1}(\text{int}(B))$. Therefore $f^{-1}(\text{int}(B)) \subseteq \text{rgwint}(f^{-1}(B))$.

**Definition 3.18:** Let $(X, \tau)$ be topological space and $\tau_{\text{rgw}} = \{V \subseteq X : \alpha\text{rgwcl}(V) = V\}$, $\tau_{\text{rgw}}$ is topology on X.

**Definition 3.19:** (i) A space $(X, \tau)$ is called $\alpha$rgw-space if every $\alpha$rgw-closed is closed.

(ii) A space $(X, \tau)$ is called $\alpha$rgw -space if every $\alpha$rgw-closed set is $\alpha$-closed set.
Theorem 3.20: Let f: X→Y be a function. Let (X,τ) and (Y,ω) be any two spaces such that τrgwa is a topology on X. Then the following statements are equivalent:
(i) For every subset A of X, f(rgwacl(A)) ⊆ cl(f(A)) holds,
(ii) f(X, τrgwa)→(Y,ω) is continuous.
Proof: Suppose (i) holds. Let A be closed in Y. By hypothesis f(rgwacl(f(A))) ⊆ cl(f(f(A))) ⊆ cl(A) = A. i.e. rgwacl(f(A))⊆f(A). Also f(rgwacl(A))⊆rgwacl(f(A)). Hence rgwacl(f(A))= f1(A). This implies f1(A)∈ τrgwa.
Thus f1(A) is closed in (X, τrgwa) and so f is continuous. This proves (ii).
Suppose (ii) holds. For every subset A of X, cl(f(A)) is closed in Y. Since f(X, τrgwa)→(Y,ω) is continuous, f1(cl(A)) is closed in (X, τrgwa) which implies by definition 3.22 rgwacl(f1(cl(A))) = f1(cl(f(A))). Now we have, A ⊆ f1(cl(A)) ⊆ f1(cl(f(A))) and by rgwa-closure, rgwacl(A)⊆rgwacl(f1(cl(A))= f1(cl(f(A))). Therefore f(rgwacl(A))⊆ cl(f(A)). This proves (i).

Remark 3.21: The Composition of two rgwa-continuous maps need not be rgwa-continuous map and this can be shown by the following example.
Example 3.22 : Let X=Y=Z={a,b,c}, τ = {X, φ, {a}, {b}, {a,b}, {a}, {c}, {a,c}, η ={Z, φ, {a}, {b}, {a,b}, {a}, {c}} and a maps f : X→Y is defined as f(a)=b, f(b)=c, f(c)=a, and g : Y→Z is defined as g(a)=b, g(b)=a, g(c)=c. Both f and g are rgwa-continuous maps. But g◦f not rgwa-continuous map, since closed set V={b,c} in Z, (f◦g)−1(V)= f1(g−1(V)) = f1{b,c} = {a,b} which is not rgwa-closed set in X.

Theorem 3.23: Let f: X→Y be rgwa-continuous function and g: Y→Z is continuous function then gof: X→Z is rgwa-continuous.
Proof: Let g be continuous function and V be any open set in Z, then g−1(V) is open in Y. Since f is rgwa-continuous, f−1(g(V)) = (g◦f)−1(V) is rgwa-open in X. Hence g◦f is rgwa-continuous.

Theorem 3.24: Let f: X→Y is rgwa-continuous function and g: Y→Z is rgwa-continuous function and Y is τrgwa-space, then gof: X→Z is rgwa-continuous.
Proof: Let g be rgwa-continuous function and V be any open set in Z then g−1(V) is rgwa-open in Y and Y is τrgwa-space, then g−1(V) is open in Y. Since f is rgwa-continuous, f−1(g−1(V)) = (g◦f)−1(V) is rgwa-open in X. Hence g◦f is rgwa-continuous.

Theorem 3.25: If a map f: X→Y is completely-continuous, then it is rgwa-continuous.
Proof: Suppose that a map f: (X, τ)→(Y ,ω) is completely-continuous. Let F closed set in Y. Then f−1(F) is regular closed in X and hence f−1(F) is is rgwa-closed in X. Thus f is rgwa-continuous.

Theorem 3.26: If a map f: X→Y is α-irresolute, then it is rgwaα-continuous.
Proof: Suppose that a map f: (X,τ)→(Y,σ) is α-irresolute. Let V be an open set in Y. Then V is α-open in Y. Since f is α-irresolute, f−1(V) is α-open and hence rgwa-open in X. Thus f is rgwa-continuous.

4. Perfectly rgwaα-Continuous and rgwa*α-Continuous Functions.

Definition 4.1: A function f from a topological space X into a topological space Y is called perfectly regular generalized weakly α-continuous if f is continuous. Let F be a open set in Y, then f1(F) is is rgwa-open in X.

Theorem 4.2: If a map f: X→Y is continuous, then the following holds.
(i) If f is perfectly rgwa-continuous, then f is rgwa-continuous.
(ii) If f is perfectly rgwa-continuous, then f is gβ-continuous.
Proof: Let F be a open set in Y, as every open is rgwa-open in Y, since f is perfectly rgwa-continuous, then f1(F) is both closed and open in X, as every open is rgwa-open, hence f is rgwa-continuous.

(ii) Let F be a open set in Y, as every open is rgwa-open in Y, since f is perfectly rgwa-continuous, then f1(F) is both closed and open in X, as every open is rgwa-open that implies is gβ-open, hence f is gβ-continuous.

Definition 4.3: A function f from a topological space X into a topological space Y is called regular generalized weakly α*-continuous (briefly rgwa*-continuous) if f−1(V) is is rgwa-closed set in X for every α-closed set V in Y.

Theorem 4.4: If a map f: X→Y is α*-continuous then it is rgwa*-continuous.
Proof: Let F be any α-closed set in Y, then F is rgwa-closed in Y. Since f is rgwa*-irresolute, the inverse image f−1(F) is rgwa-closed set in X. Therefore f is rgwa*-continuous.

(ii) Let f: X→Y be rgwa*-continuous. Let F be any closed set in Y, then F is α-closed in Y. Since f is rgwa*-continuous, the inverse image f−1(F) is rgwa-closed set in X. Therefore f is rgwa*-continuous.
Theorem 4.5: If a bijection \( f: (X, \tau) \rightarrow (Y, \sigma) \) is wa*-open and rgwa*-continuous, then it is rgwa-irresolute.

**Proof:** Let \( A \) be rgwa-closed in \( Y \). Let \( f^1(A) \subseteq U \) where \( U \) is wa-open set in \( X \). Since \( f \) is wa*-open map, \( f(U) \) is wa-open set in \( Y \). \( A \subseteq f(U) \) implies \( \text{rl}(A) \subseteq f(U) \). That is, \( f^1(\text{rl}(A)) \subseteq U \). Since \( f \) is rgwa*-continuous, \( \text{rl}(f^1(\text{rl}(A))) \subseteq U \). and so \( \text{rl}(f^1(A)) \subseteq U \). This shows \( f^1(A) \) is rgwa-closed in \( X \). Hence \( f \) is rgwa-irresolute.

**Theorem 4.6:** If \( f: (X, \tau) \rightarrow (Y, \alpha) \) is rgwa-continuous and wa*-closed and if \( A \) is rgwa-open (or rgwa-closed) subset of \( (Y, \alpha) \) and \( (Y, \alpha) \) is \( \alpha \)-space, then \( f(A) \) is rgwa-open (or rgwa-closed) in \( (X, \tau) \).

**Proof:** Let \( A \) be a rgwa-open set in \( (Y, \alpha) \) and \( G \) be any wa-closed set in \( (X, \tau) \) such that \( G \subseteq f^1(A) \). Then \( f(G) \subseteq A \). By hypothesis \( f(G) \) is wa-closed and \( A \) is rgwa-open in \( (Y, \alpha) \). Therefore \( f(G) \subseteq \text{rl}(A) \) by and so \( G \subseteq f^1(\text{rl}(A)) \). Since \( f \) is rgwa-continuous, \( \text{rl}(A) \) is \( \alpha \)-open in \( (Y, \alpha) \) and \( \alpha \)-space, so \( \text{rl}(A) \) is open in \( (Y, \alpha) \). Therefore \( f^1(\text{rl}(A)) \) is rgwa-open in \( (X, \tau) \). Thus \( G \subseteq f^1(\text{rl}(A)) \subseteq \text{rl}(f^1(A)) \); that is, \( G \subseteq \text{rl}(f^1(A)) \). \( f^1(A) \) is rgwa-open in \( (X, \tau) \). By taking the complements we can show that if \( A \) is wa-closed in \( (Y, \alpha) \), then \( f^1(A) \) is rgwa-closed in \( (X, \tau) \).

**Theorem 4.7:** Let \( (X, \tau) \) be a discrete topological space and \( (Y, \alpha) \) be any topological space. Let \( f: (X, \tau) \rightarrow (Y, \alpha) \) be a map. Then the following statements are equivalent: (i) \( f \) is strongly rgwa-continuous. (ii) \( f \) is perfectly rgwa-continuous.

**Proof:** (i) \( \Rightarrow \) (ii) Let \( U \) be any rgwa-open set in \( (Y, \alpha) \). By hypothesis \( f^1(U) \) is open in \( (X, \tau) \). Since \( (X, \tau) \) is a discrete space, \( f^1(U) \) is also in \( (X, \tau) \). \( f(U) \) is both open and closed in \( (X, \tau) \). Hence \( f \) is perfectly rgwa-continuous. (ii) \( \Rightarrow \) (i) Let \( U \) be any rgwa-open set in \( (Y, \alpha) \). Then \( f(U) \) is both open and closed in \( (X, \tau) \). Hence \( f \) is strongly rgwa-continuous.

5. **rgwa-Irresolute and Strongly rgwa-Continuous Functions.**

**Definition 5.1:** A function \( f \) from a topological space \( X \) into a topological space \( Y \) is called regular generalized weakly \( \alpha \)-irreversible (briefly rgwa-irreversible) map if \( f^1(V) \) is rgwa-closed in \( X \) for every rgwa-closed set \( V \) in \( Y \).

**Definition 5.2:** A function \( f \) from a topological space \( X \) into a topological space \( Y \) is called strongly regular generalized weakly \( \alpha \)-continuous (strongly rgwa-continuous) map if \( f^1(V) \) is closed set in \( X \) for every rgwa-closed set \( V \) in \( Y \).

**Theorem 5.3:** If a map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is rgwa-irresolute, then it is rgwa-continuous but not conversely.

**Proof:** Let \( f: X \rightarrow Y \) be rgwa-irresolute. Let \( F \) be any closed set in \( X \). Then \( F \) is rgwa-closed in \( X \). Since \( f \) is rgwa-irresolute, the inverse image \( f^{-1}(F) \) is rgwa-closed in \( X \). Therefore \( f \) is rgwa-continuous. The converse theorem need not be true as seen from the following example.

**Example 5.4:** \( X = \{a, b, c, d, e\} \), \( Y = \{a, b, c, d\} \), \( \tau = \{X, \phi, \{a\}, \{d\}, \{e\}, \{a, d\}, \{a, e\}, \{d, e\}, \{a, d, e\}\} \), \( \sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \). Let map \( f: X \rightarrow Y \) defined by \( f(a) = b \), \( f(b) = c \), \( f(c) = d \), \( f(d) = a \), \( f(e) = d \). Then \( f \) is rgwa-continuous but \( f \) is not rgwa-irresolute, as \( \text{rl}(f^{-1}(\{d\})) \) is rgwa-closed set in \( X \). and \( \{a\} \in X \), then \( f^{-1}(\{a\}) = \{a\} \) in \( X \), which is not rgwa-closed set in \( X \).

**Theorem 5.5:** If a map \( f: (X, \tau) \rightarrow (Y, \sigma) \) is rgwa-irresolute, if and only if the inverse image \( f^{-1}(V) \) is rgwa-open set in \( X \) for every rgwa-open set \( V \) in \( Y \).

**Proof:** Assume that \( f: X \rightarrow Y \) is rgwa-irresolute. Let \( G \) be rgwa-open in \( Y \). Then \( f^{-1}(G) \) is rgwa-closed in \( X \). If \( f^{-1}(G) \) is rgwa-continuous in \( X \), then \( f \) is rgwa-irresolute.

**Example 5.6:** If \( A \subseteq X \) then consider \( \text{rl}(f(A)) \) which is rgwa-closed in \( Y \). Since \( f \) is rgwa-irresolute, \( f^1(\text{rl}(f(A))) \) is rgwa-closed in \( X \). Furthermore, since \( f^1(\text{rl}(f(A))) \subseteq \text{rl}(f^1(f(A))) \), then \( f \) is rgwa-irresolute.

**Theorem 5.7:** Let \( f: (X, \tau) \rightarrow (Y, \alpha) \) and \( g: (Y, \sigma) \rightarrow (Z, \eta) \) be any two functions. Then (i) \( g \circ f: (X, \tau) \rightarrow (Z, \eta) \) is rgwa-continuous if \( g \) is rgwa-continuous and \( f \) is rgwa-irresolute. (ii) \( g \circ f: (X, \tau) \rightarrow (Z, \eta) \) is rgwa-irresolute if \( g \) is rgwa-irresolute and \( f \) is rgwa-irresolute. (iii) \( g \circ f: (X, \tau) \rightarrow (Z, \eta) \) is rgwa-continuous if \( g \) is continuous and \( f \) is rgwa-irresolute.
Proof: (i) Let U be a open set in (Z, η). Since g is r-continuous, g(U) is r-open in (Y, σ). Since every r-open is rgwa-open then g(U) is rgwa-open in Y, since f is rgwa-irresolute f(U) is an rgwa-open set in (X, τ). Thus (gof)^(-1)(U) = f^(-1)(g(U)) is an rgwa-open set in (X, τ) and hence gof is rgwa-continuous.

(ii) Let U be a rgwa-open set in (Z, η). Since g is rgwa-irresolute, g^(-1)(U) is rgwa-open set in (Y, σ). Since f is rgwa-irresolute, f^(-1)(g(U)) is an rgwa-open set in (X, τ). Thus (gof)^(-1)(U) = f^(-1)(g(U)) is an rgwa-open set in (X, τ) and hence gof is rgwa-continuous.

(iii) Let U be a open set in (Z, η). Since g is continuous, g^(-1)(U) is open set in (Y, σ). As every open set is rgwa-open, g^(-1)(U) is rgwa-open set in (Y, σ). Since f is rgwa-irresolute, f^(-1)(g(U)) is an rgwa-open set in (X, τ). Thus (gof)^(-1)(U) = f^(-1)(g(U)) is an rgwa-open set in (X, τ) and hence gof is rgwa-continuous.

Theorem 5.8: Let f: (X, τ) → (Y, σ) be strongly rgwa-continuous then it is continuous.

Proof: Assume that f: (X, τ) → (Y, σ) is strongly rgwa-continuous. Let F be closed set in Y. As every closed is rgwa-closed, F is rgwa-closed in Y. since f is strongly rgwa-continuous then f(F) is closed set in X. Therefore f is continuous.

Theorem 5.9: Let f: (X, τ) → (Y, σ) be strongly rgwa-continuous then it is strongly α-continuous but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is strongly rgwa-continuous. Let F be α-closed set in Y. As every α-closed is rgwa-closed, F is rgwa-closed in Y. since f is strongly rgwa-continuous then f(F) is closed set in Y. Therefore f is strongly α-continuous.

The converse of the above theorem 5.9 need not be true as seen from the following example

Example 5.10: Let X=Y={a,b,c,d}, τ = {X, φ, {a}, {b}, {a,b}, {a,b,c},} and σ = {Y, φ, {a}, {b}, {a,b}, {a,b,c}}. Let map f: X→Y defined by f(a)=b, f(b)=a, f(c)=d, f(d)=c, then f is strongly α-continuous but not continuous and not strongly rgwa-continuous, as closed set F={b,c,d} in Y, then f(F)={a,c,d} in X which is not closed set in X.

Theorem 5.11: Let f: (X, τ)→ (Y, σ) be strongly rgwa-continuous if and only if f(G) is open set in X for every rgwa-open set G in Y.

Proof: Assume that f: X→Y is strongly rgwa-continuous. Let G be rgwa-open in Y. The G is rgwa-closed in Y. Since f is strongly rgwa-continuous, f(G) is closed in X. But f(G) = X∩f(G). Thus f(G) is open in X. Conversely, Assume that the inverse image of each open set in Y is rgwa-open in X. Let F be any rgwa-closed set in Y. By assumption F is rgwa-open in X. But f(F) = X∩f(F). Thus X∩f(F) is open in X and so f(F) is closed in X. Therefore f is strongly rgwa-continuous.

Theorem 5.12: Let f: (X, τ) → (Y, σ) be strongly continuous then it is strongly rgwa-continuous.

Proof: Assume that f: X→Y is strongly continuous. Let G be rgwa-open in Y and also it is any subset of Y since f is strongly continuous, f(G) is open and also it is any subset of Y. Therefore f is strongly rgwa-continuous.

Theorem 5.13: Let f: (X, τ)→ (Y, σ) be strongly rgwa-continuous then it is rgwa-continuous.

Proof: Let G be open in Y, every open is rgwa-open, G is rgwa-open in Y. Hence f is strongly rgwa-continuous, f(G) is open in X and therefore f(G) is rgwa-open in X. Hence f is rgwa-continuous.

Theorem 5.14: In discrete space, a map f: (X, τ)→(Y, σ) is strongly rgwa-continuous then it is strongly continuous. Proof: f any subset of Y, in discrete space, Every subset F in Y is both open and closed, then subset F is both rgwa-open or rgwa-closed, i) let F is rgwa-closed in Y, since f is strongly rgwa-continuous, then f(F) is closed in X. ii) let F is rgwa-open in Y, since f is strongly rgwa-continuous, then f(F) is open in X. Therefore f(F) is closed and open in X. Hence f is strongly continuous.

Theorem 5.15: Let f: (X, τ)→(Y, σ) and g: (Y, σ)→ (Z, η) be any two functions. Then

(i) g o f : (X, τ)→(Z, η) is strongly rgwa-continuous if g is strongly rgwa-continuous and f is strongly rgwa-continuous.

(ii) g o f : (X, τ)→(Z, η) is strongly rgwa-continuous if g is strongly rgwa-continuous and f is continuous.

(iii) g o f : (X, τ)→(Z, η) is rgwa-irresolute if g is strongly rgwa-continuous and f is rgwa-continuous.

(iv) g o f : (X, τ)→(Z, η) is continuous if g is rgwa-continuous and f is strongly rgwa-continuous.

Proof:

(i) Let U be a rgwa-open set in (Z, η). Since g is strongly rgwa-continuous, g(U) is open set in (Y, σ). As every open set is rgwa-open, g(U) is rgwa-open set in (Y, σ). Since f is strongly rgwa-continuous f(G) is open set in (X, τ). Thus (gof)(U) = f(g(U)) is an open set in (X, τ) and hence gof is strongly rgwa-continuous.

(ii) Let U be a rgwa-open set in (Z, η). Since g is strongly rgwa-continuous, g(U) is open set in (Y, σ). Since f is continuous f(g(U)) is an open set in (X, τ). Thus (gof)(U) = f(g(U)) is an open set in (X, τ) and hence gof is strongly rgwa-continuous.

(iii) Let U be a rgwa-open set in (Z, η). Since g is strongly rgwa-continuous, g(U) is open set in (Y, σ). Since f is rgwa-continuous f(g(U)) is an rgwa-open
set in \((X, \tau)\). Thus \((gof) \cdot (U) = f'(g(U))\) is an rgwα-open set in \((X, \tau)\) and hence \(gof\) is rgwα-irresolute.

(iv) Let \(U\) be open set in \((Z, \eta)\). Since \(g\) is rgwα-continuous, \(g'(U)\) is rgwα-open set in \((Y, \sigma)\). Since \(f\) is strongly rgwα-continuous \(f'\) \((g'(U))\) is an open set in \((X, \tau)\). Thus \((gof)'(U) = f'(g(U))\) is an open set in \((X, \tau)\) and hence \(gof\) is continuous.

**Theorem 5.16:** Let \(f: (X, \tau) \to (Y, \sigma)\) and \(g: (Y, \sigma) \to (Z, \eta)\) be any two functions. Then

1. \(g \circ f: (X, \tau) \to (Z, \eta)\) is strongly rgwα-continuous if \(g\) is perfectly rgwα-continuous and \(f\) is continuous.
2. \(g \circ f: (X, \tau) \to (Z, \eta)\) is perfectly rgwα-continuous if \(g\) is strongly rgwα-continuous and \(f\) is perfectly rgwα-continuous.

**Proof:**

1. Let \(U\) be a rgwα-open set in \((Z, \eta)\). Since \(g\) is perfectly rgwα-continuous, \(g'(U)\) is open set in \((Y, \sigma)\). Since \(f\) is continuous \(f'(g'(U))\) is an open set in \((X, \tau)\). Thus \((gof)'(U) = f'(g(U))\) is an open set in \((X, \tau)\) and hence \(gof\) is continuous.

2. Let \(U\) be a rgwα-open set in \((Z, \eta)\). Since \(g\) is strongly rgwα-continuous, \(g'(U)\) is open set in \((Y, \sigma)\). Since \(f\) is perfectly rgwα-continuous, \(f'(g'(U))\) is an open set in \((X, \tau)\). Thus \((gof)'(U) = f'(g(U))\) is an open set in \((X, \tau)\) and hence \(gof\) is perfectly rgwα-continuous.

**Theorem 5.17:** If \(A\) map \(f: (X, \tau) \to (Y, \sigma)\) is strongly rgwα-continuous and \(A\) is open subset of \(X\) then the restriction \(f/A: A \to Y\) is strongly rgwα-continuous.

**Proof:** Let \(V\) be any rgwα-open set of \(Y\), since \(f\) is strongly rgwα-continuous \(f'(V)\) is open in \(X\). Since \(A\) is open in \(X\), \(f/A \cdot (V) = A \cap f(V)\) is open in \(A\). Hence \(f/A\) is strongly rgwα-continuous.

**Theorem 5.18** Let \((X, \tau)\) be any topological space and \((Y, \sigma)\) be a \(T_{rgwα}\)-space and \(f: (X, \tau) \to (Y, \sigma)\) be a map. Then the following are equivalent: (i) \(f\) is strongly rgwα-continuous, (ii) \(f\) is continuous.

**Proof:** (i) \(\Rightarrow\) (ii) Let \(U\) be any open set in \((Y, \sigma)\). Since every open set is rgwα-open, \(U\) is rgwα-open in \((Y, \sigma)\). Then \(f'(U)\) is open in \((X, \tau)\). Hence \(f\) is continuous.

(ii) \(\Rightarrow\) (i) Let \(U\) be any rgwα-open set in \((Y, \sigma)\). Since \((Y, \sigma)\) is a \(T_{rgwα}\)-space, \(U\) is open in \((Y, \sigma)\). Since \(f\) is continuous. Then \(f'(U)\) is open in \((X, \tau)\). Hence \(f\) is strongly rgwα-continuous.

**Theorem 5.19:** Let \((X, \tau)\) be a discrete topological space and \((Y, \sigma)\) be any topological space. Let \(f: (X, \tau) \to (Y, \sigma)\) be a map. Then the following statements are equivalent: (i) \(f\) is strongly rgwα-continuous, (ii) \(f\) is perfectly rgwα-continuous.

**Proof:** (i) \(\Rightarrow\) (ii) Let \(U\) be any rgwα-open set in \((Y, \sigma)\). By hypothesis \(f'(U)\) is open in \((X, \tau)\). Since\((X, \tau)\) is a discrete space, \(f'(U)\) is also closed in \((X, \tau)\). \(f'(U)\) is both open and closed in \((X, \tau)\). Hence \(f\) is perfectly rgwα-continuous, (ii) \(\Rightarrow\) (i) Let \(U\) be any rgwα-open set in \((Y, \sigma)\). Then \(f'(U)\) is both open and closed in \((X, \tau)\). Hence \(f\) is strongly rgwα-continuous.

**Theorem 5.20:** Let \(f: (X, \tau) \to (Y, \sigma)\) be a map. Both \((X, \tau)\) and \((Y, \sigma)\) are \(T_{rgwα}\)-spaces. Then the following are equivalent:

1. \(f\) is rgwα-irresolute.
2. \(f\) is strongly rgwα-continuous.
3. \(f\) is continuous.
4. \(f\) is rgwα-continuous.

**Proof:** Straight forward.

**Theorem 5.21:** Let \(X\) and \(Y\) be \(T_{rgwα}\)-spaces, then for a function \(f: (X, \tau) \to (Y, \sigma)\), the following are equivalent: (i) \(f\) is \(\alpha\)-irresolute, (ii) \(f\) is rgwα-irresolute.

**Proof:** (i) \(\Rightarrow\) (ii) Let \(f: (X, \tau) \to (Y, \sigma)\) be a \(\alpha\)-irresolute. Let \(V\) be a rgwα-closed set in \(Y\). As \((Y, \sigma)\) is \(T_{rgwα}\)-space, \(V\) be a \(\alpha\)-closed set in \(Y\). Since \(f\) is \(\alpha\)-irresolute, \(f'(V)\) is \(\alpha\)-closed in \(X\). But every \(\alpha\)-closed set is rgwα-closed in \(X\) and hence \(f'(V)\) is a rgwα-closed in \(X\). Therefore, \(f\) is rgwα-irresolute.

(ii) \(\Rightarrow\) (i) Let \(f: (X, \tau) \to (Y, \sigma)\) be a rgwα-irresolute. Let \(V\) be a \(\alpha\)-closed set in \(Y\). But every \(\alpha\)-closed set is rgwα-closed and hence \(V\) is rgwα-closed in \(Y\) and \(f\) is rgwα-irresolute implies \(f'(V)\) is rgwα-closed in \(X\). But \((X, \tau)\) is \(T_{rgwα}\)-space and hence \(f'(V)\) is \(\alpha\)-closed in \(X\). Thus, \(f\) is \(\alpha\)-irresolute.

6. Conclusion

In this paper we have introduced and studied the properties of rgwα-continuous and rgwα-irresolute maps. Our future extension is rgwα-continuous and rgwα-irresolute in Fuzzy Topological Spaces.

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