A Procedure for Division by Some Small Primes

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Abstract

This note gives a procedure for division of a number by some small primes up to 151.

1. Introduction

While dealing with division in early school days, we are told that a positive integer $N$ is divisible by 3 (respectively 9) if and only if the sum of the digits forming the number is divisible by 3 (respectively 9). We are not normally told why this happens although there is a very valid reason behind it. In the same way we also learn that $N$ is divisible by 11 if and only if the sum and difference of the digits forming $N$ taken alternatively in order is divisible by 11.

Divisibility of $N$ by 2 or 5 is simple. For 2 the digit in the units place of $N$ is divisible by 2 while for 5 this digit has to be either 0 or 5. In [2] there is given a procedure for the division of $N$ by 19. Proceeding on the lines of the procedure given in [2], we give procedures for division of $N$ by several small primes up to 151.

2. The Procedure

Let $N$ be a positive integer, $x$ the number of tens (and not the digit in the tens place) in it and $y$ be the digit in the units place in $N$. Then $N = 10x + y$. Let $p > 1$ and $k$ be positive integers such that $p$ divides $10k + 1$.

Theorem 1. Let $N_1 = x - ky$. Then $p$ divides $N$ if and only if $p$ divides $N_1$.

Proof. Observe that $N - 10N_1 = 10x + y - 10(x - ky) = (10k + 1)y$. Therefore $p$ divides $N - 10N_1$. Since $\gcd(10k + 1, 10) = 1$ and $p$ divides $10k + 1$, $\gcd(10, p) = 1$. If $p$ divides $N$, then $p$ divides $N - (N - 10N_1) = 10N_1$ and, therefore, $p$ divides $N_1$. On the other hand, if $p$ divides $N_1$, then $p$ divides $N - 10N_1 + 10N_1 = N$.

Corollary 2.

(a) 31 divides $N$ if and only if 31 divides $x - 3y$;

(b) 41 divides $N$ if and only if 41 divides $x - 4y$;

(c) 17 divides $N$ if and only if 17 divides $x - 5y$;
(d) 61 divides $N$ if and only if 61 divides $x - 6y$;
(e) 71 divides $N$ if and only if 71 divides $x - 7y$;
(f) 101 divides $N$ if and only if 101 divides $x - 10y$;
(g) 37 divides $N$ if and only if 37 divides $x - 11y$;
(h) 131 divides $N$ if and only if 131 divides $x - 13y$;
(i) 47 divides $N$ if and only if 47 divides $x - 14y$;
(j) 151 divides $N$ if and only if 151 divides $x - 15y$;

Now suppose that $p > 1, k$ are positive integers such that $p$ divides $10k - 1$. With $N, x, y$ as already stated, let $N_2 = x + ky$. Proceeding precisely on the lines of proof of Theorem 1, we can also prove the following:

**Theorem 3.** The number $p$ divides $N$ if and only if $p$ divides $N_2$.

**Corollary 4.**

(a) 29 divides $N$ if and only if 29 divides $x + 3y$;
(b) 13 divides $N$ if and only if 13 divides $x + 4y$;
(c) 7 divides $N$ if and only if 7 divides $x + 5y$;
(d) 59 divides $N$ if and only if 59 divides $x + 6y$;
(e) 79 divides $N$ if and only if 79 divides $x + 8y$;
(f) 89 divides $N$ if and only if 89 divides $x + 9y$;
(g) 109 divides $N$ if and only if 109 divides $x + 11y$;
(h) 17 divides $N$ if and only if 17 divides $x + 12y$
(i) 43 divides $N$ if and only if 43 divides $x + 13y$;
(j) 139 divides $N$ if and only if 139 divides $x + 14y$;
(k) 149 divides $N$ if and only if 1493 divides $x + 15y$.

3. Some Examples
As illustrations on the use of Theorems 1 and 3 or their corollaries and using recursive argument we decide the divisibility of some numbers by 17, 37, 29 and 47.

**Example** Decide if the number 8179341256 is divisible by (a) 17, (b) 29, (c) 37, (d) 47.

**Solution.** (a) Adding $12 \times (units \; digit)$ recursively, we get

\[
N_2 = 817934125 + 6 \times 12 = 817934197 \rightarrow 81793419 + 7 \times 12 = 81793503 \\
\rightarrow 8179350 + 3 \times 12 = 8179386 \rightarrow 817938 + 6 \times 12 = 818010 \rightarrow 81801 \\
\rightarrow 8180 + 1 \times 12 = 8192 \rightarrow 819 + 2 \times 12 = 843 \rightarrow 84 + 3 \times 12 = 120 \rightarrow 12
\]

Since the last value of $N_2$ is 12 which is not divisible by 17, the given number is not divisible by 17.

(b) Adding $3 \times (units \; digit)$ recursively, we get

\[
N_2 = 817934125 + 6 \times 3 = 817934143 \rightarrow 81793414 + 3 \times 3 = 81793423 \rightarrow 8179342 + \\
3 \times 3 = 8179351 \rightarrow 817935 + 1 \times 3 = 817938 \rightarrow 81793 + 8 \times 3 = 81817 \rightarrow 8181 + 7 \times \\
3 = 8202 \rightarrow 820 + 2 \times 3 = 826 \rightarrow 82 + 6 \times 3 = 100 \rightarrow 10.
\]

Since the last value of $N_2$ is 10 which is not divisible by 29, the given number is not divisible by 29.

(c) Subtracting $11 \times (units \; digit)$ recursively, we get

\[
N_1 = 817934125 - 6 \times 11 = 817934059 \rightarrow 81793405 - 9 \times 11 = 81793406 \\
\rightarrow 8179340 - 6 \times 11 = 8179274 \rightarrow 817927 - 4 \times 11 = 817883 \\
\rightarrow 81788 - 3 \times 11 = 81755 \rightarrow 8175 - 5 \times 11 = 8120 \rightarrow 812 - 2 \times 11 \\
= 59.
\]

The last value of $N_1$ is 59 which is not divisible by 37. Therefore the given number is not divisible by 37.

(d) Subtracting $14 \times (units \; digit)$ recursively, we get

\[
N_1 = 817934125 - 6 \times 14 = 817934041 \rightarrow 8179340 - 1 \times 14 = 8179326 \\
\rightarrow 817932 - 6 \times 14 = 817848 \rightarrow 81784 - 8 \times 14 = 81672 \\
\rightarrow 8167 - 2 \times 14 = 8139 \rightarrow 813 - 9 \times 14 = 687 \rightarrow 68 - 7 \times 14 = -30.
\]

The last value of $N_1$ is $-30$ which is not divisible by 47. Therefore the given number is not divisible by 47.
The procedure of division as discussed above involves multiplication of a single digit number by a number of 1 or 2 digits and then addition of the result in a number. The processes of addition and (such a simple) multiplication are much simpler than the process of usual division. As such the procedures for checking divisibility given here are preferable to the usual process of division.

References
