Selberg integral involving a certain extension of the Hurwitz-Lerch Zeta function, a

class of polynomials the multivariable I-function

and multivariable Aleph-functions

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ABSTRACT

In the present paper we evaluate the Selberg integral involving , a certain extension of the Hurwitz-Lerch Zeta function a multivariable Aleph-function, the multivariable I-function defined by Prasad [3] and general class of polynomials of several variables. The importance of the result established in this paper lies in the fact they involve the Aleph-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializing the parameters their in.

Keywords: Multivariable Aleph-function, general class of polynomials, Selberg integral, General sequence of functions, multivariable I-function, multivariable H-function

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1. Introduction and preliminaries.

The function Aleph of several variables generalize the multivariable I-function recently study by C.K. Sharma and Ahmad [4], itself is an a generalisation of G and H-functions of multiple variables defined by Srivastava et al [6]. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We define:

\[ N\left(z_1, \ldots, z_r\right) = N^{0,N;M_1,N_1, \ldots, M_r,N_r}_{P_1,Q_1}; \tau_1; \cdots; P^{(1)}_1, Q^{(1)}_1; \tau^{(1)}_1; R^{(1)}; \cdots; P^{(r)}_r, Q^{(r)}_r; \tau^{(r)}_r; R^{(r)} \]

\[
\left[ \begin{array}{c}
\left( \alpha_j^{(1)}; \alpha_j^{(r)} \right)_{1,N_1} \\cdots \\left( \alpha_j^{(1)}; \alpha_j^{(r)} \right)_{N+1,P_1} \\
\left( \tau_1^{(1)}; \cdots; \tau_1^{(r)} \right)_{1,N_1} \\cdots \\left( \tau_1^{(1)}; \cdots; \tau_1^{(r)} \right)_{M+1,Q_1}
\end{array} \right]
\]

\[
\left[ \begin{array}{c}
\left( \gamma_j^{(1)}; \gamma_j^{(r)} \right)_{1,N_1} \\cdots \\left( \gamma_j^{(1)}; \gamma_j^{(r)} \right)_{N+1,P_1} \\
\left( \delta_j^{(1)}; \delta_j^{(r)} \right)_{1,M_1} \\cdots \\left( \delta_j^{(1)}; \delta_j^{(r)} \right)_{M+1,Q_1}
\end{array} \right]
\]

\[
= \frac{1}{(2\pi \omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \ldots, s_r) \prod_{k=1}^r g_k^{s_k} ds_1 \cdots ds_r
\]

(1.1)

with \( \omega = \sqrt{-1} \)

\[
\psi(s_1, \ldots, s_r) = \frac{\prod_{j=1}^N \Gamma(1 - a_j + \sum_{k=1}^r \alpha_j^{(k)} s_k)}{\sum_{i=1}^R \prod_{j=N+1}^{P_i} \Gamma(a_{ji} - \sum_{k=1}^r \alpha_{ji}^{(k)} s_k) \prod_{j=1}^{Q_i} \Gamma(1 - b_{ji} + \sum_{k=1}^r \beta_{ji}^{(k)} s_k)}
\]

(1.2)
Suppose, as usual, that the parameters

\[ \beta_j, \gamma_j = 1, \ldots, Q, \alpha_j, \gamma_j = 1, \ldots, P; \]

\[ c^{(k)}_{j+i}, j = n_k + 1, \ldots, P; \]

\[ d^{(k)}_{j+i}, j = M_k + 1, \ldots, Q; \]

with \( k = 1, \ldots, r, i = 1, \ldots, R, \gamma^{(k)} = 1, \ldots, R^{(k)} \)

are complex numbers, and the \( \alpha' s, \beta' s, \gamma' s \) and \( \delta' s \) are assumed to be positive real numbers for standardization purposes such that

\[ U^{(k)}_i = \sum_{j=1}^{N} \alpha_j^{(k)} + \tau_i \sum_{j=N+1}^{P} \alpha_j^{(k)} + \sum_{j=1}^{N_n} \gamma_j^{(k)} + \tau_i \sum_{j=N_k+1}^{P} \gamma_j^{(k)} - \tau_i \sum_{j=1}^{Q} \beta_j^{(k)} - \sum_{j=1}^{M_k} \delta_j^{(k)} \]

\[ -\tau_i \sum_{j=M_k+1}^{Q} \delta_j^{(k)} j_i \leq 0 \] (1.4)

The real numbers \( \tau_i \) are positive for \( i = 1 \) to \( R \), \( \tau_i^{(k)} \) are positive for \( i^{(k)} = 1 \) to \( R^{(k)} \).

The contour \( L_k \) is in the \( s_k \)-plane and runs from \( \sigma - i \infty \) to \( \sigma + i \infty \) where \( \sigma \) is a real number with loop, if necessary ensure that the poles of \( \Gamma(\delta_j^{(k)} - \gamma_j^{(k)} s_k) \) with \( j = 1 \) to \( m_k \) are separated from those of \( \Gamma(1 - \alpha_j + \sum_{i=1}^{r} \alpha_j^{(k)} s_k) \) with \( j = 1 \) to \( N \) and \( \Gamma(1 - \gamma_j^{(k)} + \gamma_j^{(k)} s_k) \) with \( j = 1 \) to \( N_k \) to the left of the contour \( L_k \). The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as:

\[ \arg z_k \leq \frac{1}{2} A^{(k)}_i \pi, \text{ where} \]

\[ A^{(k)}_i = \sum_{j=1}^{N} \alpha_j^{(k)} - \tau_i \sum_{j=N+1}^{P} \alpha_j^{(k)} - \tau_i \sum_{j=1}^{Q} \beta_j^{(k)} + \sum_{j=1}^{N_k} \gamma_j^{(k)} - \tau_i \sum_{j=N_k+1}^{Q} \gamma_j^{(k)} \]

\[ + \sum_{j=1}^{M_k} \delta_j^{(k)} - \tau_i^{(k)} \sum_{j=M_k+1}^{Q} \delta_j^{(k)} \geq 0, \text{ with } k = 1 \cdots, r, i = 1 \cdots, R, i^{(k)} = 1 \cdots, R^{(k)} \] (1.5)

The complex numbers \( z_k \) are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the following asymptotic expansion in the following convenient form:

\[ N(y_1, \ldots, y_r) = \max \{|y_1|^{\alpha_1} \ldots |y_r|^{\alpha_r} , 0\} \rightarrow 0 \]

\[ N(y_1, \ldots, y_r) = \min \{|y_1|^{\beta_1} \ldots |y_r|^{\beta_r} , 0\} \rightarrow \infty \]
where, with \( k = 1, \ldots, r \) : \( \alpha_k = \min \{ \text{Re}(d_j^{(k)}/\delta_j^{(k)}) \} \), \( j = 1, \ldots, M_k \) and
\[
\beta_k = \max \{ \text{Re}((c_j^{(k)}) - 1)/\gamma_j^{(k)} \} , j = 1, \ldots, N_k
\]

Serie representation of Aleph-function of several variables is given by
\[
N(y_1, \ldots, y_r) = \sum_{G_1, \ldots, G_r = 0}^{\infty} \sum_{g_1 = 0}^{M_1} \cdots \sum_{g_r = 0}^{M_r} \frac{(-)^{G_1 + \cdots + G_r}}{\prod_{g_i = 0}^{G_i} \Gamma(g_i + 1)} \psi(\eta_{G_1, g_1}, \ldots, \eta_{G_r, g_r})
\times \theta_1(\eta_{G_1, g_1}) \cdots \theta_r(\eta_{G_r, g_r}) y_1^{-\eta_{G_1, g_1}} \cdots y_r^{-\eta_{G_r, g_r}}
\]

(1.6)

where \( \psi(\cdot, \ldots, \cdot), \theta_i(\cdot), i = 1, \ldots, r \) are given respectively in (1.2), (1.3) and
\[
\eta_{G_i, g_i} = \frac{d_{g_i}^{(i)} + G_i}{\delta_{g_i}^{(i)}} , \ldots, \eta_{G_r, g_r} = \frac{d_{g_r}^{(r)} + G_r}{\delta_{g_r}^{(r)}}
\]

which is valid under the conditions
\[
\delta_{g_i}^{(i)} [d_j^{(i)} + p_i] \neq \delta_{j}^{(i)} [d_{g_i}^{(i)} + G_i]
\]

(1.7)

for \( j \neq M_1, M_i = 1, \ldots \eta_{G_i, g_i} ; P_i, N_i = 0, 1, 2, \ldots ; y_i \neq 0, i = 1, \ldots, r \)

In the document, we will note:
\[
G(\eta_{G_1, g_1}, \ldots, \eta_{G_r, g_r}) = \phi(\eta_{G_1, g_1}, \ldots, \eta_{G_r, g_r}) \theta_1(\eta_{G_1, g_1}) \cdots \theta_r(\eta_{G_r, g_r})
\]

(1.9)

where \( \phi(\eta_{G_1, g_1}, \ldots, \eta_{G_r, g_r}) , \theta_1(\eta_{G_1, g_1}) , \ldots, \theta_r(\eta_{G_r, g_r}) \) are given respectively in (1.2) and (1.3)

We will note the Aleph-function of \( r \) variables
\[
N_{u,v}^{0, N,v}(z_1, \ldots, z_r)
\]

(1.10)

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral:
\[
I(z_1, z_2, \ldots z_s) = \mathcal{I}^{0, n_2, 0, n_3, \ldots 0, n_r, m_1, m_2, \ldots, m_s, n_1, n_2, \ldots, n_s}_{p_2, q_2, p_3, q_3, \ldots, p_r, q_r, p', q', \ldots, p', q'}(z_1, \ldots, z_s)
\]

(1.11)

\[
(a_{s_1}; \alpha_{s_1}', s_1)_{1,p_1} : (a_{s_2}; \alpha_{s_2}', s_2)_{1,p_2'} : \cdots ; (a_{s_r}; \alpha_{s_r}', s_r)_{1,p_r'};
\]

\[
(b_{s_1}; \beta_{s_1}', s_1)_{1,q_1} : (b_{s_2}; \beta_{s_2}', s_2)_{1,q_2'} : \cdots ; (b_{s_r}; \beta_{s_r}', s_r)_{1,q_r'}
\]
\[ \frac{1}{(2\pi \omega)^s} \int_{L_1} \cdots \int_{L_s} \xi(t_1, \cdots, t_s) \prod_{i=1}^{s} \phi_i(t_i) z_i^{a_i} dt_1 \cdots dt_s \]  

(1.12)

The defined integral of the above function, the existence and convergence conditions, see Y, N Prasad [4]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as:

\[ |\arg z_k| < \frac{1}{2} \Omega_i^{(k)} \pi, \text{ where} \]

\[ \Omega_i^{(k)} = \sum_{k=1}^{n_i} \alpha_k^{(i)} - \sum_{k=n_i+1}^{p_i} \alpha_k^{(i)} + \sum_{k=1}^{q_i} \beta_k^{(i)} - \sum_{k=q_i+1}^{p_2} \beta_k^{(i)} + \left( \sum_{k=1}^{n_2} \alpha^{(i)}_{2k} - \sum_{k=n_2+1}^{p_2} \alpha^{(i)}_{2k} \right) + \cdots \]

\[ \left( \sum_{k=1}^{n_s} \alpha^{(i)}_{sk} - \sum_{k=n_s+1}^{p_s} \alpha^{(i)}_{sk} \right) - \left( \sum_{k=1}^{q_2} \eta^{(i)}_{2k} + \sum_{k=1}^{q_3} \eta^{(i)}_{3k} + \cdots + \sum_{k=1}^{q_s} \eta^{(i)}_{sk} \right) \]  

(1.13)

where \( i = 1, \cdots, s \)

The complex numbers \( z_i \) are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the asymptotic expansion in the following convenient form:

\[ N(z_1, \cdots, z_s) = 0(|z_1|^{\alpha'} \cdots |z_s|^{\alpha'}), \max(|z_1| \cdots |z_s|) \to 0 \]

\[ N(z_1, \cdots, z_s) = 0(|z_1|^{\beta'} \cdots |z_s|^{\beta'}), \min(|z_1| \cdots |z_s|) \to \infty \]

where, with \( k = 1, \cdots, z: \alpha_k' = \min[\operatorname{Re}(b_j^{(k)} / \beta_j^{(k)})], j = 1, \cdots, m_k \) and

\[ \beta_k' = \max[\operatorname{Re}((\alpha_j^{(k)} - 1) / \alpha_j^{(k)}), j = 1, \cdots, n_k] \]

We will use the following notations in this paper:

\[ U = p_2, q_2, p_3, q_3; \cdots; p_{s-1}, q_{s-1}; V = 0, n_2; 0, n_3; \cdots; 0, n_{s-1} \]  

(1.14)

\[ W = (p', q'); \cdots; (p^{(s)}, q^{(s)}); X = (m', n'); \cdots; (m^{(s)}, n^{(s)}) \]

(1.15)

\[ A = (\alpha_{2k}, \alpha_{2k}', \alpha_{2k}''), \cdots; (a_{(s-1)k}, \alpha_{(s-1)k}', \alpha_{(s-1)k}''), \cdots, \alpha_{(s-1)k}')_{1,p_s} \]

(1.16)

\[ B = (b_{2k}, \beta_{2k}', \beta_{2k}''), \cdots; (b_{(s-1)k}, \beta_{(s-1)k}', \beta_{(s-1)k}''), \cdots, \beta_{(s-1)k}')_{1,q_s} \]

(1.17)

\[ a = (a_{sk}, \alpha_{sk}, \alpha_{sk}''', \cdots, \alpha_{sk}^{s})_{1,p_s}; \mathcal{B} = (b_{sk}, \beta_{sk}', \beta_{sk}'', \cdots, \beta_{sk}^{s})_{1,q_s} \]  

(1.18)

[Note: The text is not a complete representation of the page as the content is too long to be fully transcribed accurately. The formatting and symbols do not align perfectly with the natural text representation.]
The multivariable I-function write:

\[
I(z_1, \ldots, z_s) = I_{V;0,n_r;W}^{U;0,n_s;X} \left( \begin{array}{c}
\mathbf{A} \cdot \mathbf{A}' \\
\mathbf{B} \cdot \mathbf{B}'
\end{array} \right)
\]

The generalized polynomials defined by Srivastava [5], is given in the following manner:

\[
S_{N_1', \ldots, N_t'}^{M_1', \ldots, M_t'}[y_1, \ldots, y_t] = \sum_{K_1=0}^{[N_1'/M_1']} \cdots \sum_{K_t=0}^{[N_t'/M_t']} \frac{(-N_1')_{M_1'K_1}}{K_1!} \cdots \frac{(-N_t')_{M_t'K_t}}{K_t!}
\]

Where \( M_1', \ldots, M_t' \) are arbitrary positive integers and the coefficients \( A[N_1', K_1; \ldots; N_t', K_t] \) are arbitrary constants, real or complex. In the present paper, we use the following notation

\[
a_K = \frac{(-N_1')_{M_1'K_1}}{K_1!} \cdots \frac{(-N_t')_{M_t'K_t}}{K_t!} A[N_1', K_1; \ldots; N_t', K_t]
\]

In the document, we note:

\[
G(\eta_{G_1, g_1}, \ldots, \eta_{G_r, g_r}) = \phi(\eta_{G_1, g_1}, \ldots, \eta_{G_r, g_r}) \theta_1(\eta_{G_1, g_1}) \cdots \theta_r(\eta_{G_r, g_r})
\]

where \( \phi(\eta_{G_1, g_1}, \ldots, \eta_{G_r, g_r}), \theta_1(\eta_{G_1, g_1}), \ldots, \theta_r(\eta_{G_r, g_r}) \) are given respectively in (1.2) and (1.3)

2. Generalization of the Hurwitz-Lerch Zeta function

Parmar et al [2] have defined the Generalization of the Hurwitz-Lerch Zeta function

\[
\phi_{\lambda, \mu, v}(z, s', a) = \sum_{m=0}^{\infty} \frac{\lambda_m (\mu)_m}{v_m m!} \frac{z^m}{(m+a)^{s'}}
\]

\[\lambda, \mu \in \mathbb{C}, v, a \in \mathbb{C} \setminus \mathbb{Z}^{-}; s' \in \mathbb{C} \text{ when } |z| < 1; Re(s' + v - \lambda - \mu) > 1 \text{ when } |z| = 1\]

3. Required integral

We have the following result, see (Beals et al [1], page 54)

\[
S(a, b, c) = \int_0^1 \cdots \int_0^1 \prod_{i=1}^n x_i^{a-1}(1-x_i)^{b-1} \prod_{1 \leq j < k \leq n} |x_j - x_k|^{2c} \ dx_1 \cdots dx_n =
\]

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\[= \prod_{j=0}^{n-1} \frac{\Gamma(a + jc)\Gamma(b + jc)\Gamma(1 + (j + 1)c)}{\Gamma(a + b + (n - 1 + j)c)\Gamma(1 + c)} \]  
\[\text{(3.1)}\]

with \(\text{Re}(a) > 0, \text{Re}(b) > 0, \text{Re}(c) > \text{Max} \left\{ \frac{1}{n}, \frac{\text{Re}(a)}{n - 1}, \frac{\text{Re}(b)}{n - 1} \right\} \)

\(S(a, b, c)\) is the Selberg integral with three parameters

4. Main integral

Let \(X_{u,v,w} = \prod_{i=1}^{n} x_i^u(1 - x_i)^v \prod_{1 \leq j < k \leq n} |x_j - x_k|^{2w}\)

We have the following formula

**Theorem**

\[
\int_{0}^{1} \cdots \int_{0}^{1} \prod_{i=1}^{n} x_i^{a-1}(1 - x_i)^{b-1} \prod_{1 \leq j < k \leq n} |x_j - x_k|^{2c} \phi_{\lambda,\mu,\nu}(zX_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta}) \]

\[S_{N_1, \ldots, N_t}^{M_1, \ldots, M_t}(y_1 X_{\alpha_1, \beta_1, \gamma_1}, \ldots, y_t X_{\alpha_t, \beta_t, \gamma_t}) N_{u,w}^{\alpha, \nu}(z_1 X_{\delta_1, \epsilon_1}, \ldots, z_r X_{\delta_r, \epsilon_r, \phi_r}) \]

\[J_{U,v,q,w}^{\alpha, \nu}(X_{\delta_1, \epsilon_1}, \ldots, Z_{s} X_{\epsilon_s, \eta_s, \zeta_s}) \]

\[
\sum_{m=0}^{\infty} \sum_{g_1, \ldots, g_r=0}^{M_1} \sum_{K_1=0}^{[N_1'/M_1']} \cdots \sum_{K_r=0}^{[N_r'/M_r']} \sum_{g_t=0}^{M_t} \sum_{K_t=0}^{[N_t'/M_t']} \sum_{g_r=0}^{M_r} \sum_{K_r=0}^{[N_r'/M_r']} \sum_{g_s=0}^{M_s} \sum_{K_s=0}^{[N_s'/M_s']} \alpha_{K_1} \cdots \alpha_{K_r} \delta_{g_1} \cdots \delta_{g_r} G_{r}! \cdots \delta_{g_r} G_{r}! \delta_{g_r} G_{r}! \nu_{m}! \nu_{m}! (m + a)^s \]

\[G(\eta_{G_1,g_1}, \ldots, \eta_{G_r,g_r}) \]

\[y_1^{K_1} \cdots y_t^{K_t} \gamma_1 \cdots \gamma_t \zeta_1 \cdots \zeta_t \]

\[\int_{U,v,q,w}^{\alpha, \nu}(X_{\delta_1, \epsilon_1}, \ldots, Z_{s} X_{\epsilon_s, \eta_s, \zeta_s}) \]

\[1-a \alpha m - \sum_{i=1}^{t} K_i \alpha_i - \sum_{i=1}^{r} \gamma_i \delta_i - j(c + \gamma m + \sum_{i=1}^{t} \gamma_i K_i + \sum_{i=1}^{r} \phi_i \eta_{G_i,g_i}, \epsilon_1 + j \zeta_1, \ldots, \epsilon_s + j \zeta_s)_{0,n-1} \]

\[(-c - \gamma m + \sum_{i=1}^{t} K_i \gamma_i - \sum_{i=1}^{r} \phi_i \eta_{G_i,g_i}, \zeta_1, \ldots, \zeta_s), \ldots, \]

\[1-b \beta m - \sum_{i=1}^{t} K_i \beta_i - \sum_{i=1}^{r} \gamma_i \psi_i - j(c + \gamma m + \sum_{i=1}^{t} \gamma_i K_i + \sum_{i=1}^{r} \phi_i \eta_{G_i,g_i}, \eta_1 + j \zeta_1, \ldots, \eta_s + j \zeta_s)_{0,n-1} \]

\[(-c - \gamma m - \sum_{i=1}^{t} \gamma_i K_i - \sum_{i=1}^{r} \phi_i \eta_{G_i,g_i}, \zeta_1, \ldots, \zeta_s), B_{i} \]
\[-(j+1)(c+\gamma m + \sum_{i=1}^{t} \gamma_i K_i + \sum_{i=1}^{r} \phi_i \eta G_i, g_i), j \zeta_1, \cdots, j \zeta_s \}_{0, n-1}, \mathfrak{A} : A'\] (4.1)

where \(B_1 = [1 - a - b - (\alpha + \beta)m - \sum_{i=1}^{t} K_i(\alpha_i + \beta_i) - \sum_{i=1}^{r} (\delta_i + \phi_i)\eta G_i, g_i]n - (n - 1 + j)\times \]

\[(c + \gamma m + \sum_{i=1}^{t} K_i\gamma_i + \sum_{i=1}^{r} \phi_i \eta G_i, g_i)\epsilon_1 + \eta_1 + j \zeta_1, \cdots, \epsilon_s + \eta_s + j \zeta_s \}_{0, n-1} \quad (4.2)\]

Provided that

a) \(\min\{\alpha, \beta, \gamma, \alpha_i, \beta_i, \gamma_i, \delta_j, \psi_j, \phi_j, \epsilon_k, \eta_k, \zeta_k\} > 0, i = 1, \cdots, t, j = 1, \cdots, r, k = 1, \cdots, s,\)

b) \(A = \text{Re}[a + \alpha m + \sum_{i=1}^{r} \delta_i \min_{1 \leq j \leq M_i} \frac{d_i(j)}{\delta_i(j)} + \sum_{i=1}^{s} \epsilon_i \min_{1 \leq j \leq m(i)} \frac{b_i(j)}{\beta_j}] > 0\)

c) \(B = \text{Re}[b + \beta m + \sum_{i=1}^{r} \psi_i \min_{1 \leq j \leq M_i} \frac{d_i(j)}{\delta_i(j)} + \sum_{i=1}^{s} \eta_i \min_{1 \leq j \leq m(i)} \frac{b_i(j)}{\beta_j}] > 0\)

d) \(C = \text{Re}[c + \gamma m + \sum_{i=1}^{r} \phi_i \min_{1 \leq j \leq M_i} \frac{d_i(j)}{\delta_i(j)} + \sum_{i=1}^{s} \zeta_i \min_{1 \leq j \leq m(i)} \frac{b_i(j)}{\beta_j}] > \text{Max} \left\{ -\frac{1}{n}, -\frac{A}{n-1}, -\frac{B}{n-1} \right\} \)

e) \(|\arg z_k| < \frac{1}{2} A^{(k)}(\pi), \text{ where } A^{(k)}(\pi) \text{ is defined by (1.5); } i = 1, \cdots, r\)

f) \(|\arg Z_k| < \frac{1}{2} \Omega^{(k)}(\pi), \text{ where } \Omega^{(k)}(\pi) \text{ is defined by (1.13); } i = 1, \cdots, s\)

g) The multiple series occurring on the right-hand side of (4.1) is absolutely and uniformly convergent.

h) \(\lambda, \mu \in \mathbb{C}, \nu, a \in \mathbb{C}, a \notin \mathbb{Z}^+, s' \in \mathbb{C} \text{ when } |z| < 1; \text{Re}(s' + \nu - \lambda - \mu) > 1 \text{ when } |z| = 1\)

Proof

First, expressing the generalized sequence of functions \(\phi_{\lambda, \mu, v}(\cdot, s', a)\) in serie with the help of equation (2.1), the Aleph-function of \(r\) variables in serie with the help of equation (1.6), the general class of polynomial of several variables \(S_{M_1, \cdots, M_r}^{N_1, \cdots, N_s} \{y_1, \cdots, y_l\}\) with the help of equation (1.21) and the I-function of \(s\) variables in defined by Prasad [4] in Mellin-Barnes contour integral with the help of equation (1.12), changing the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process). Now evaluating the resulting Selberg integral with the help of equation (3.1). Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

5. Particular case

If \(U = V = A = B = 0\), the multivariable I-function defined by Prasad degenerates in multivariable H-function defined by Srivastava et al [6]. We have the following result.

Corollary
6. Conclusion

In this paper we have evaluated a Selberg integral involving the multivariable Aleph-function, the multivariable I-function defined by Prasad [2], a class of polynomials of several variables and a certain extension of the Hurwitz-Lerch Zeta function. The integral established in this paper is of very general nature as it contains multivariable Aleph-function, and multivariable I-function defined by Prasad [3] which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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