Regarding Edge Domination in Hypergraph

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Abstract: In this paper we further prove more results about edge domination in hypergraphs. In particular we prove necessary & sufficient conditions under which the edge domination number of a hypergraph increases or decreases when an edge is added or removed from the hypergraph. We have proved that if \(\gamma_e(G - v) > \gamma_e(G)\) & if \(F \subseteq E(G)\) then there is an edge \(e\) containing \(v\) \(\not\in F\) \& \(\text{Prm}(e, F)\) contains two distinct edges also we have proved that if \(\gamma_e(G + h) < \gamma_e(G)\) then there are at least two vertices \(x\) & \(y\) in \(h\) \(\not\in F\) \& all the edges containing \(x\) or \(y\) except \(h\) are in the complement of \(F\). Where \(F\) is any minimum edge dominating set of \(G + h\).

Keywords: Hypergraph, Dominating Set in Hypergraph, Edge Dominating Set, Edge Dominating Number, Minimal Edge Dominating Set, Minimum Edge Dominating Set, Edge Degree, Edge Neighbourhood, Sub Hypergraph, Partial Sub Hypergraph, Dual Hypergraph, edge addition, edge removal.

AMS Subject Classification (2010): 05C15, 05C69, 05C65

1. Introduction

Edge dominating set & edge domination number have been explored by several authors [5, 6]. The concept of edge domination requires the adjacency relation among the edges of a graph. The same relation is also available in hypergraphs and therefore we have considered edge domination in hypergraphs [7].

The change in the edge domination number when an edge is added to the hypergraph or an edge is removed from the hypergraph has been studied here.

We have considered the operation of vertex removal from hypergraph in [8]. Here also the edge domination number may increase, decrease or remains unchanged when a vertex is removed from the hypergraph.

2. Preliminaries

Definition 2.1 Hypergraph: [4] A hypergraph \(G\) is an ordered pair \((V(G), E(G))\) where \(V(G)\) is a non-empty finite set \& \(E(G)\) is a family of non-empty subsets of \(V(G)\) \(\cup\) their union = \(V(G)\). The elements of \(V(G)\) are called vertices & the members of \(E(G)\) are called edges of the hypergraph \(G\).

We make the following assumption about the hypergraph.

(1) Any two distinct edges intersect in at most one vertex.

(2) If \(e_1\) and \(e_2\) are distinct edges with \(|e_1|, |e_2| > 1\) then \(e_1 \not\subset e_2 \& e_2 \not\subset e_1\).

Definition 2.2 Edge Degree: [4] Let \(G\) be a hypergraph \& \(v \in V(G)\) then the edge degree of \(v = \Delta_0(v) = \text{number of edges containing the vertex } v\). The minimum edge degree among all the vertices of \(G\) is denoted as \(\Delta_0(G)\) and the maximum edge degree is denoted as \(\Delta_1(G)\).

Definition 2.3 Dual Hypergraph: [4] Let \(G\) be a hypergraph. For every \(v \in V(G)\) define \(\overline{v}\) as follows.

\[\overline{v} = \{e \in E(G) / v \not\in e\}.\]

Let \(E(G^*) = \{\overline{v} / v \in V(G)\}\) and let \(V(G^*) = E(G)\). Then the dual hypergraph of the given hypergraph \(G\) is the hypergraph \(G^*\) whose vertex set is \(V(G^*)\) \& the edge set is \(E(G^*)\). We will write \(G^* = (V(G^*), E(G^*))\).

Definition 2.4 Dominating Set in Hypergraph: [1] Let \(G\) be a hypergraph \& \(S \subseteq V(G)\) then \(S\) is said to be a dominating set of \(G\) if for every \(v \in V(G) - S\) there is some \(u \in S\) \& \(u\) and \(v\) are adjacent vertices.

A dominating set with minimum cardinality is called minimum dominating set and cardinality of such a set is called domination number of \(G\) and it is denoted as \(\gamma(G)\).

Definition 2.5 Edge Dominating Set: [7] Let \(G\) be a hypergraph \& \(S \subseteq E(G)\) then \(S\) is said to be an edge dominating set of \(G\) if for every \(e \in E(G) - S\) there is some \(f \in S \not\subset e\) \& \(f\) are adjacent edges.

An edge dominating set with minimum cardinality is called a minimum edge dominating set and cardinality of such a set is called edge domination number of \(G\) and it is denoted as \(\gamma_e(G)\).

Definition 2.6 Minimal Edge Dominating Set: [7] Let \(G\) be a hypergraph \& \(F \subseteq E(G)\) then \(F\) is said to be a minimal edge dominating set if

(1) \(F\) is an edge dominating set
(2) No proper subset of \(F\) is an edge dominating set of \(G\).

Definition 2.7 Sub hypergraph and Partial sub hypergraph: [3] Let \(G\) be a hypergraph \& \(v \in V(G)\). Consider the subset \(V(G) - \{v\}\) of \(V(G)\). This set will induce two types of hypergraphs from \(G\).
(1) First type of hypergraph: Here the vertex set $V(G) - \{v\}$ and the edge set $\{ e' / e' = e - \{v\} \text{ for some } e \in E(G) \}$. This hypergraph is called the sub hypergraph of $G$ & it is denoted as $G - \{v\}$.

(2) Second type of hypergraph: Here also the vertex set $V(G) - \{v\}$ and edges in this hypergraph are those edges of $G$ which do not contain the vertex $v$. This hypergraph is called the partial sub hypergraph of $G$.

Definition 2.8 Edge Neighbourhood: [3] Let $G$ be a hypergraph & $e$ be any edge of $G$ then

- Open edge neighbourhood of $e = N(e) = \{ f \in E(G) / f \text{ is adjacent to } e \}$.
- Close edge neighbourhood of $e = N[e] = N(e) - \{e\}$.

Definition 2.9 Private Neighbourhood of an edge: [3] Let $G$ be a hypergraph. $F$ be a set of edges & $e \in F$, then the private neighbourhood of $e$ with respect to set $F = \text{Prn}[e, F] = \{ f \in E(G) / N[f] \cap F = \{e\} \}$

3. Edge Removal from the Hypergraph:

Definition 3.1 Edge Removal in Hypergraph: Let $G$ be a hypergraph & $e$ be an edge of $G$ then $G - e$ will denote the partial hypergraph whose vertex set is $V(G)$ & edge set is $E(G) - \{e\}$. (Now, we assume that in $G - \{e\}$ there are no isolated vertices, which is the requirement of any hypergraph)

Now, we consider the effect of removing an edge from a hypergraph $G$. The following examples show that the edge domination number may increase, decrease or remain unchanged.

Example 3.2:

Here, $\gamma_{E}(G) = 2$, When any edge is removed from this graph $\gamma_{E}(G - e) = 1$ in resultant graph.

Thus $\gamma_{E}(G - e) < \gamma_{E}(G)$.

Theorem 3.3: Let $G$ be a hypergraph & $e \in E(G)$ then $\gamma_{E}(G - e) < \gamma_{E}(G)$ iff there is a minimum edge dominating set $F$ of $G$ containing $\therefore \text{Prn}[e, F] = \{e\}$

Proof: Suppose $\gamma_{E}(G - e) < \gamma_{E}(G)$

Let $F_{1}$ be a minimum edge dominating set of $G - e$. Then $F_{1}$ cannot be an edge dominating set of $G$. Therefore, $e$ cannot intersect any member of $F_{1}$. Let $F = F_{1} - \{e\}$. It is obvious that $F$ is a minimum edge dominating set of $G$ & $e \in F$. Since, $e$ is not adjacent with any other member of $F$, $e \notin \text{Prn}[e, F]$. Let $f$ be any edge of $G \ni f \neq e$ then $f$ is an edge of $G - e$. Since $F_{1}$ is an edge dominating set of $G - e$, $F$ intersect some member of $F_{1}$. Therefore, $f \notin \text{Prn}[e, F]$. Therefore, $\text{Prn}[e, F] = \{e\}$.

Conversely suppose there is a minimum edge dominating set $F$ of $G \ni f \in F$ & $\text{Prn}[e, F] = \{e\}$. Let $F_{1} = F - \{e\}$ then $F_{1}$ is a set of edges of $G - e$. Let $f$ be any edge of $G - e \ni f \notin F_{1}$ then $f \notin F$. Now, $f$ is adjacent to some member of $F$. Suppose $f$ is adjacent
to e. Now by assumption f ∈ Prn[e, F]. Therefore, f must be adjacent to some other member h of F, then h ∈ F1. Thus f is adjacent to some member of F1. Thus, F1 is an edge dominating set of G − e.

\[ γ_e(G − e) ≤ |F1| ≤ |F| = γ_e(G) \]
\[ γ_e(G − e) < γ_e(G) \]

**Corollary 3.4:** Let G be a hypergraph & e ∈ E(G) if γ_e(G − e) < γ_e(G) then γ_e(G − e) = γ_e(G) − 1

**Proof:** Obvious

**Remark 3.5:** Let G be a hypergraph & e be an edge of G such that edge degree of x ≥ 2 for every x in e. Now, consider hypergraph (G − e)* & G* − e.

The vertices of (G − e)* are those edges of G which are different from e. The vertices of G* − e are those edges of G which are different from e. Therefore, V(G* − e) = V((G − e)*). Let x be a vertex of G containing x ∈ e. Let \( x_0 = \{ f ∈ E(G) / x ∈ f & f ∉ e \} \) then it is obvious that \( x_0 \) is an edge of (G − e)*. Suppose x ∉ e & let \( x_0 = \{ f ∈ E(G) / x ∈ f & f ∉ e \} \) then it is obvious that \( x_0 = x − \{ e \} \) if x ∈ e & \( x_0 = \bar{x} \) if x ∉ e.

Thus, we have \( \{ x_0 / x ∈ V(G) \} \) which is the edge set of (G − e)*. On the other hand if we consider the sub hypergraph G* − e then \( \bar{x} \) & \( \bar{x} \) are \( x \) & \( x \) (i.e. x ∉ e & e ∉ x) & \( \bar{x} = x − \{ e \} \) if e ∈ x (i.e. x ∈ e). Thus, we observed that \( \bar{x} = \bar{x} \) for every x ∈ V(G).

\[ \therefore \] The edge set of (G − e)* = The edge set of sub hypergraph G* − e.

\[ \therefore \] The dual hypergraph (G − e)* = The sub hypergraph G* − e of G*.

**Theorem 3.6:** [8] Let G be a hypergraph & v ∈ V(G) such that [v] is not an edge of G then γ(G − v) > γ(G) if the following two conditions are satisfied.

(1) \( v \in S \) for every minimum dominating set S of G.

(2) There is no subset S of G − v \( v \in S \cap N[v] = \emptyset, [S] \leq γ(G) \) & S is a dominating set of G − v. (Here G − v is the sub hypergraph of G.)

**Theorem 3.7:** Let G be a hypergraph & e ∈ E(G) \( v \in V(G) \) edge degree of x ≥ 2 for every x in e. Then γ_e(G − e) > γ_e(G) if the following conditions are satisfied

(1) e ∈ F for every minimum edge dominating set F of G.

(2) There is no subset F of E(G) − \{ e \} \( |F| ≤ γ_e(G) \), F \( F \cap N[e] = \emptyset \) & F is an edge dominating set of G − e.

**Proof:** Suppose γ_e(G − e) > γ_e(G)

(1) Suppose F is a minimum edge dominating set of G then F is a minimum dominating set of G. Since γ_e(G − e) > γ_e(G) it follows that (γ_e(G − e))* > γ(G*) but (G − e)* = G* − e. Thus, condition (1) is satisfied.

(2) Suppose there is an edge dominating set F of G − e \( \geq |F| ≤ γ_e(G) \), F \( F \cap N[e] = \emptyset \). Then F is a dominating set of (G − e)*. Thus, condition (2) is satisfied.

Thus, by above theorem e ∈ F. Thus, condition (1) is satisfied.

First suppose that e is not adjacent to any edge in F then \( F \cap N[e] = \emptyset \) & F is an edge dominating set of G − e. This contradicts (2). Suppose that e is not adjacent to any edge in F then \( F \cap N[e] = \emptyset \) & F is an edge dominating set of G − e. This contradicts (2).

\[ γ_e(G − e) ≤ γ_e(G) \] is also not possible.

Hence, γ_e(G − e) > γ_e(G).

In the following theorem we consider the partial sub hypergraph.

**Theorem 3.8:** [8] Let G be a hypergraph & v ∈ V(G) such that [v] is not an edge of G then γ_e(G − v) > γ_e(G) if there is a minimum edge dominating set F & edge e containing v of G \( G \in F \) & the following two conditions are satisfied.

(1) e ∈ Prn[e, F]

(2) Prn[e, F] is a subset of N_e(v)

**Corollary 3.9:** Let G be a hypergraph & e be an edge of G. If γ_e(G − e) > γ_e(G) then γ_e(G − e) < γ_e(G) for all x in e. (We consider here partial sub hypergraph G − x).

**Proof:** Since γ_e(G − e) > γ_e(G), there is a minimum edge dominating set F of G − e \( G \in F \) & Prn[e, F] = [e]. Let x ∈ e. Then F is a minimum edge dominating set containing some edge (namely e) containing x \( x \in Prn[e, F] \) contains e & Prn[e, F] contains only those edges which contain the vertex x (Which is empty set in this case).

Thus by above theorem γ_e(G − x) < γ_e(G).
Theorem 3.11[8]: Let G be a hypergraph & v ∈ V(G) such that \{v\} is not an edge of G if \(\gamma_E(G - v) > \gamma_E(G)\) & if F is a minimum edge dominating set of G then there is an edge e containing v \(\exists e \in F\) & Prn[e, F] contains two distinct edges.

Corollary 3.12: Let G be a hypergraph & v ∈ V(G) such that \{v\} is not an edge of G. If \(\gamma_E(G - v) > \gamma_E(G)\) then there is no edge e containing v \(\exists \gamma_E(G - e) \leq \gamma_E(G)\).

Proof: Suppose there is an edge e containing v \(\exists \gamma_E(G - e) > \gamma_E(G)\). Let F₁ be a minimum edge dominating set of G then by the above theorem there is an edge f containing v \(\exists f \in F₁\) & Prn[f, F₁] contains two distinct edges. Since \(\gamma_E(G - e) > \gamma_E(G)\) there is a minimum edge dominating set F₁ such that e \(\exists \gamma_E(G - e) < \gamma_E(G)\).

Now, we state & prove a necessary & sufficient condition under which the edge domination number of a hypergraph increases when an edge h is added to the hypergraph G.

Theorem 4.3: Let G be a hypergraph & h be a subset of V(G) such that h \(\exists e \in E(G)\) is at most one vertex for every edge e of G then the following statements are equivalent.

(1) \(\gamma_E(G + h) > \gamma_E(G)\)
(2) There is a minimum edge dominating set F₁ of G + h \(\exists h \in F₁\) & Prn[h, F₁] = \{h\}
(3) For every minimum edge dominating set F of G, \(h \cap e = \emptyset\) for every e \(\in F\).
(4) There is a minimum edge dominating set F₁ of G + h \(\exists F₁ = F \cup \{h\}\) for some minimum edge dominating set F of G.

Proof: (1) \(\Rightarrow\) (3)
Let F be a minimum edge dominating set of G. Since \(|F| = \gamma_E(G) < \gamma_E(G + h)\), F cannot be an edge dominating set of G + h. Since every edge of G intersect some member of F, h does not intersect any member of F.

Thus (1) \(\Rightarrow\) (3) is proved.

(3) \(\Rightarrow\) (2)
Let F be a minimum edge dominating set of G & h does not intersect any member of F.

Let F₁ = F \(\cup \{h\}\). Obviously, F₁ is an edge dominating set of G + h & h \(\in F₁\). Since h does not intersect any other member of F₁, Prn[h, F₁] = \{h\}

Thus (3) \(\Rightarrow\) (2) is proved.

(2) \(\Rightarrow\) (1)
Let F₁ = F \(\cup \{h\}\) then F is a set of edges of G & |F| < |F₁|. Let f be any edge of G \(\exists f \notin F\) then f \(\notin F₁\). Suppose f \(\cap h \neq \emptyset\).

Now, f \(\notin Prn[h, F₁]\), f \(\cap g \neq \emptyset\) for some g in F₁ with g \(\neq h\). Then g \(\in F\) & f \(\cap g \neq \emptyset\).

Suppose f \(\cap h = \emptyset\).
Since $F_1$ is an edge dominating set of $G + h$, $f$ must be adjacent with some other edge $h'$ of $F_1$. Thus, $h' \in F$ & $f \cap h' \neq \phi$.

Thus in either case $f$ is adjacent to some member of $F$ & therefore $F$ is an edge dominating set of $G$.

Thus, $e_1 \Rightarrow (\sum_{e \in h} m) \times \text{edge degree of } x$ minimum edge dominating set of $G + h$.

Now, consider the possibility that the edge domination number of a hypergraph decreases when an edge $h$ is added to the hypergraph.

**Theorem 4.6**: Let $G$ be a hypergraph & $h$ be as above. If $\gamma_e(G + h) < \gamma_e(G)$ then $h \in F$ for every minimum edge dominating set $F$ of $G + h$.

**Proof**: Suppose there is a minimum edge dominating set $F$ of $G + h$. Then $F$ contains edges of $G$. If $f$ is any edge of $G$ then $f \notin F$. Now, $f$ is also an edge of $G + h$ therefore it is adjacent to some member of $F$. Thus, $F$ is an edge dominating set of $G + h$.

Thus, $\gamma_e(G + h) \leq \gamma_e(G)$. This is a contradiction. Thus, $h \in F$ for every minimum edge dominating set $F$ of $G + h$.

**Proposition 4.7**: Let $G$ be a hypergraph & $h$ be as above. Let $F$ be any minimum edge dominating set of $G + h$. If $\gamma_e(G + h) < \gamma_e(G)$ then there are at least two vertices $x$ & $y$ in $h$ such that all the edges containing $x$ or $y$ except $h$ are in the complement of $F$.

**Proof**: From above theorem $h \in F$. Let $F_1 = F \setminus \{h\}$ then $F_1$ cannot be an edge dominating set of $G$ because $|F_1| < |F| < \gamma_e(G)$. Therefore, there is an edge $f$ of $G$ such that $f$ does not intersect any member of $F_1$ but $F$ is an edge dominating set of $G + h$ & $f$ is an edge of $G + h$. Therefore, $f$ intersects some unique member of $F$. This member of $F$ must be $h$.

Now, suppose $f \cap h = \{x\}$. Suppose there is some edge $e_i$ containing $x \not\in \{x\}$ & $e_i \not\in F$. Then it means that $f$ intersects some member of $F_1$. This is a contradiction. Thus, all the edges containing $x$ except $h$ are in the complement of $F$.

Select one edge say $e_i$ containing $x$ with $e_i \not\in F$. Let $F_2 = F_1 \cup \{e_i\}$. Now, $|F_2| = |F| < \gamma_e(G)$. Therefore $F_2$ cannot be an edge dominating set of $G$. Therefore, $g$ must intersect $h$. Let $g \cap h = \{y\}$ then $y \in h$. Again, if there is an edge $e_j$ containing $y \not\in \{y\}$ & $e_j \not\in F$ then $e_j \in F_2$ & $g \cap e_i \neq \phi$. This is again a contradiction. Therefore, there is no edge $e_j$ containing $y \not\in \{y\}$ & $e_j \not\in F$.

Thus, all the edges containing $y$ except $h$ are in the complement of $F$.

**Theorem 4.8**: Let $G$ be a hypergraph & $h$ be as above. Suppose there is a minimum edge dominating set $F$ of $G$. Then there are two distinct vertices $x$ & $y$ in $h$
& there are two distinct edges e₁, e₂ containing x & y respectively in every edge adjacent to e₁ or e₂ in F or is adjacent to some member of F then γₑ(G + h) ≤ γₑ(G).

**Proof:** Let F₁ = (F – {e₁, e₂}) ∪ {h} then |F₁| < |F|. Let f be any edge of G + h ∋ f ∉ F₁. If f is adjacent to e₁ in F (in G) then by our assumption f is adjacent to some member of F different from e₁. This means that f is adjacent to some member of F₁. Similarly, if f is adjacent to e₂ then also it adjacent to some member of F₁.

If f is adjacent to some edge g of F with g ≠ e₁, g ≠ e₂ then g ∉ F₁ & in this case also f is adjacent to some member of F₁. Thus, F₁ is an edge dominating set of G + h.

\[ γₑ(G + h) ≤ |F₁| < |F| = γₑ(G). \]

\[ γₑ(G + h) < γₑ(G). \]

**Theorem 4.9:** Let G be a hypergraph & h be as above. Suppose γₑ(G + h) < γₑ(G) then γₑ(G) - |h| + 1 ≤ γₑ(G + h) < γₑ(G).

**Proof:** Let F₀ be a minimum edge dominating set of G + h. Let h = \{x₁, x₂, ……, xₖ\}. Consider any edge eᵢ, of G containing xᵢ, (1) eᵢ ≠ eᵢ & (2) eᵢ ≠ h (i = 1, 2, ……, k)

Let F = F₀ ∪ \{e₁, e₂, ……, eₖ\} – {h} then F is an edge dominating set of G + h.

\[ |F| = |F₀| + |h| - 1 ≥ γₑ(G). \]

\[ |F₀| ≥ γₑ(G) - |h| + 1 \]

Thus, γₑ(G) - |h| + 1 ≤ γₑ(G + h) < γₑ(G).

**Theorem 4.10:** Let G be a hypergraph & h be as above then γₑ(G + h) = γₑ(G) - |h| + 1 iff there is a minimum edge dominating set of F of G & distinct edges e₁, e₂, ……, eₖ (Where k = |h|) in F such that all the edges incident at xᵢ & every edge f which is adjacent to eᵢ is either in F or is adjacent to some member of F.

**Proof:** Suppose the condition holds. Let F be a minimum edge dominating set of G which the above condition is satisfied for F.

Let F₀ = F – \{e₁, e₂, ……, eₖ\} ∪ {h} then |F₀| = γₑ(G) - |h| + 1.

Now, we prove that F₀ is a minimum edge dominating set of G + h. Let g be any edge of G + h \∉ F₀ then g ∉ F also.

Also g is an edge of G. Therefore, g is adjacent to some member of F. If g is adjacent to eᵢ for some i then by our assumption there is some edge f in F ∋ g is adjacent to f. Note that f is edge of G + h also. Therefore, g is adjacent to some member of F₀. We may note that if g ∩ eᵢ = h ∩ eᵢ then g is adjacent to h, which is a member of F₀.

In other cases it is obvious that g is adjacent to some member of F₀.

Thus, F₀ is an edge dominating set of G + h.

By the above theorem F₀ is a minimum edge dominating set of G + h.

\[ |F₀| = γₑ(G) - |h| + 1 = γₑ(G + h) \]

Conversely suppose γₑ(G + h) = γₑ(G) - |h| + 1. Let F₀ be a minimum edge dominating set of G + h then h ∈ F₀. Let h = \{x₁, x₂, ……, xₖ\} then by proposition 4.7 there are two distinct vertices say x₁ & x₂ such that all the edges incident at x₁ except h & all the edges incident at x₂ except h are in the complement of F₀.

Let e₁ & e₃ be two distinct edges containing x₁ & x₂ respectively.

Let F₁ = F₀ ∪ \{e₁, e₃\} – {h}. Suppose |h| = 2. Let f be any edge of G which is not in F₁. If f is adjacent to h at x₁ then f is adjacent to e₁ & similarly f is adjacent to h at x₂ then f is adjacent to e₃. For other edges which are not in F₁ it can be verified that they are adjacent to some member of F₁.

Let g be any edge of G \∉ G & suppose g ∉ F₀. Since e₁ ∉ F₀ there is an edge incident at e₁ which is in F₀. Therefore, the condition is satisfied. Similarly, if g is adjacent to e₃ then g is adjacent to some member of F₀. Thus, the condition is satisfied.

Suppose |h| ≥ k. Then again there are two vertices x₁ & x₂ in F₀ all the edges containing x₁ or x₂ are in the complement of F₀. Select two edges e₁ & e₃ containing x₁ & x₂ respectively & eᵢ ≠ h for i = 1, 2. Let F₁ = F₀ ∪ \{x₁, x₂\} – {h} then |F₁| = γₑ(G) (because γₑ(G) = γₑ(G + h) + |h| - 1, |h| ≥ 3). Therefore, F₁ cannot be an edge dominating set of G. Therefore, there is an edge e’ of G \∉ G’ which is not adjacent with any member of F₁ but e’ must be adjacent with some member of F₀. Therefore, e’ is adjacent with only one member of F₀ namely h. Then e’ ∩ h = \{x₃\} (Say).

Now, all the edges containing x₁ except h are in the complement of F₁. Select an edge e₄ containing x₃ \∉ F₀ & e₁ ≠ h. Now, let F₂ = F₁ ∪ \{e₄\}. If |h| = 3 then |F₂| = γₑ(G). Here also we can prove that F₂ is an edge dominating set of G & required conditions are satisfied.

In general if k > 3 then by selecting edges e₁, e₂, ……, eₖ containing x₁, x₂, ……, xₖ respectively which are not in F₀ & by considering the set F = F₀ ∪ \{e₁, e₂, ……, eₖ\} - {h} we can prove that F is an edge dominating set of G also |F| = |F₀| + |h| - 1 = γₑ(G + h) + |h| - 1 = γₑ(G).

\[ F \text{ is a minimum edge dominating set of G. Also the conditions are satisfied by } F. \]

ISSN: 2231-5373  http://www.ijettjournal.org  Page 113
5. Conclusions

In this paper we have established the conditions under which the edge domination number increases or decreases when an edge is removed or added to the hypergraph. Further one can consider the following problems:

(1) What is the minimum number of edges which must be removed or added to increases or decreases the edge domination number of a hypergraph.

(2) One can also study the minimal edge dominating set with maximum cardinality in hypergraph.

Acknowledgment

The authors are thankful to the referees for their valuable suggestions to improve the quality of this paper.

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