A Note on soShearEnergy of Jahangir graphs
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Abstract:
Let $G = (V, E)$ be a finite connected graph. A set $D \subset V$ is a dominating set of $G$ if every vertex in $V - D$ is adjacent to some vertex in $D$. A dominating set $D$ of $G$ is called a minimal dominating set if no proper subset of $D$ is a dominating set. The notion of shear Energy in terms of idegree and odegree has been introduced in the paper [2]. In this paper soShearEnergy of Jahangir graphs $J_{2,m}$ is calculated for all possible minimal dominating set. Energy cure for those graphs are plotted. Hardihood $+$ and - of the Jahangir graph $J_{2,m}$ is also calculated.

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1 introduction

Let $G = (V, E)$ be a finite connected graph. A set $D \subset V$ is a dominating set of $G$ if every vertex in $V - D$ is adjacent to some vertex in $D$. A dominating set $D$ of $G$ is called a minimal dominating set if no proper subset of $D$ is a dominating set. The notion of shear Energy in terms of idegree and odegree has been introduced in the paper [2]. In this paper soShearEnergy of Jahangir graphs $J_{2,m}$ is calculated for all possible minimal dominating set. Energy cure for those graphs are plotted. Hardihood $+$ and - of the Jahangir graph $J_{2,m}$ is also calculated.

A dominating set $D$ of $G$ is called a minimum dominating set if $D$ is a dominating set with minimum cardinality of a minimal dominating sets of $G$. A dominating set $D$ of $G$ is called an independent dominating set if the vertices in $D$ are independent. A dominating set $D$ of $G$ is called a maximum independent dominating set if $D$ is a independent dominating set with maximum cardinality. If its cardinality is minimum, it is minimum independent dominating set. Independent sets with some other properties also evolved and soShearEnergy is calculated for these minimal dominating sets.
2 Preliminaries

Definition 2.1.

Let $G$ be a graph and $S$ be a subset of $V(G)$. Let $v \in V - S$, the **idegree** of $v$ with respect to $S$ is the number of neighbours of $v$ in $V - S$ and it is denoted by $id_S(v)$.

Definition 2.2.

Let $G$ be a graph and $S$ be a subset of $V(G)$. Let $v \in V - S$, the **odegree** of $v$ with respect to $S$ is the number of neighbours of $v$ in $V - S$ be a minimal dominating set, the number of edges which join the vertices of $S$ and is denoted as $od_S(v)$.

Definition 2.3.

Let $G$ be a graph and $S$ be a subset of $V(G)$. Let $v \in V - S$, the **oidegree** of $v$ with respect to $S$ is $od_S(v) - id_S(v)$ if $od > id$ and it is denoted by $oid_S(v)$.

Definition 2.4.

Let $G$ be a graph and $D$ be a dominating set, $oShearEnergy$ of a graph with respect to $D$ is the summation of all oid if $od > id$ or otherwise zero.

Definition 2.5.

Let $G$ be a graph and $D$ be a minimal dominating set, then $ShearEnergy$ curve is the curve obtained by joining the oShearEnergies of $D_{i-1}$ and $D_i$ for $1 \leq i \leq n$, taking the number of vertices of $D_i$ along the x axis and the oShearEnergy along the y axis.

Definition 2.6.

Let $G$ be a graph and $D$ be a minimal dominating set, $soShearEnergy$ of a graph with respect to $D$ is

$$\sum_{0}^{V-D} os\epsilon_{D_i}(G)$$

where $D_{i+1} = D_i \cup V_{i+1}, V_{i+1}$ is a singleton vertex with minimum oidegree of $V - D_i$ and $D_0$ is a minimal dominating set where $0 \leq i \leq |V - D|$, it is denoted by $so\epsilon_D(G)$.

Definition 2.7.

Let $G$ be a graph and $MDS(G)$ is the set of all minimal dominating set of $G$, then $Hardihood^+$ of a graph $G$ is $\max \{so\epsilon_{(MDS(G))}(G)\}$ is denoted as $HD^+(G)$.

Definition 2.8.

Let $G$ be a graph and $MDS(G)$ is the set of all minimal dominating set of $G$, then $Hardihood^-$ of a graph $G$ is $\min \{so\epsilon_{(MDS(G))}(G)\}$ is denoted as $HD^-(G)$.

Given below is the modified steps in the algorithm find in [2] to find the soShearEnergy of any given graph with respect to the given dominating set. Using this algorithm soSherEnergy is found out.
Algorithm
5. Find the vertex with minimum positive oidegree and the number of vertices with minimum positive oidegree.
6.a) If the number of vertices with minimum oidegree is 1, then shift the vertex to the dominating set
Else
Find the vertex with maximum idegree among the vertex with minimum oidegree and shift it to the dominating set.

b) If no such positive oidegree exists shift a vertex with oidegree 0 that also has the maximum idegree among the vertex with oidegree zero to the set D otherwise shift a vertex with minimum negative oidegree to the dominating set.

Remark 2.9. The number of iterations needed to find the soEnergy of a graph is $|V - D| + 1$.

3 Minimal dominating sets of Jahangir graphs

Definition 3.1. Jahangir graph $J_{n,m}$ for $m \geq 3$, is a graph on $nm+1$ vertices consisting of a cycle $C_{nm}$ with one additional vertex which is adjacent to $m$ vertices of $C_{nm}$ at distance $n$ to each other on $C_{nm}$.

Let $v_{2m+1}$ be the center vertex and $v_1, v_2, ..., v_{2m}$ be vertices of the cycle $C_{2m}$ where the vertex $v_s$ with $s \equiv 1 \ (mod \ 2)$ is incident with the center vertex $v_{2m+1}$.

Let $S_0 = \{v_1, v_5, ..., v_{2m-3}, v_{2m+1}\}$ be the connected dominating set, $S_1 = \{v_2, v_4, ... v_{2m}, v_{2m+1}\}$ be the maximum independent dominating set. Independent dominating set without the center point, Minimum independent dominating set can also be calculated. These are possible minimal dominating sets of Jahangir graph $J_{2,m}$.

Theorem 3.2. Let $J_{n,m}$ for $m \geq 3$ be the graph and $D$ be the minimum connected dominating set, then the number of iterations is $nm - \gamma + 2$.

Proof: Let $J_{n,m}$ be the Jahangir graph. The number of vertices in the Jahangir graph is $pn = nm + 1$
i.e., $p = nm + 1$
$\implies p - \gamma = nm + 1 - \gamma$
$\implies p - |D| = nm - \gamma + 1$
$\implies |V - D| = nm - \gamma + 1$
$\implies |V - D| + 1 = nm - \gamma + 2$
Hence by remark 1.10 the number of iterations in a Jahangir graph is $nm - \gamma + 2$.

Corollary 3.3. Let $J_{2s,m}$ for $m \geq 3$ be the graph and $D$ be the minimum connected dominating set, then the number of iterations is $2(sm + 1) - \gamma$.

Proof: Let $J_{n,m}$ be the Jahangir graph. Let $n=2s$, $s=1,2,3,...$
By theorem 4.1, number of iterations in
a Jahangir graph is \( nm - \gamma + 2 \) 

Since \( n = 2s \), number of iterations of \( J_{2s,m} \) is \( 2sm - \gamma + 2 \)

number of iterations of \( J_{2s,m} \) is \( 2(sm + 1) - \gamma \).

**Corollary 3.4.** Let \( J_{2,m} \) for \( m \geq 3 \) be the graph and \( D \) be the minimum connected dominating set, then the number of iterations is \( 2(m + 1) - \gamma \).

Proof: By putting \( s = 1 \) in corollary 2.3, number of iteration in a Jahangir graph \( J_{2,m} \) is \( 2(m + 1) - \gamma \). □

**Example**

In this example soShearEnergy of \( J_{2,5} \) is calculated using the algorithm for all possible minimal dominating sets.

The minimum connected dominating set of \( J_{2,5} \) is \( D = \{ v_1, v_5, v_7, v_11 \} \) and \( V - D = \{ v_2, v_3, v_4, v_6, v_8, v_9, v_10 \} \). Let us consider \( D_0 = D \), then \( oid \) s are 0,-1,0,2,0,-1,0. Hence \( ose(D_0)(J_{2,5}) = 2 \).

By step 3 of the algorithm, the vertex with minimum positive \( oid \) 2 is \( v_6 \). So vertex \( v_6 \) is shifted to the set \( D \). Now \( D_1 = \{ v_1, v_5, v_6, v_7, v_11 \} \). \( oid \) s are 0,-1,0,0,-1,0. \( ose_{D_1}(J_{2,5}) = 0 \).

By step 4 of the algorithm, all the vertices have zero or negative value, hence oenergy is 0 and by step 6, the vertex with \( oid \) zero is \( v_2 \) is shifted to the set \( D \).

\( D_2 = \{ v_1, v_2, v_5, v_6, v_7, v_11 \} \) \( oid_{V-D_2} \) are 1,0,0,-1,0. \( ose_{D_2}(J_{2,5}) = 1 \).

By step 3 of the algorithm, the vertex with minimum positive \( oid \) is \( v_3 \). So vertex \( v_3 \) is shifted to the set \( D \).

\( D_3 = \{ v_1, v_2, v_3, v_5, v_6, v_7, v_11 \} \) \( oid \) s are 2,0,-1,0. \( ose_{D_3}(J_{2,5}) = 2 \).

By step 3 of the algorithm, the vertex with minimum positive \( oid \) is \( v_4 \). So vertex \( v_4 \) is shifted to the set \( D \).

\( D_4 = \{ v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_11 \} \) \( oid \) s are 0,-1,0. \( ose_{D_4}(J_{2,5}) = 0 \).

By step 4 of the algorithm, all the vertices have zero or negative values, vertex with zero \( oid \) is \( v_8 \). So vertex \( v_8 \) is shifted to the set \( D \).

\( D_5 = \{ 1, 2, 3, 4, 5, 6, 7, 8, 11 \} \) \( oid \) s are 1,0. \( ose_{D_5}(J_{2,5}) = 1 \).

By step 3 of the algorithm, the vertex with minimum positive \( oid \) is \( v_9 \). So vertex \( v_9 \) is shifted to the set \( D \).

\( D_6 = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 11 \} \) \( oid \) s are 1,0. \( ose_{D_6}(J_{2,5}) = 0 \).

By step 3 of the algorithm, the first vertex with maximum positive \( oid \) is \( v_10 \). So vertex \( v_10 \) is also shifted to the set \( D \).

\( D_7 = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 \} \) now the set \( V-D \) is empty. So \( ose_D(J_{2,5}) = 0 \).

By step 9, \( sos\epsilon(D)(J_{2,5}) = 2+0+1+2+0+1+2+0=2+2(1+2+0) = 8 \).

\( sos\epsilon(D)(J_{2,5}) = 8 \), when \( D \) is minimum connected dominating set.

The maximal independent dominating set of \( J_{2,5} \) is \( D = \{ 2, 4, 6, 8, 10, 11 \} \) and \( V-D = \{ 1, 3, 5, 7, 9 \} \). The \( oid \)’s 3 are all the vertices, since id are zero and od are 3 for all the vertices. Since the number of vertices in \( V-D \) is 5, \( ose_D = 5(3) \).
As the number of vertices in \( V - D \) decreases till zero. The \( sos_\epsilon_D(J_{2,5}) = 3(5 + 4 + 3 + 2 + 1) = 45 \).

The minimum independent dominating set of \( J_{2,5} \) is \( D = \{2, 5, 8, 10\} \) and \( V - D = \{1, 3, 4, 6, 7, 9, 111\} \). Applying the algorithm \( sos_\epsilon_D(J_{2,5}) = 2 + 1 + 1 + 2 + 3 + 1 + 2 + 0 = 12 \).

The independent dominating set which is not minimal nor maximum of \( J_{2,5} \) with all the vertex with degree 2 is \( D = \{1, 3, 5, 7, 9\} \) and \( V - D = \{2, 4, 6, 8, 10, 11\} \). Applying the algorithm \( sos_\epsilon_D(J_{2,5}) = 15 + 13 + 11 + 9 + 7 + 5 + 0 = 60 \).

By step 10 of the algorithm, fixing the x axis with the number of vertices of \( V - D \) and the Y axis with the oShearEnergy in each stage, the graphs obtained for all the minimal dominating sets are given below.

**Energy Curves of Jahangir graph \( J_{2,m} \)**
4 soEnergy of Jahangir graph $J_{2,m}$ with respect to the connected dominating set

**Theorem 4.1.** Let $J_{2,m}$ for $m \geq 3$ be the graph and $D$ be the minimum connected dominating set then, $\omega\varepsilon_{J_{2,m}}(D) \in \{0, 1, 2\} = z_3$.

*Proof:* Let $G$ be the graph $J_{2,m}$ and $D$ be the minimum connected dominating set with the center vertex. Case(i) Let $m$ be an odd number, then the minimal connected dominating set have the vertex $v_{2m-5}$ and $v_{2m-3}$. Therefore the vertex $v_{2m-4} \in V - D$ have the $id = 0$, $od = 2$ and $oid$ as 2. All other even vertices have $id$ and $od$ 1, all the odd vertices have $id$ 2, $od$ 1 and $oid$ -1, and $\omega\varepsilon = 2$. As the vertex $v_{2m-4}$ is shifted to $D$, there is no change in the $id$ and $od$. Hence oids are 0 and -1. At this stage $\omega\varepsilon = 0$. The first vertex with oid value zero is shifted to $D$, i.e., vertex $v_2$ is shifted to $D$. Now $id(v_3) = 1$, $od(v_3) = 2$, $oid(v_3) = 1$, and for all other odd vertices $iod$ is -1 and even vertices it is 0. At this stage $\omega\varepsilon = 1$. This process is repeated till cardinality of V-D is zero. Therefore the possible $\omega\varepsilon$ are 0, 1 and 2.

Case(ii) Let $m$ be an even number. All the even labeled vertices have $id$, $od$ 1, and $iod = 0$ and odd labeled vertices have $id$ 2, $od$ 1 and $oid$ -1. Therefore $\omega\varepsilon = 0$. At this stage the first vertex with oid zero is shifted to $D$. So the vertex $v_3$ have $id$ 1, $od$ 2 and $oid$ 1. All other vertices remains the same. Therefore $\omega\varepsilon = 1$. At this stage the vertex $v_3$ is shifted to $D$. The vertex $v_4$ have $id$ 0, $od$ 2 and $oid$ 2. All other vertices remains the same. Therefore $\omega\varepsilon = 2$. This process is repeated till cardinality of V-D is zero. Therefore the possible $\omega\varepsilon$ are 0, 1, and 2. Therefore $\omega\varepsilon_{J_{2,m}}(D) \in \{0, 1, 2\} = z_3$. ■

**Theorem 4.2.** Let $J_{2,m}$ for $m \geq 3$ be the graph and $D$ be the minimum connected dominating set, then

$$sos\varepsilon_D(J_{2,m}) = \begin{cases} \frac{3m}{2} & \text{if } m \text{ is even and } m \geq 4 \\ 2 + \frac{3m}{2} & \text{if } m \text{ is odd and } m \geq 3. \end{cases}$$

*Proof:* Let $J_{2,m}$ be the graph, $v_{2m+1}$ be the center vertex and $v_1, v_2, \ldots, v_{2m}$ be vertices of the cycle $C_{2m}$ where the vertex $v_s$ with $s \equiv 1(mod2)$ is incident with the center vertex $v_{2m+1}$. Let $D$ be a minimum connected dominating set containing the center vertex $v_{2m+1}$, i.e., $\{v_1, v_5, \ldots, v_{2m-3}, v_{2m+1}\}$. We know that in Jahangir graph $J_{2,m}$, degree of center vertex is $m$, degrees of vertices $\{v_1, v_3, \ldots, v_{2m-2}\}$ are 3 and for other vertices it is 2.
(i) Let \( m \) be odd number. By the theorem 4.3, for \( m \) is odd the \( o\epsilon \) begin with 2. Then it continues as 0, 1 and 2. Two \( c_n \) are involved for the occurrence of these \( o\epsilon \).

Therefore \( sos\epsilon_D(J_{2,m}) = 2 + \frac{m}{2}(0 + 1 + 2) = 2 + \frac{3m}{2} \).

(ii) Let \( m \) be even number. By theorem 4.3 \( o\epsilon \) s are 0, 1 and 2. Two \( C_n \) are needed for this formation. Therefore

\[
sos\epsilon_D(J_{2,m}) = \begin{cases} 
\frac{3m}{2} & \text{if } m \text{ is even and } m \geq 4 \\
2 + \frac{3m}{2} & \text{if } m \text{ is odd and } m \geq 3.
\end{cases}
\]

\[\blacksquare\]

**Corollary 4.3.** Let \( J_{2,m} \) for \( m \geq 3 \) be the graph and \( D \) be the minimum connected dominating set, then

\[
sos\epsilon_{J_{2,m+1}}(D) = 2 + sos\epsilon_{J_{2,m}}(D), \text{ where } m \text{ is odd and } m \geq 3.
\]

Proof: From theorem 3.2, It is clear that \( sos\epsilon_{J_{2,m+1}}(D) = 2 + \frac{3m}{2}, \text{ where } m \text{ is odd and } m \geq 3 \).

By the theorem 3.2 we know that \( sos\epsilon_{J_{2,m}}(D) = \frac{3m}{2}, \text{ if } m \text{ is even} \).

\[
\therefore sos\epsilon_{J_{2,m+1}}(D) = 2 + sos\epsilon_{J_{2,m}}(D), \text{ where } m \text{ is odd and } m \geq 4
\]

\[\blacksquare\]

### 5 ShearEnergy of \( J_{2,m} \) with respect to independent dominating set

Let \( D \) be the maximal independent dominating set and \( D = \{v_2, v_4, v_6, \ldots, v_{2m}, v_{2m+1}\} \).

**Theorem 5.1.** Let \( J_{2,m} \) for \( m \geq 3 \) be the graph and \( D \) be the maximal independent dominating set, then \( os\epsilon_{J_{2,m}}(D) \in \{0, 3, 6, \ldots 3m\} \).

Proof: Let \( G \) be the graph \( J_{2,m} \) and \( D \) be the maximal independent dominating set. Since \( D \) is the maximal independent dominating set \( D = \{v_2, v_4, v_6, \ldots, v_{2m}, v_{2m+1}\} \). Then \( V - D = \{v_1, v_3, \ldots, v_{2m-1}\} \). From the construction itself it is clear that \( od \) of all the odd labeled vertices are 3 and \( id \) is zero. Since \( id \) is zero for all the vertices, \( oid \) is 3 for all the vertices. It is clear that cardinality of \( V-D \) is \( m \). Therefore \( os\epsilon = 3m \). As the vertices get shifted to the set \( D \) \( os\epsilon = 3(m-1) \) and so on. The process continues till cardinality of \( V-D \) is zero. When the cardinality is zero \( os\epsilon = 0 \) Therefore \( os\epsilon_{J_{2,m}}(D) \in \{0, 3, 6, \ldots 3m\} \)

\[\blacksquare\]

**Theorem 5.2.** Let \( J_{2,m} \) for \( m \geq 3 \) be the graph and \( D \) be the maximal independent dominating set then, \( sos\epsilon_{J_{2,m}}(D) = \frac{3}{2}m(m+1) \).
Proof: Let $J_{2,m}$ be the graph, $v_{2m+1}$ be the center vertex and $v_1, v_2, \ldots, V_{2m}$ be vertices that incident clockwise on cycle $C_{2m}$ so that $\text{deg}(v_1) = 3$.

Let $D$ be the maximal independent dominating set containing the center vertex $v_{2m+1}$ also, i.e., $\{v_2, v_4, \ldots, V_{2m}, v_{2m+1}\}$ then $V - D = \{v_1, v_3, \ldots, v_{2m-1}\}$. As all these vertices are independent, degree of these vertices are zero. From the label it is clear that all the odd labeled vertices have odegree 3. oid of these vertices are 3. As the cardinality of $V - D$ is $m$, $o\varepsilon_D(J_{2,m}) = 3m$.

As the algorithm proceeds in each and every step, the vertices are shifted one by one to the set $D$, then the value of $m$ decreases three by three till cardinality of $V - D$ is zero. Therefore the value of $o\varepsilon_D(J_{2,m})$ are $3(m - 1), 3(m - 2), \ldots, 3, 0$.

By definition, $o\varepsilon_{J_{2,m}}(D) = 3m + 3(m - 1) + 3(m - 2) + \ldots + 3 + 0 = 3(m + (m - 1) + (m - 2) + \ldots + 2 + 1) = \frac{3}{2}m(m + 1)$.

Let IDWOC be the independent dominating set with out the center vertex and it is not the minimum dominating set. Then the soShearEnergy of Jahangir graph with respect to the dominating set IDWOC is calculated in the formula given below.

**Theorem 5.3.** Let $J_{2,m}$ for $m \geq 3$ be the graph and IDWOC be the dominating set then, $sos\varepsilon_{J_{2,m}}(\text{IDWOC}) = \sum_{i=0}^{m}(3m - 2i)$.

Proof: Let IDWOC be an independent dominating set without center vertex i.e, $\{v_1, v_4, \ldots, V_{2m-1}\}$ then $V - D = \{v_2, v_5, \ldots, v_{2m}, v_{2m+1}\}$. As all these vertices are independent, degree of these vertices are zero. From the label it is clear that all the even labeled vertices have odegree 2, and the center vertex have odegree $m$. oid of all the even vertices are 2 and the center vertex is $m$. As the cardinality of $V - D$ is $m+1$, $o\varepsilon_D(J_{2,m}) = m + 2m$.

As the algorithm proceeds the vertex of minimum oid is shifted to $D$ and the $o\varepsilon_D(J_{2,m}) = 3m - 2$. As the algorithm proceeds in each and every step the vertices are shifted one by one to the set $D$, then the value of $m$ decreases two by two till cardinality of $V - D$ is zero. Therefore the value of $o\varepsilon_D(J_{2,m})$ are $(3m - 2), (3m - 4), \ldots, (m + 2), m, 0$.

By definition, $o\varepsilon_{J_{2,m}}(ISWC) = (m + 2m) + (3m - 2) + (3m - 4) + \ldots + (m + 2) + m + 0$.

On Simplification we get, $o\varepsilon_{J_{2,m}}(ISWC) = \sum_{i=0}^{m}(3m - 2i)$.

Let $^4D$ be the minimum independent dominating set. The vertices in the dominating set forms three groups, they are as follows,

1. If $m \mod 3 = 0$, then $D = \{v_2, v_5, v_8, \ldots, v_{2m-4}, v_{2m-1}\}$.
2. If $m \mod 3 = 1$ and $m$
mod 3 = 2, then $D = \{v_2, v_5, v_8, \ldots, v_{2m-3}, v_{2m}\}$.

**Definition 5.4.** Let $G$ be a Jahangir graph, $D$ the dominating set and $(\text{oShearEnergy}_D)$ be the oShearEnergy sequence of the graph $G$ with respect to the minimal dominating set. The sum of the subsequence of $(\text{oShearEnergy}_D)$ given by $\sum_{v_{2m+1} \leq 0} (\text{oShearEnergy}_D)$ is known as $\omega$-index and denoted by $\omega(m)$. The sum of the subsequence of $(\text{oShearEnergy}_D)$ given by $\sum_{v_{2m+1} > 0} (\text{oShearEnergy}_D)$ is known as $\Omega$-index and denoted by $\Omega(m)$.

**Result 5.5.** Let $J_{2,m}$ for $m \geq 3$ be the graph, and $D$ the minimal dominating set, then series of $\omega$-index is given by

1. Let $m \text{ mod } 3 = 0$, then
   (a) If $m \text{ mod } 2 = 0$, then the sequence begin with 2 followed by $\frac{m}{6}$ numbers of $(1,0)$
   (b) If $m \text{ mod } 2 = 1$, then the sequence begin with 2 followed by $\lfloor \frac{m}{6} \rfloor$ numbers of $(1,0)$ number of times and then by $a (1,1)$.

2. Let $m \text{ mod } 3 = 1$, then
   (a) If $m \text{ mod } 2 = 0$, then the sequence begin with $\lfloor \frac{m}{6} \rfloor + 1$ numbers of $(1,0)$
   (b) If $m \text{ mod } 2 = 1$, then the sequence begin with $\lfloor \frac{m}{6} \rfloor$ numbers of $(1,0)$ number of times and then by $a (1,1)$.

3. Let $m \text{ mod } 3 = 2$, then
   (a) If $m \text{ mod } 2 = 0$, then the sequence begin with 2 followed by $\frac{3}{2}$ numbers of $(1,0)$
   (b) If $m \text{ mod } 2 = 1$, then the sequence begin with 2 followed by $\lfloor \frac{m}{6} \rfloor$ numbers of $(1,0)$ number of times and then by $a (1,1)$.

**Result 5.6.** Let $J_{2,m}$ for $m \geq 3$ be the graph, and $D$ the minimal dominating set, then $\omega$-index is given by $\sum (2i^2) + (2i-1)^2 + \ldots$ where $i = 1, 2, \ldots$. Further i’s are bunch which contains two consecutive m’s such as when $i=1$, $m=4,5$.

**Result 5.7.** Let $J_{2,m}$ for $m \geq 3$ be the graph, and $D$ the minimal dominating set, then $\sigma = \omega(m) + \Omega(m)$.

6 Hardihood of Jahangir graph $J_{2,m}$

**Theorem 6.1.** Let $J_{2,m}$ for $m \geq 3$ be the graph,

$$HD^+(G) = (m+2m)+(3m-2)+(3m-4)+\ldots+(m+2)+m+0 \text{ when } D \text{ is the ISWC dominating set}$$

$$HD^-(G) = \sigma = \omega(m) + \Omega(m) \text{ when } D \text{ is the minimum connected dominating set}.$$  

proof: Let G be the Jahangir graph $J_{2,m}$. It is clear that $\frac{3}{2}(m(m+1)) \leq (m+2m)+(3m-2)+(3m-4)+\ldots+$
Therefore \( s_{os}(J_{2,m}(D)) \) is maximum when \( D \) is ISWC dominating set.

Hence \( HD^+(G) = (m+2m)+(3m-2)+(3m-4)+...+(m+2)+m+0 \) when \( D \) is the ISWC dominating set. When \( D \) is minimal dominating set, all the vertices will not have the value 3 as the iodegree so the value is less than \( \frac{3}{2}m(m+1) \). Another possible minimal dominating set is connected dominating set which has the value \( \frac{m}{2}(0+1+2) \) or \( 2 + \frac{m}{2}(0+1+2) \)

References


