\( I_{gm} \) - Closed Sets

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Abstract

We define \( I_{gm} \) - closed sets in \((X, M, I)\) and discuss their properties.

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1 Introduction and preliminaries

An ideal \( I \) on a topological space \((X, \tau)\) is a non empty collection of subsets of \( X \) which satisfies (i) \( A \in I \) and \( B \subset A \) implies \( B \in I \) and (ii) \( A, B \in I \) implies \( A \cup B \in I \). Given a topological space \((X, \tau)\) with an ideal \( I \) on \( X \) and if \( P(X) \) is the set of all subsets of \( X \), a set operator \((\cdot)^* : P(X) \rightarrow P(X)\) called a local function \([5]\) of \( A \) with respect to \( \tau \) and \( I \) is defined as follows: for \( A \subset X, A^*(X, \tau) = \{x \in X/U \cap A \notin I, \text{ for every } U \in \tau(x)\}, \) where \( \tau(x) = \{U \in \tau/x \in U\}. \)
A Kuratowski closure operator $cl^*(\cdot)$ for a topology $\tau^*(I, \tau)$ called the $*-$ topology, finer than $\tau$, is defined by $cl^*(A) = A \cup A^*(I, \tau)$ [8]. When there is no confusion we will simply write $A^*$ for $A^*(I, \tau)$ and $\tau^*$ for $\tau^*(I, \tau)$. If $I$ is an ideal on $X$, then $(X, \tau, I)$ is called an ideal space. A subset $A$ of an ideal space $(X, \tau, I)$ is said to be $*-$ closed [4] if $A^* \subset A$ and $*-$ dense in itself if $A \subset A^*$ [3]. A subset $A$ of an ideal space $(X, \tau, I)$ is said to be $I_g-$ closed [2] if $A^* \subset U$ whenever $A \subset U$ and $U$ is open. A subset $A$ of an ideal space $(X, \tau, I)$ is said to be $I-$ locally $*-$ closed [7] if there exists an open set $U$ and a $*-$ closed set $F$ such that $A = U \cap F$.

By a space, we always mean a topological space $(X, \tau)$ with no separation properties assumed. If $A \subset X, cl(A)$ and $int(A)$ will respectively, denote the closure and interior of $A$ in $(X, \tau)$ and $int^*(A)$ will denote the interior of $A$ in $(X, \tau^*)$. A subset $A$ of a topological space $(X, \tau)$ is said to be a $g-$ closed set [6] if $cl(A) \subset U$ whenever $A \subset U$ and $U$ is open. A subset $A$ of a topological space $(X, \tau)$ is said to be a $g-$ open set if $X - A$ is a $g-$ closed set. A sub collection $M$ of $P(X)$ is called a minimal structure [1] on $X$, if (i) $\phi, X \in M$ and (ii) $M$ is closed under finite intersection.

$(X, M)$ is called a minimal space. If $I$ is an ideal on $X$, $(X, M, I)$ is called an ideal minimal space. If $U \in M$, $U$ is said to be a $m-$ open set. The complement of a $m-$ open set is called $m-$ closed set. We set $mint(A) = \cup\{U \in M/U \subset A\}$ and $mcl(A) = \cap\{F/A \subset F \text{ and } X - F \in M\}$. A subset $A$ of $(X, M)$ is said to be $mg-$ closed [1] if $mvl(A) \subset U$ whenever $A \subset U$ and $U \in M$. The complement of a $mg-$ closed set is called $mg-$ open set.
2 $I_{gm}$ - closed sets

If $(X, M)$ is a $m$-space, we denote the topology generated by $M$ by $\tau_m$. If $(X, M, I)$ is an ideal $m$-space, then $(X, \tau_m, I)$ is an ideal topological space. We denote the $\star$-topology generated by $I$ and $\tau_m$ on $X$ by $\tau_m^\star$.

For a subset $A$ of $X$, we denote the local function of $A$ with respect to $I$ and $\tau_m$ by $A^\star$ and closure of $A$ in $\tau_m$ and $\tau_m^\star$ by $cl(A)$ and $cl^\star(A)$ respectively.

A subset $A$ of an ideal $m$-space $(X, M, I)$ is said to be $I_{gm}$-closed if $A^\star \subset U$ whenever $A \subset U$ and $U \in M$. The complement of an $I_{gm}$-closed set is called an $I_{gm}$-open set.

A subset $A$ of $(X, \tau_m)$ is said to be $g_m$-closed if $cl(A) \subset U$ whenever $A \subset U$ and $U \in M$. The complement of a $g_m$-closed set is called a $g_m$-open set.

Since $cl^\star(A) \subset cl(A) \subset mcl(A)$ and $M \subset \tau_m$ we have the following diagram.

\[
\begin{array}{cccc}
m - closed & \rightarrow & closed & \rightarrow \star - closed \\
\downarrow & \downarrow & \downarrow \\
mg - closed & \rightarrow & g - closed & \rightarrow I_g - closed \\
\downarrow & \downarrow \\
gm - closed & \rightarrow I_{gm} - closed \\
\end{array}
\]

If $M = \tau$, a topology on $X$, then $\tau_m = \tau$, $cl(A) = mcl(A)$ and hence the concepts $mg - closed$, $g - closed$ and $gm - closed$ are coincide and the concepts $I_g - closed$ and $I_{gm} - closed$ are coincide.

The Theorems 2.1 and 2.2 gives characterizations for $I_{gm}$-closed sets.

**Theorem 2.1.** A subset $A$ of an ideal $m$-space $(X, M, I)$ is $I_{gm}$-closed if and only if $cl^\star(A) \subset U$ whenever $A \subset U$ and $U \in M$.

**Proof.** Suppose that $A$ is $I_{gm}$-closed. Then $A^\star \subset U$ whenever $A \subset U$ and $U \in M$. Therefore, $A \cup A^\star \subset U$ whenever $A \subset U$ and $U \in M$. (ie) $cl^\star(A) \subset U$
whenever $A \subset U$ and $U \in M$. Converse follows from the fact that $A^* \subset \text{Cl}^*(A)$.

For a subset $A$ of an ideal $m$–space $(X, M, I)$, define $\Lambda_m(A) = \cap\{U \in M/ A \subset U\}$. $A$ is said to be a $\Lambda_m$–set if $\Lambda_m(A) = A$.

**Theorem 2.2.** A subset $A$ of an ideal $m$–space $(X, M, I)$ is $I_{gm}$–closed if and only if $\text{cl}^*(A) \subset \Lambda_m(A)$.

**Proof.** Suppose $A$ is $I_{gm}$–closed. Let $U \in M$ be such that $A \subset U$. Then $\text{cl}^*(A) \subset U$. Therefore $\text{cl}^*(A) \subset \cap\{U \in M/ A \subset U\}$. (ie) $\text{cl}^*(A) \subset \Lambda_m(A)$.

Conversely, suppose $\text{cl}^*(A) \subset \Lambda_m(A)$. If $A \subset U$ and $U \in M$ then $\Lambda_m(A) \subset U$ and hence $\text{cl}^*(A) \subset U$. Therefore $A$ is $I_{gm}$–closed.

The Theorem 2.3 gives some properties of $I_{gm}$–closed sets and Example 2.4 shows that the converse need not be true.

**Theorem 2.3.** Let $(X, M, I)$ be an ideal $m$–space If $A \subset X$ is $I_{gm}$–closed, then the following properties hold.

(a) $\text{cl}^*(A) – A$ contains no non empty $m$–closed set.

(b) $A^* – A$ contains no non empty $m$–closed set.

**Proof.** Let $A$ be $I_{gm}$–closed set.

(a). Suppose $V \subset \text{cl}^*(A) – A$ and $V$ is $m$–closed. since $A$ is $I_{gm}$–closed and $X – V$ is a $m$–open, set containing $A$, $\text{cl}^*(A) \subset X – V$. Hence $V \subset X – \text{Cl}^*(A)$. Since $V \subset \text{Cl}^*(A)$ and $V \subset \text{cl}^*(A) – A$, we get $V = \phi$.

(b) If $A$ is $I_{gm}$–closed, then by (a) $\text{cl}^*(A) – A$ contains no non empty closed set. But $\text{cl}^*(A) – A = A^* – A$. Therefore (b) follows.

**Example 2.4.** Let $X = \{a, b, c\}, M = \{\phi, \{a\}, \{b\}, \{b, c\}, X\}$ and $I = \{\phi, \{a\}\}$. Then $\tau_m = \{\phi, \{a\}, \{b\}\{a, b\}, \{b, c\}, X\}$

If $A = \{b\}$, then $A^* – A = \{c\}$, which contains no non empty $m$–closed sets. But $A$ is not $I_{gm}$–closed.
Theorem 2.5. Suppose a subset $A$ of an ideal $m$ – space is both $I_{gm}$ – closed and $m$ – open. Then it is $\star$ – closed.

**Proof.** Since $A$ is $I_{gm}$ – closed. $A \subset A$ and $A \in M$ implies that $cl^*(A) \subset A$. Hence $A$ is $\star$ – closed.

Theorem 2.6. Let $(X, M, I)$ be an ideal $m$ – space. Then every subset of $X$ is $I_{gm}$ – closed if and only if every $m$ – open set is $\star$ – closed.

**Proof.** Suppose every subset of $X$ is $I_{gm}$ – closed. Let $U$ be an $m$ – open set. Since $U \subset U$, from the definition of $I_{gm}$ – closed sets, $U^* \subset U$ and hence $cl^*(U) \subset U$. Therefore $U$ is $\star$ – closed.

Conversely, suppose that every $m$ – open set is $\star$ – closed. If $A$ is any subset of $X$ and $A \subset U$, $U$ is $m$ – open, then $A^* \subset U^* \subset cl^*(U) = U$ and hence $A$ is $I_{gm}$ – closed.

Theorem 2.7. If $A$ is an $I_{gm}$ – closed subset of an ideal $m$ – space $(X, M, I)$, then the following properties are equivalent.

(a) $A$ is a $\star$ – closed set

(b) $cl^*(A) – A$ is a $m$ – closed set

(c) $A^* – A$ is a $m$ – closed set.

**Proof.** (a) $\Rightarrow$ (b). If $A$ is $\star$ – closed then $cl^*(A) = A$ and hence $cl^*(A) – A = \phi$, which is $m$ – closed.

(b) $\Rightarrow$ (c). Since $cl^*(A) – A = A^* – A, A^* – A$ is $m$ – closed.

(c) $\Rightarrow$ (a). Suppose $A^* – A$ is $m$ – closed. Since $A$ is $I_{gm}$ – closed, by Theorem 2.3, $A^* – A$ contains no non empty $m$ – closed set. Therefore, $A^* – A = \phi$ and hence $A^* \subset A$. So $A$ is $\star$ – closed.

Theorem 2.8. Let $(X, M, I)$ be an ideal $m$ – space. Then a subset $A$ of $X$ is $\star$ – closed if and only if $A^* – A$ is $m$ – closed and $A$ is $I_{gm}$ – closed.
Proof. Suppose $A$ is $\star$–closed. Then $A^* - A = cl^*(A) - A = \phi$, which is $m$–closed. Also every $\star$–closed set is $I_{gm}$–closed. Hence $A$ is $I_{gm}$–closed.

Conversely, suppose $A^* - A$ is $m$–closed and $A$ is $I_{gm}$–closed. Then by Theorem 2.7, $A$ is $\star$–closed.

Theorem 2.9. Let $(X, M, I)$ be an ideal $m$–space. If $A$ is $\star$–dense in itself, then $A^* = mcl(A^*) = mcl(A)$

Proof. Clearly, $A^* \subset mcl(A^*)$. It $x \notin A^*$, then there exist $U \in \tau_m$ such that $x \in U$ and $U \cap A \in I$. Since $\tau_m$ is generated by $M$, there exist $V \in M$ such that $x \in V \subset U$. Since $V \cap A \cup U \cap A \in I$, we have $V \cap A \in I$. If $x \in V$, then $x \in U$ and $U \cap A \in I$ and hence $x \notin A^*$. Therefore $V \cap A^* = \phi$. So $x \notin mcl(A^*)$. This proves that $mcl(A^*) \subset A^*$. Therefore $A^* = mcl(A^*)$. Since $A$ is $\star$–dense in itself, $A \subset A^*$ and hence $mcl(A) \subset mcl(A^*)$.

On the other hand, $A^* \subset cl^*(A) \subset cl(A) \subset mcl(A)$ and hence $mcl(A^*) \subset mcl(A)$. Therefore $mcl(A^*) = mcl(A)$.

In general $I_{gm}$–closed sets need not be $mg$–closed. The following Theorem 2.10 gives a condition where it is $mg$–closed.

Theorem 2.10. Let $(X, M, I)$ be an ideal $m$–space and $A$ is a subset of $X$. If $A$ is $\star$–dense in itself and $I_{gm}$–closed, then $A$ is $mg$–closed.

Proof. $A$ is $\star$–dense in itself. So $A \subset A^*$. Therefore, $cl^*(A) = A \cup A^* = A^*$. Suppose $U \in M$ and $A \subset U$. Since $A$ is $I_{gm}$–closed, by Theorem 2.1, $cl^*(A) \subset U$. Therefore $A^* \subset U$. Since $A$ is $\star$–dense in itself, by Theorem 2.9, $A^* = mcl(A)$. Therefore $mcl(A) \subset U$ and hence $A$ is $mg$–closed.

Theorem 2.11. Let $(X, M, I)$ be an ideal $m$–space and $A, B$ be subsets of $X$. If $A$ is $I_{gm}$–closed and $A \subset B \subset cl^*(A)$, then $B$ is also $I_{gm}$–closed.
Proof. Suppose $B \subset U$ and $U$ is $m$–open. Then $A \subset U$. Since $A$ is $I_{gm}$–closed, $\text{cl}^*(A) \subset U$. Since $A \subset B \subset \text{cl}^*(A)$, $\text{cl}^*(B) \subset \text{cl}^*(\text{cl}^*(A)) = \text{cl}^*(A)$.

Hence $\text{cl}^*(A) = \text{cl}^*(B)$. Therefore, $\text{cl}^*(B) \subset U$ and hence $B$ is $I_{gm}$–closed.

Theorem 2.12. Union of two $I_{gm}$–closed sets is an $I_{gm}$–closed set.

Proof. Let $(X, M, I)$ be an ideal $m$–space and let $A$ and $B$ be $I_{gm}$–closed sets in $X$. Suppose $A \cup B \subset U$ and $U$ is $m$–open. Since $A$ is $I_{gm}$–closed and $A \subset U$, we have $\text{cl}^*(A) \subset U$. Similarly $\text{cl}^*(B) \subset U$. Therefore $\text{cl}^*(A \cup B) = \text{cl}^*(A) \cup \text{cl}^*(B) \subset U$. Hence $A \cup B$ is $I_{gm}$–closed.

The following Example 2.13 shows that intersection of two $I_{gm}$–closed sets need not be $I_{gm}$–closed.

Example 2.13. Let $X = \{a, b, c\}$, $M = \{\phi, \{a\}, \{b, c\}, X\}$ and $I = \{\phi, \{a\}\}$. Then $\tau_m = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$.

Let $A = \{a, b\}$ and $B = \{b, c\}$. Since $X$ is the only $m$–open set containing $A$, $A$ is $I_{gm}$–closed. Since, $B^* = \{b, c\}$, $B$ is $\star$–closed and hence $I_{gm}$–closed. Now $A \cap B = \{b\}$, which is $m$–open and $(A \cap B)^* = \{b, c\}$. Therefore $A \cap B$ is not $I_{gm}$–closed.

Theorem 2.14. Let $(X, M, I)$ be an ideal $m$–space and $A, B$ be subsets of $X$. If $A \subset B \subset A^*$ and $A$ is $I_{gm}$–closed, then $B$ is $mg$–closed.

Proof. Since $A \subset B \subset A^*$, we have $A^* \subset B^* \subset (A^*)^* = A^*$. Therefore $A^* = B^*$ and hence $A$ and $B$ are $\star$–dense in itself. Since $A \subset B \subset A^* \subset \text{cl}^*(A)$ and $A$ is $I_{gm}$–closed, by Theorem 2.11, $B$ is $I_{gm}$–closed. Since $B$ is $\star$–dense in itself and $I_{gm}$–closed, by Theorem 2.10, $B$ is $mg$–closed.

Theorem 2.15. Let $(X, M, I)$ be an ideal $m$–space and $I = \{\phi\}$. Then $A$ is $I_{gm}$–closed if and only if $A$ is $mg$–closed.
Proof. Since $I = \{\phi\}$, $cl(A) = cl^*(A)$, for every subset of $X$. Therefore $cl^*(A) \subset U$ if and only if $cl(A) \subset U$. Therefore $A$ is $I_{gm} - closed$ if and only if $A$ is $mg - closed$.

Theorem 2.16. Let $(X, M, I)$ be an ideal $m$ - space. For every $x \in X$, the set $X - \{x\}$ is $I_{gm} - closed$ or $m - open$.

Proof. Suppose $X - \{x\}$ is not $m - open$. Then $X$ is the only $m - open$ set contains $X - \{x\}$ and $(X - \{x\})^* \subset X$. Hence $X - \{x\}$ is $I_{gm} - closed$.

The following theorem 2.17, gives a characterization for $I_{gm} - open$ sets.

Theorem 2.17. Let $(X, M, I)$ be an ideal $m$ - space and $A \subset X$. Then $A$ is $I_{gm} - open$ if and only if $F \subset int^*(A)$ whenever $F$ is $m - closed$ and $F \subset A$.

Proof. Suppose $A$ is $I_{gm} - open$ and $F \subset A$, $F$ is $m - closed$. Then $X - A \subset X - F$, $X - F$ is $m - open$ and $X - A$ is $I_{gm} - closed$. Therefore $cl^*(X - A) \subset X - F$. Therefore $F \subset X - cl^*(X - A) = int^*(A)$.

Conversely, suppose $F \subset int^*(A)$ whenever $F \subset A$ and $F$ is $m - closed$. If $X - A \subset U$ and $U$ is $m - open$, then $X - U \subset A$ and $X - U$ is $m - closed$. Therefore, by hypothesis, $X - U \subset int^*(A)$ and hence $cl^*(X - A) = X - int^*(A) \subset U$. Therefore $X - A$ is $I_{gm} - closed$ and hence $A$ is $I_{gm} - open$.

Since every $m - closed$ set is $I_{gm} - closed$, every $m - open$ set is $I_{gm} - open$.

Theorem 2.18. Let $(X, M, I)$ be an ideal $m$ - space and $A, B$ be subsets of $X$. If $A$ is $I_{gm} - open$ and $int^*(A) \subset B \subset A$, then $B$ is $I_{gm} - open$.

The proof follows from the Theorem 2.11.

Theorem 2.19. Intersection of two $I_{gm} - open$ sets is an $I_{gm} - open$ set.

The proof follows from the Theorem 2.12.

Theorem 2.20. If a subset $A$ of an ideal $m$ - space $(X, M, I)$ is $I_{gm} - closed$ then $A \cup (X - A^*)$ is also $I_{gm} - closed$. 
Proof. Suppose $A$ is $I_{gm} - closed$. If $A \cup (X - A^*) \subset U$ and $U$ is $m - open$, then $X - U \subset X - [A \cup (X - A^*)]$ and hence $X - U$ is $m - closed$. Since $A$ is $I_{gm} - closed$, by Theorem 2.3, $X - U = \phi$ and hence $X = U$. Therefore $X$ is the only $m - open$ set containing $A \cup (X - A^*)$ and hence $A \cup (X - A^*)$ is $I_{gm} - closed$.

Theorem 2.21. Let $(X,M,I)$ be an ideal $m - space$. Then the following are equivalent.

(a) Every $I_{gm} - closed$ set $\star - closed$

(b) Every singleton of $X$ is either $m - closed$ or $\star - open$.

Proof. $(a) \Rightarrow (b)$. Let $x \in X$. If $\{x\}$ is not $m - closed$, then $X - \{x\}$ is not $m - open$. Therefore $X$ is the only $m - open$ set containing $X - \{x\}$ and $X - \{x\}$ is $I_{gm} - closed$ set. By hypothesis, $X - \{x\}$ is $\star - closed$ and hence $\{x\}$ is $\star - open$.

$(b) \Rightarrow (a)$. Let $A$ be an $I_{gm} - closed$ set and $x \in A^*$. We have to prove that $x \in A$.

Case (i). If $\{x\}$ is $m - closed$ and $x \notin A$, then $A \subset X - \{x\}$ and $X - \{x\}$ is $m - open$. Since $A$ is $I_{gm} - closed$, $A^* \subset X - \{x\}$ and hence $x \notin A^*$, which is a contradiction.

Case (ii). If $\{x\}$ is $\star - open$, since $x \in A^*$ we have $x \in cl^*(A)$ and hence $\{x\} \cap A \neq \phi$. (ie) $x \in A$. Therefore $A^* \subset A$ and hence $A$ is $\star - closed$.

A subset $A$ of an ideal $m - space (X,M,I)$ is said to be $m - locally \star - closed$ if there exist a $m - open$ set $U$ and a $\star - closed$ set $F$ of $(X,\tau^*_m)$ such that $A = U \cap F$. The set $A$ is said to be $m - locally$ closed if there exist a $m - open$ set $U$ and a closed set $F$ of $(X,\tau_m)$ such that $A = U \cap F$.

If $I = \{\phi\}$, then the concept $m - locally \star - closed$ sets coincide with $m - locally$ closed set.
**Theorem 2.22.** Let \((X, M, I)\) be an ideal \(m\) – space and \(A\) be a subset of \(X\). Then the following statements are equivalent.

(a) \(A\) is \(m\) – locally \(*\) – closed

(b) \(A = U \cap \text{cl}^*(A)\), for some \(m\) – open set \(U\).

**Proof.** (a) \(\Rightarrow\) (b). If \(A\) is \(m\) – locally \(*\) – closed then there exist a \(m\) – open set \(U\) and a \(*\) – closed set \(F\) such that \(A = U \cap F\). Clearly \(A \subset U \cap \text{cl}^*(A)\). On the other hand, since \(F\) is \(*\) – closed, \(A \subset F\) implies that \(\text{cl}^*(A) \subset \text{cl}^*(F) = F\) and so \(U \cap \text{cl}^*(A) \subset U \cap F = A\). Therefore \(A = U \cap \text{cl}^*(A)\)

(b) \(\Rightarrow\) (a) is clear.

**Theorem 2.23.** Let \((X, M, I)\) be an ideal \(m\) – space and \(A\) be a \(m\) – locally \(*\) – closed subset of \(X\). Then the following properties hold.

(a) \(A^* - A\) is closed

(b) \((X - A^*) \cup A = A \cup (X - \text{cl}^*(A))\) is open

(c) \(A \subset \text{int}(A \cup (X - A^*))\)

(d) \(A\) is \(I\) – locally \(*\) – closed in \((X, \tau^*_m)\).

**Proof.** (a) Since \(A\) is \(m\) – locally \(*\) – closed, by Theorem 2.21, \(A = U \cap \text{cl}^*(A)\), for some \(m\) – open set \(U\). Then, \(A^* - A = A^* \cap (X - A) = A^* \cap [X - (U \cap \text{cl}^*(A))]\)

\[A^* \cap [(X - U) \cup (X - \text{cl}^*(A))] = (A^* \cap (X - U)) \cup (A^* \cap (X - \text{cl}^*(A))) = A^* \cap (X - U),\] which is closed (b). \(X - (A^* - A) = X - [A^* \cap (X - A)] = (X - A^*) \cup A\)

By (a), \((X - A^*) \cup A\) is open. Also \((X - A^*) \cup A = A \cup (X - \text{cl}^*(A))\).

(c) Since \(A\) is \(m\) – locally \(*\) – closed, by (b), \(A \cup (X - A^*)\) is open.

Therefore \(A \cup (X - A^*) \subset \text{int}[A \cup (X - A^*)]\) and hence \(A \subset \text{int}[A \cup (X - A^*)]\).

(d) The proof follows from the fact that every \(m\) – open set is open.
Theorem 2.24. Let \((X,M,I)\) be an ideal \(m\) – space and \(A\) is a \(m\) – locally \(*\) – closed and \(I\) – dense subset of \(X\). Then \(A\) is open.

Proof. If \(A\) is \(m\) – locally \(*\) – closed, then by Theorem 2.23(d), \(A\) is \(I\) – locally \(*\) – closed. Therefore, by Theorem 3.1 (e)[7], \(A \subset \text{int}[A \cup (X - A^*)]\). Since \(A\) is \(I\) – dense, \(A^* = X\) and so \(A \subset \text{int}(A)\). Therefore \(A\) is open.

References


