Commutative Monoids in Intuitionistic Fuzzy Sets

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Abstract — *In this paper, various operations in Intuitionistic Fuzzy Sets are discussed. Some theorems are proved for commutative Monoids using these operations with respect to different Intuitionistic Fuzzy Sets*

Keywords — *Fuzzy Sets, Intuitionistic Fuzzy Sets, Commutative Monoids*

I. Introduction

L.A.Zadeh [5] introduced the notion of a Fuzzy sub set μ of a Set X as a function from X to [0,1]. After the introduction of Fuzzy sets by L.A.Zadeh [5], the Fuzzy concept has been introduced in almost all branches of Mathematics. Then the concept of Intuitionistic Fuzzy Set (IFS) was introduced by K.T. Atanassov [1] as a generalization of the notation of a Fuzzy set. Here, we discuss the algebraic nature of Intuitionistic Fuzzy operations and prove some results on the commutative Monoid.

II. Preliminaries

For any two IFSs A and B, the following relations and operations can be defined [2, 3, 4] as follows.

A. *Definition 1.1*

Let S be any non empty set, A mapping μ from S to [0,1] is called a Fuzzy sub set of S.

B. *Definition 1.2*

An Intuitionistic Fuzzy Set A in a non empty set X is an object having the form A= {<x, μA (x), γA (x)>/xϵX} where the functions μA : X→[0.1] and γA : X→[0.1] denote the degrees of membership and non membership of the element xϵX to A respectively and satisfy 0⩽ μA (x)+ γA (x) ⩽ 1 for all xϵX. The family of all intuitionistic fuzzy sets in X denoted by IFS (X).

C. *Definition 1.3*

For every two IFSs A and B the following operations and relations can be defined as

A∩B iff (for all xϵE)(μA (x) ⩽ μB (x) and γA (x)⩾ γ B (x)

A=B iff (for all xϵE)(μA (x) = μB (x))and(γA (x)=γB (x))

A∩B = {[x,min(μA (x), μB (x)),max(γA (x),γB (x)]/xϵE}

A∪B = {[x,max((μA (x), μB (x)),min(γA (x),γB (x))]/xϵE}

A+B= {[x, (μA (x)+ μB (x)-μA (x) μB (x), γA (x).γB (x)]/ xϵE}

A.B= {[x, (μA (x) μB (x), γA (x)+γB (x)- γA (x).γB (x)]/xϵE}

A@B= { [x, μA (x)+ μB (x)/2, γA (x)+γB (x)/2]/xϵE}.

D. *Definition 1.4*

Let us define the following special IFSs

O*= {(x,0,1)/xϵE}

E*= {(x,1,0)/ xϵE}

U*= {(x,0,0)/ xϵE}

Then P (E*) = {A/A= {(x, μA (x), γA (x))/ xϵE}

P (U*) = {B/B= {(x,0 , γA (x))/ xϵE}

P (O*)={O*}.

E. *Definition 1.5*

Let M be a fixed set, let e*ϵM be a unitary element of M and let * be an operation. < M,*, e*> is said to be a commutative monoid, if

(i) a*b ϵ M for all a,b ϵM.

(ii) (a*b)*c = a*(b*c) for all a,b ϵM

(iii) a*e^*=e^** a for all a ϵM

(iv) a*b=b*a for all a,b ϵM.

III. PROOF OF THEOREMS

A. *Theorem: 2.1*

(P (E), ∩,E*) is a commutative monoid.*

Let A,B ϵ P(E*)

1) *Axiom 1: Closure Property*

Consider A ∩ B = {<x,min { μA (x),μB (x)},max{ γA (x),γB (x) } >/xϵE }

= {<x,μA (x),γA (x)>/xϵE} or

{<x,μA (x),γB (x)>/xϵE} or

{<x,μB (x),γA (x)>/xϵE} or

{<x,μB (x),γB (x)>/xϵE} ϵ P(E*) for all A,B ϵ P(E*)

Therefore A∩B ϵ P (E*)

=> Axiom1 is satisfied.

In all the cases A∩B ϵ P (E*)

=> Closure is satisfied

For example $A,B \in P(E^*)$ where $A=\{<x,\mu A(x)=0.6, \gamma B(x)=0.3>\}$ and $B=\{<x,\mu A(x)=0.5, \gamma B(x)=0.2>\}$

Then $A \cap B = \{<x,\min\{0.6,0.5\},\max\{0.3,0.2\}>\} = \{<x,0.5,0.3>\} \in P(E^{\wedge *})$

2) Axiom 2: Associative Property

Consider $(A \cap B) \cap C$

$= \{<x,\min\{\mu A(x),\mu B(x)\},\max\{\gamma A(x),\gamma B(x)\}> / x \in E\} \cap \{<x,\mu C(x),\gamma C(x)>/x \in E\}$

$= \{<x,\min\{\mu A(x),\mu B(x), \mu C(x)\},\max\{\gamma A(x),\gamma B(x),\gamma C(x)\}> / x \in E\}$

$= \{<x,\mu A(x),\gamma A(x)>/x \in E\} \cap \{<x,\min\{\mu B(x),\mu C(x)\},\max\{\gamma B(x),\gamma C(x)\}>/x \in E\}$

$= A \cap (B \cap C)$

=> $(A \cap B) \cap C = A \cap (B \cap C)$ for all $A,B,C \in P(E^{\wedge *})$

=> Axiom2 is satisfied.

=> Associative property is satisfied.

3) Axiom 3: Identity Property

$E^{\wedge *}$ is the identity element with respect to '∩'.

Consider $A \cap E^{\wedge *} = \{<x,\mu A(x),\gamma A(x)>/x \in E\} \cap \{<x,1,0>/x \in E\}$

$= \{<x,\max\{\mu A(x),1\},\min\{\gamma A(x),0\}>/x \in E\}$

$= \{<x,1.0>/x \in E\}$

$= E^{\wedge *}$ (by definition of $E^{\wedge *}$)

$A \cap E^{\wedge *} = E^{\wedge *}$, for all $A \in P(E^{\wedge *})$

=> $E^{\wedge *}$ is the identity element of $P(E^{\wedge *})$ with respect to the operation '∩'.

=> Axiom3 is satisfied.

=> Identity is satisfied.

From Axiom1, Axiom2 and Axiom3 => $<P(E^{\wedge *}), \cap, E^{\wedge *}>$ is monoid.

4) Axiom 4: Commutative Property

Consider $A \cap B = \{<x,\max\{\mu A(x),\mu B(x)\},\min\{\gamma A(x),\gamma B(x)\}>/x \in E\}$

$= \{<x,\max\{\mu B(x),\mu A(x)\},\min\{\gamma B(x),\gamma A(x)\}>/x \in E\}$

$= B \cap A$

=> $A \cap B = B \cap A$ for all $A,B \in P(E^{\wedge *})$

Hence Axiom4 is satisfied.

=> Commutative is satisfied.

=> $<P(E^{\wedge *}), \cap, E^{\wedge *}>$ is a commutative monoid.

B. Theorem 2.2

$(P(E^*), \cup, O^*)$ is a commutative monoid.

Let $A,B \in P(E^*)$

1) Axiom 1: Closure Property

Consider $A \cup B = \{<x,\max\{\mu A(x),\mu B(x)\},\min\{\gamma A(x),\gamma B(x)\}>/x \in E\}$

$= \{<x,\mu A(x),\gamma A(x)>/x \in E\}$ or

$\{<x,\mu A(x),\gamma B(x)>/x \in E\}$ or

$\{<x,\mu B(x),\gamma A(x)>/x \in E\}$ or

$\{<x,\mu B(x),\gamma B(x)>/x \in E\}$

In all the cases $A \cap B \in P(E^{\wedge *})$ (by its definition)

Therefore $A \cap B \in P(E^{\wedge *})$, for all $A,B \in P(E^{\wedge *})$

Closure property is satisfied.

=> Axiom1 is satisfied.

2) Axiom2: Associative Property

Let $A,B,C \in P(E^{\wedge *})$

Consider $(A \cup B) \cup C$

$= \{<x,\max\{\mu A(x),\mu B(x)\},\min\{\gamma A(x),\gamma B(x)\}>/x \in E\} \cup \{<x,\mu C(x)\ \gamma C(x)>/x \in E\}$

$= \{<x,\max\{\mu A(x),\mu B(x), \mu C(x)\},\min\{\gamma A(x),\gamma B(x),\gamma C(x)\}>/x \in E\}$

$= \{<x,\mu A(x),\gamma A(x)>\} \cup \{<x,\max\{\mu B(x),\mu C(x)\},\min\{\gamma B(x),\gamma C(x)\}>/x \in E\}$

$= A \cup (B \cup C)$

=> $(A \cup B) \cup C = A \cup (B \cup C)$

=> Associative property is satisfied.

=> Axiom 2 is satisfied.

3) Axiom 3: Identity Property

$0^{\wedge *}$ is the identity element of $P(E^{\wedge *})$ with respect to '∪'.

Consider $A \cup 0^{\wedge *} = \{<x,\max\{\mu A(x),0\},\min\{\gamma A(x),1\}>/x \in E\}$

$= \{<x,\mu A(x),\gamma A(x)>/x \in E\}$.

$= A$

=> Identity property is satisfied.

=> Axiom 3 is satisfied.

From Axiom1, Axiom2 and Axiom3 => $<P(E^{\wedge *}), \cup, 0^{\wedge *}>$ is a monoid.

4) Axiom 4: Commutative Property

Consider $A \cup B = \{<x,\max\{\mu A(x),\mu B(x)\},\min\{\gamma A(x),\gamma B(x)\}>/x \in E\}$

$= \{<x,\max\{\mu B(x),\mu A(x)\},\min\{\gamma B(x),\gamma A(x)\}>/x \in E\}$

$= B \cup A$

Therefore $A \cup B = B \cup A$

=> Commutative property is satisfied.

=>Axiom 4 is satisfied.

Hence $<P(E^{\wedge *}), \cup, 0^{\wedge *}>$ is a commutative monoid.

C. Theorem 2.3

$(P(U^*), \cap, U^*)$ is a commutative monoid.

Let $A,B \in P(U^{\wedge *})$

1) Axiom 1: Closure Property

$A+B = \{<x,\mu A(x)+\mu B(x)- \mu A(x).\mu B(x),\gamma A(x).\gamma B(x)>/x \in E\}$

$= \{<x,0,\gamma(A+B)(x)>/x \in E\}$

$A+B \in P(U^{\wedge *})$

Therefore Axiom1 is satisfied.

2) Axiom 2: Associative Property

$(A+B)+C = A+(B+C)$

LHS $= (A+B)+C$

$= \{<x,\mu A(x)+\mu B(x)- \mu A(x).\mu B(x),\gamma A(x).\gamma B(x)>/x \in E\}+\{<x,\gamma C(x),\gamma C(x)>/x \in E\}$

$= \{<x,0,\gamma(A+B+C)(x)>/x \in E\}$

$= \{<x,0,\gamma A(x)>/x \in E\}+\{<x,0,\gamma(B+C)(x)>/x \in E\}$

$= A+(B+C) =$ RHS

Associative property is true.

Axiom 2 is satisfied.

3) Axiom 3: Identity Property

$A+0^{\wedge *} = \{<x,\mu A(x)+\mu(0^{\wedge *})(x)- \mu A(x).\mu(0^{\wedge *})(x),\gamma A(x).\gamma(0^{\wedge *})(x)>/x \in E\}$

$= \{<x,0+0-0,\gamma A(x).1>/x \in E\}$

$= \{<x,0,\gamma A\ (x)>/x\epsilon E\}$

$= A$

Axiom3 is satisfied.

4) ***Axiom 4: Commutative Property***

$A+B = B+A$

$A+B = \{<x,\mu A\ (x)+\mu B\ (x)-\ \mu A\ (x).\mu B\ (x),\gamma A\ (x).\gamma B\ (x)>/x\epsilon E\}$

$= \{<x,\mu B\ (x)+\mu A\ (x)-\ \mu B\ (x).\mu A\ (x),\gamma B\ (x).\gamma A\ (x)>/x\epsilon E\}$

$= B+A$

Axiom 4 is satisfied.

D. Theorem 2.4

(P (U*),U,O^*) is commutative monoid.

1) ***Axiom 1: Closure Property***

Let $A,B \epsilon P(U^*) =\{ B/B = \{< x,0, \gamma B\ (x)\ >/\ x\epsilon E\ \}\}$

$A \cup B = \{<x,\max\{\mu A\ (x),\mu B\ (x)\},\min\{\gamma A\ (x),\gamma B\ (x)\}>/x\epsilon E\}$

Here $\mu A\ (x)=\mu B\ (x) = 0$

$=\{<x,\max\{0,0\},\max\{\gamma A\ (x),\gamma B\ (x)\}>/x\epsilon E\}$

$= \{< x,0, \gamma A\ (x)\ >/\ x\epsilon E\ \}$, if $\gamma A\ (x)<\gamma B\ (x)$

$= \{< x,0, \gamma B\ (x)\ >/\ x\epsilon E\ \}$, if $\gamma B\ (x)<\gamma A\ (x)$

In both cases $A \cup B \ \epsilon\ P(U^*)$

∴ Axiom1 is satisfied.

2) ***Axiom 2: Associative Property***

$(A \cup B) \cup C = \{<x,\max\{0,0\},\min\{\gamma A\ (x),\gamma B\ (x)\}>\} \cup \{<x,0,\gamma C\ (x)>/x\epsilon E\}$

$= \{< x,0, 〖\min\{\gamma〗\ A\ (x),\gamma B\ (x),\gamma C\ (x)\}\ >/\ x\epsilon E\ \}$

$= \{< x,0,\gamma A\ (x)\ >/\ x\epsilon E\ \} \cup \{<x,\min\{\gamma B\ (x),\gamma C\ (x)\}/x\epsilon E\}$

$= A \cup (B \cup C)$

=>Associative property is satisfied.

∴ Axiom2 is satisfied.

3) ***Axiom 3: Identity Property***

$A \cup O^{\wedge *} = \{<x,\max\{0,0\},\min\{\gamma A\ (x),1\}>/x\epsilon E\}$

$= \{<x,0,\gamma A\ (x)>/x\epsilon E\} = A$

$=> A \cup O^{\wedge *} = A$

=>O^* is the identity for (P(U*),U)

∴ Axiom3 is satisfied.

4) ***Axiom 4: Commutative Property***

$A \cup B = \{<x,\max\{0,0\},\min\{\gamma A\ (x),\gamma B\ (x)\}>/x\epsilon E\}$

$= \{<x,0,\min\{\gamma B\ (x),\gamma A\ (x)\}>/x\epsilon E\}$

$= B \cup A$

$=> A \cup B= B \cup A$

∴ Commutative property is satisfied.

∴ Axiom 4 is satisfied.

E. Theorem 2.5

(P (U*),.,U*) is a commutative monoid.

1) ***Axiom 1: Closure Property***

$A,B\ \epsilon\ P(U^*)$

$A \cdot B = \{<x,\mu A\ (x).\mu B\ (x),\gamma A\ (x)+\gamma B\ (x)-\gamma A\ (x)\ \gamma B\ (x)>/x\epsilon E\}$

$= \{<x,0,\gamma A\ (x)+\gamma B\ (x)-\ \gamma A\ (x)\ \ \gamma B\ (x)>/x\epsilon E\ \}$

$A \cdot B\ \epsilon\ P(U^*)$ (by its definition)

∴ Axiom1 is satisfied.

Therefore closure is satisfied.

2) ***Axiom 2: Associative Property***

$A,B,C\ \epsilon\ P(U^*)$

$(A \cdot B) \cdot C = \{<x,0,\gamma_A\ (x)+\gamma_B\ (x)-\ \gamma_A\ (x)\ \ \gamma_B\ (x)>/x\epsilon E\ \} \cdot \{<x,0,\gamma_C\ (x)>/x\epsilon E\ \}$

$= \{<x,0,\gamma_A\ (x)+\gamma_B\ (x)-\ \gamma_A\ (x)\ \ \gamma_B\ (x)+\gamma_C\ (x)-\ \gamma_A\ (x)\ \ \gamma_C\ (x)-\ \gamma_B\ (x)\ \ \gamma_C\ (x)+\ \gamma_A\ (x)\ \gamma_B\ (x)\ \gamma_C\ (x)>/x\epsilon E\ \}$ (1)

$A \cdot (B \cdot C) = \{<x,0,\gamma A\ (x)>/x\epsilon E\ \} \cdot \{<x,0,\gamma_B\ (x)+\gamma_C\ (x)-\ \gamma_B\ (x) \cdot \gamma_C\ (x)>/x\epsilon E\ \}$

$= \{<x,0, \gamma_A\ (x)+ \gamma_B\ (x)\ + \gamma_C\ (x)- \gamma_A\ (x)+ \gamma_B\ (x)\ + \gamma_C\ (x)- \gamma_A\ (x)\ \ \gamma_B\ (x)- \gamma_A\ (x)\ \gamma_C\ (x)+ \gamma_A\ (x)\ \ \gamma_B\ (x)\ \gamma_C\ (x)\}$ ..(2)

(1) = (2)

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

∴ Axiom2 is satisfied.

∴ Associative property is satisfied.

3) ***Axiom 3: Identity Property***

$(A \cdot U^*) = \{<x,0,\gamma_A\ (x)>/x\epsilon E\ \} \cdot \{<x,0,0>/x\epsilon E\ \}$

$= \{<x,0,\gamma_A\ (x)+0-\gamma_A\ (x) \cdot 0>/x\epsilon E\ \}$

$= \{<x,0,\gamma_A\ (x)>/x\epsilon E\ \} = A$

$=> A \cdot U^* = A$, for all $A\ \epsilon\ P(U^*)$

∴ Axiom3 is satisfied.

=> U* is the identity for (P (U*),.,U*)

4) ***Axiom 4: Commutative Property***

$A \cdot B = B\ \cdot A$

Let $A,B\ \epsilon\ P(U^*)$

Consider $A \cdot B = \{<x,0,\gamma_A\ (x)+\gamma_B\ (x)-\ \gamma_A\ (x)\ \ \cdot\gamma_B\ (x)>/x\epsilon E\ \}$

$= \{<x,0,\gamma_B\ (x)+\gamma_A\ (x)-\ \gamma_B\ (x)\ \ \cdot \gamma_A\ (x)>/x\epsilon E\ \}$

$= B\ \cdot A$

$=> A \cdot B = B\ \cdot A$, for all $A,B\ \epsilon\ P(U^*)$

∴ Axiom4 is satisfied.

=>Commutative property is satisfied.

=>(P (U*),.,U*) is a commutative monoid.

F. Theorem 2.6

(P (E*),.,E*) is a commutative monoid.

1) ***Axiom 1: Closure Property***

Let $A,B\ \epsilon\ P(E^*)$

$A \cdot B = \{<x,\mu_A\ (x)\ \mu_B\ (x),\gamma_A\ (x)+\gamma_B\ (x)-\ \gamma_A\ (x)\ \ \gamma_B\ (x)>/x\epsilon E\ \}$

$=> A \cdot B\ \epsilon\ P(E^*)$

∴ Axiom1 is satisfied.

=> Closure property is satisfied

2) ***Axiom 2: Associative Property***

$(A \cdot B) \cdot C$

$=\{<x,\mu_A\ (x)\ \mu_B\ (x),\gamma_A\ (x)\ \ + \gamma_B\ (x)-\ \gamma_A\ (x)\ \ \ \gamma_B\ (x)>/x\epsilon E\ \} \cdot \{<x, \mu_C\ (x)\gamma_C\ (x)>/x\epsilon E\ \}$

$= \{<x,\mu_A\ (x)\ \mu_B\ (x)\ \mu_C\ (x),\gamma_A\ (x)\ \ + \gamma_B\ (x)+\mu_C\ (x)-\gamma_A\ (x)\ \ \gamma_B\ (x)-\gamma_A\ (x)\ \ \gamma_C\ (x)-\gamma_B\ (x)\ \ \gamma_C\ (x)+\gamma_A\ (x)\ \ \gamma_B\ (x)\ \gamma_C\ (x)>/x\epsilon E\ \}$ (1)

$A \cdot (B \cdot C)$

$=\{<x, \mu_A\ (x)\gamma_A\ (x)>/x\epsilon E\ \} \cdot \{<x,\mu_B\ (x)\ \mu_C\ (x),\gamma_B\ (x) + \gamma_C\ (x)-\ \gamma_B\ (x)\ \ \ \gamma_C\ (x)>/x\epsilon E\ \}$

= {<x,μ_A (x) μ_B (x) μ_C (x),γ_A (x) + γ_B (x)+μ_C (x)- γ_B (x) γ_C (x)-γ_A (x) γ_B (x)- γ_A (x) γ_C (x)+γ_A (x) γ_B (x) γ_C (x)> /xєE } (2)

=> (1) = (2)

∴ Axiom2 is satisfied.

=>Associative property is satisfied.

3) Axiom 3: Identity Property

A · B = {<x,μA (x) μB (x),γA (x) + γB (x)- γA (x) γB (x)>/xєE }

= {<x,μB (x) μA (x),γB (x) + γA (x)- γB (x) γA (x)>/xєE }

= B · A

=> A · B = B · A

∴ Axiom3 is satisfied.

=> P(E*) is commutative

=> (P (E*),.,E*) is a commutative monoid.

4) Axiom 4: Commutative Property

G. Theorem 2.7

(P (E*),+, O*) is a commutative monoid

Let A,B є P(E*)

1) Axiom 1: Closure Property

A+B = {<x,μA (x) +μB (x)-μA (x) μB (x) ,γA (x) γB (x)>/xєE }

A+B є P(E*)

∴ Axiom1 is satisfied.

=> Closure property is satisfied.

2) Axiom 2: Associative Property

(A+B)+C

= {<x,μ_A (x) +μ_B (x)-μ_A (x) μ_B (x) ,γ_A (x) γ_B (x)>/xєE }+{<x,μ(x), γ_C (x)>xєE }

= {<x,μ_A (x) +μ_B (x)+μ_C (x)-μ_A (x) μ_B (x)-μ_A (x) μ_C (x)- μ_B (x) μ_C (x) + μ_A (x) μ_B (x) μ_C (x) ,γ_A (x) γ_B (x) γ_C (x)>/xєE } (1)

A+(B+C)

={<x,μ_A (x), γ_A (x)>/xєE }+{μ(x) +μ_C (x)-μ_B (x) μ_C (x) ,γ_B (x) γ_C (x)>/xєE }

= {<x,μ_A (x) +μ_B (x)+μ_C (x)-μ_B (x) μ_C (x)-μ_A (x) μ_B (x)- μ_A (x) μ_C (x) + μ_A (x) μ_B (x) μ_C (x) ,γ_A (x) γ_B (x) γ_C (x) >/xєE } …(2)

=> (1) = (2)

=> (A+B)+C = A+(B+C)

∴ Axiom2 is satisfied.

=>Associative property is satisfied.

3) Axiom 3: Identity Property

A+O* = {<x,μ_A (x), γ_A (x)>/xєE }+ {<x,0,1>/xєE }

= {<x,μ_A (x)+0- μ_A (x).0, γ_A (x).1>/xєE }

= {<x,μ_A (x), γ_A (x)>/xєE }

=A

=> A+O* =A, for all A є P(E*)

∴ Axiom3 is satisfied.

=> Existence of Identity is proved.

4) Axiom 4: Commutative Property

A+B = {<x,μ_A (x) +μ_B (x)-μ_A (x) μ_B (x) ,γ_A (x) γ_B (x)>/xєE }

= {<x,μ_B (x) +μ_A (x)-μ_B (x) μ_A (x) ,γ_B (x) γ_A (x)>/xєE }

= B+A, for all A,B є P(E*)

∴ Axiom4 is satisfied.

=> Commutative property is satisfied.

=>(P (E*),+, O*) is a commutative monoid

H. Theorem 2.8

(P (U*),∩, U*) is a commutative monoid

Let A,B є P(E*)

1) Axiom 1: Closure Property

A ∩ B = {<x,min{μ_A (x) ,μ_B (x)},max{γ_A (x) ,γ_B (x)}>/xєE}

= {<x,min{0,0},max{γ_A (x) ,γ_B (x)}>/xєE}

={<x,min{0,0},max{γ_A (x) ,γ_B (x)}>/xєE} є P(U*)

∴ Axiom1 is satisfied.

=>Closure property is satisfied

2) Axiom 2: Associative Property

(A∩B)∩C

= {<x,min{0,0},max{γ_A (x) ,γ_B (x)}>/xєE}∩ {<x,0, γ_C (x)>/xєE}

= {<x,0,max{γ_A (x) ,γ_B (x)}>/xєE}

= {<x,0, γ_A (x)>/xєE}∩{<x,0,max{γ_A (x) ,γ_B (x)}>/xєE}

= A ∩(B∩C)

∴ Axiom2 is satisfied.

=>Commutative property is satisfied

3) Axiom 3: Identity Property

A ∩U* = {<x,0,γ_A (x)>/xєE}∩{<x,0,0>/xєE}

= {<x,min{0,0},max{γ_A (x) ,0}>/xєE}

= {<x,0,γ_A (x)>/xєE}

= A

=> A ∩U* = A

=>U* is identity

∴ Axiom3 is satisfied.

=>Existence of identity is proved

4) Axiom 4: Commutative Property

A ∩ B ={<x,min{0,0},max{γ_A (x) ,γ_A (x)}>/xєE}

= {<x,0,max{γ_A (x) ,γ_A (x)}>/xєE}

= B ∩A

=> A ∩B= B ∩A

∴ Axiom4 is satisfied.

=> Commutative property is satisfied

=>(P (U*),∩, U*) is a commutative monoid

IV. Conclusions

We have defined different operations of Intuitionistic Fussy Sets. Using these, we have proved various possible operations with a particular set as a Commutative Monoid. We hope that these results can also be extended to further algebraic systems.

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