A New Approach to (1, 2)-j-open sets in Bitopological Spaces

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Abstract
The aim of this present paper is to introduce a new class of set namely (1,2)-j-open, (1,2)-j-closed in bitopological spaces. We investigate the several properties and study their relationship with other existing sets.

Keywords: (1,2)-j-open sets, (1,2)-j-closed sets

1 Introduction
Kelly[6] initiated the study of bitopological spaces in 1963. A nonempty set X equipped with two topological spaces τ₁,τ₂ is called a bitopological space and is denoted by (X,τ₁,τ₂). Using the notation of pre open set in 1990 D. Andrijiević and M. Ganster[1] defined the concept of γ-open set in topological spaces. S.N. Maheshwariand R. Prasad[10] extended the notion of semi-open sets and semi-continuity to the bitopological setting in 1977. B.P. Dvalishvili[4] introduced concepts of (1,2)-open domain and (1,2)-boundaries in bitopological space. B. Bhattacharya and A. Paul[2] introduced γ-open set in bitopological spaces and studied their properties. The concept of pre open sets in topological space was initiated by Mashrouret. Al[11]. S. Raychaudhui and M.N. Mukherjee[12] have introduced the notion of δ-preopen sets and δ-almost continuity in topological spaces. The class of δ-preopen sets is larger than that of preopen sets The purpose of this paper is to define some properties by using (1,2)-j-open, (1,2)-j-closed in bitopological space and analyse the relationships between them.

2 Preliminaries

2.1 Definition
Let X be a non empty set and τ₁,τ₂ be the topologies on X. A triple(X,τ₁,τ₂) is said to be a bitopological space.

2.2 Definition
A subset A of a bitopological space (X,τ₁,τ₂) is called a (1,2)-semi open if A ⊆ cl₂(ś₃₁(A)) and it is (1,2)-semi closed if cl₂(ś₃₁(A)) ⊆ A

2.3 Definition
Let (X,τ₁,τ₂) be a bitopological space, A ⊆ X, A is said to be (1,2)-p-open set if A ⊆ int₂(cl₁(A)) and A is (1,2)-p-closed if X\A is (1,2)-p-open

2.4 Definition
Let (X,τ₁,τ₂) be a bitopological space, A ⊆ X, A is said to be (2,1)-p-open set if A ⊆ int₁(cl₂(A))

2.5 Definition
Let (X,τ₁,τ₂) be a bitopological space, A ⊆ X, A is said to be 1-p-open set if A ⊆ int₁(cl₁(A))

2.6 Definition
Let (X,τ₁,τ₂) be a bitopological space, A ⊆ X, A is said to be 2-p-open set if A ⊆ int₂(cl₂(A))

2.7 Definition
Let A be a subset of a bitopological space (X,τ₁,τ₂) then, the union all (1,2)-p-open sets contained in A is called (1,2)-p-int(A)

2.8 Definition
Let A be a subset of a bitopological space (X,τ₁,τ₂) then, the intersection all (1,2)-p-closed sets containing in A is called (1,2)-p-cl(A)

2.9 Definition
Let A be a subset of a bitopological space (X,τ₁,τ₂) then, A subset N of a bitopological space (X,τ₁,τ₂) is called (1,2)-p- neighbourhood of a subset A of
X if there exists on (1,2)-p-open set U such that 

\[ A \subseteq U \subseteq N \]

### 2.10 Definition

Let A be a subset of a bitopological space \((X, \tau_1, \tau_2)\) then, (1,2)-\(\gamma\)-open set if for any non empty (1,2)-p-open set B such that 

\[ A \cap B \subseteq \text{int}_1(cl_2(A \cap B)) \]

#### 3 (1,2)-j-open sets

### 3.1 Definition

Let A be a subset of a bitopological space \((X, \tau_1, \tau_2)\) then, A is said to be (1,2)-j-open set if 

\[ A \subseteq \text{int}_1(Pcl_2(A)). \]

### 3.2 Definition

Let \((X, \tau_1, \tau_2)\) be a bitopological space, \( A \subseteq X \), A is said to be (2,1)-j-open set if 

\[ A \subseteq \text{int}_2(Pcl_1(A)). \]

### 3.3 Definition

Let \((X, \tau_1, \tau_2)\) be a bitopological space, \( A \subseteq X \), A is said to be 1-j-open set if 

\[ A \subseteq \text{int}_1(Pcl_2(A)). \]

### 3.4 Definition

Let \((X, \tau_1, \tau_2)\) be a bitopological space, \( A \subseteq X \), A is said to be 2-j-open set if 

\[ A \subseteq \text{int}_2(Pcl_1(A)). \]

### 3.5 Definition

Let \( X = \{a, b, c\} \), \( \tau_1 = \{\phi, X, \{b\}, \{b, c\}\} \), \( \tau_2 = \{\phi, X, \{a\}, \{a, c\}, \{c\}\} \) then (1,2)-jO(X)=\{\phi, X, \{b\}, \{a\}, \{b, c\}, \{c\}\} and (2,1)-jO(X)=\{\phi, X, \{a\}, \{a, c\}, \{c\}\}. It is clear that \{b\} is (1,2)-jO(X) but not 2-jO(X) and \{a\} is 2-jO(X) but not (1,2)-jO(X)

### 3.6 Example

Let \( X = \{a, b, c\} \), \( \tau_1 = \{\phi, X, \{b\}, \{b, c\}\} \), \( \tau_2 = \{\phi, X, \{a\}, \{a, c\}, \{c\}\} \) then (1,2)-jO(X)=\{\phi, X, \{b\}, \{a\}, \{b, c\}, \{c\}\} and 1-jO(X)=\{\phi, X, \{b\}, \{a\}, \{b, c\}\} It is clear that \{c\} is (1,2)-jO(X) but not 1-jO(X) and \{a\} is 1-jO(X) but not (1,2)-jO(X)

### 3.7 Example

Let \( X = \{a, b, c\} . \tau_1 = \{\phi, X, \{b\}, \{b, c\}\} \), \( \tau_2 = \{\phi, X, \{a\}, \{a, c\}, \{c\}\} \) then (1,2)-jO(X)=\{\phi, X, \{b\}, \{a\}, \{b, c\}, \{c\}\} and 2-jO(X)=\{\phi, X, \{a\}, \{a, c\}, \{c\}\} It is clear that \{b\} is (1,2)-jO(X) but not 2-jO(X) and \{a\} is 2-jO(X) but not (1,2)-jO(X)

### 3.8 Theorem

Every 1-open sets is (1,2)-j-open

Every 2-open sets is (2,1)-j-open

#### Proof

Let A be any 1-open set in bitopological space \((X, \tau_1, \tau_2)\) then \( A = \text{int}_1(A) \) and also \( A \subseteq Pcl_2(A) \).

Therefore, \( A \subseteq \text{int}_1(Pcl_2(A)) \).

Let A be any 2-open set in bitopological space \((X, \tau_1, \tau_2)\) then \( A = \text{int}_2(A) \) and also \( A \subseteq Pcl_2(A) \).

Therefore, \( A \subseteq \text{int}_2(Pcl_2(A)) \).

### 4 Interior, Closure and neighbourhood in bitopological spaces

#### 4.1 Definition

Let A be a subset of a bitopological space \((X, \tau_1, \tau_2)\) then, the union all (1,2)-j-open sets contained in A is called (1,2)-j-int(A).

#### 4.2 Definition

Let A be a subset of a bitopological space \((X, \tau_1, \tau_2)\) then, the intersection all (1,2)-j-closed sets containing in A is called (1,2)-j-cl(A).

#### 4.3 Definition

Let A be a subset of a bitopological space \((X, \tau_1, \tau_2)\) then, a subset N of a bitopological space \((X, \tau_1, \tau_2)\) is called (1,2)-j-neighbourhood of \( A \) if there exists on (1,2)-j-open set U such that \( A \subseteq U \subseteq N \)

#### 4.4 Theorem

For any subset A & B of a bitopological space X, the following statements are true:

(i) The (1,2)-j-int(A) is the largest (1,2)-j-open set contained in A.
(ii) The (1,2)-j-int(A) is an (1,2)-j-open set in X contained in A. 

(iii) A is an (1,2)-j-open set iff A=(1,2)-j-int(A) 

(iv) (1,2)-j-int (φ)= φ 

(v) (1,2)-j-int (X)= X 

(vi) If A ⊆ B, the (1,2)-j-int (A) ⊆ (1,2)-j-int (B) 

(vii) (1,2)-j-int (A ∩ B) ⊆ (1,2)-j-int (A) ∩ (1,2)-j-int (B) 

(viii) (1,2)-j-int (A) ∪ (1,2)-j-int (B) ⊆ (1,2)-j-int (A ∪ B) 

Proof 

(i) Since (1,2)-j-int(A) = ∪ {G:G is (1,2)-open, GcA} contains every j-open subset G of A. It is therefore largest open subset of A.

(ii) Since (1,2)-j-int(A) is the largest (1,2)-j-open set of A and (1,2)-j-int(A) = ∪ {G:G is (1,2)-j-open set of A} so (1,2)-j-int(A) is an (1,2)-j-open set of A.

(iii) Let A=(1,2)-j-int(A) and since (1,2)-j-int(A) is an (1,2)-j-open set of A. So, A is also (1,2)-j-open.

Let A is (1,2)-j-open then A is largest (1,2)-j-open set of A. Hence A=(1,2)-j-int(A).

(vi) Let A ⊆ B. Let x∈ (1,2)-j-int (A), ∃ a (1,2)-j-open set G such that x∈ G ⊆ A. Since A ⊆ B, B is also have x∈ G ⊆ B which implies x∈ (1,2)-j-int (A). Thus x∈ (1,2)-j-int (A) ⇒ x∈ (1,2)-j-int (B). Therefore, (1,2)-j-int (A) ⊆ (1,2)-j-int (B).

(vii) Let x∈ (1,2)-j-int (A ∩ B), ∃ a (1,2)-j-open set G in X such that x∈ (A ∩ B), x∈ A and x∈ B. x∈ G ⊆ A and x∈ G ⊆ B. So, x∈ (1,2)-j-int (A) and x∈ (1,2)-j-int (B). Therefore, (1,2)-j-int (A) ∩ (1,2)-j-int (B).

(viii) Let x∈ (1,2)-j-int (A) ∪ (1,2)-j-int (B), x∈ (1,2)-j-int (A) or x∈ (1,2)-j-int (B). If x∈ (1,2)-j-int (A), ∃ (1,2)-j-open set G in X such that x∈ G ⊆ A ⇒ x∈ G ⊆ A ∪ B ⇒ x∈ (1,2)-j-int (A ∪ B). Therefore, (1,2)-j-int (A) ∩ (1,2)-j-int (B) ⊆ (1,2)-j-int (A ∪ B).

4.5 Example 

(1,2)-j-int(A) ∩ (1,2)-j-int(B) ≠ (1,2)-j-int(A ∩ B) 

Let X={a,b,c}, τ₁={φ, X, {a,b}} then (1,2)-j-int (A) = τ₂={φ, X, {a,b}}. If we take A={a,b} and B={b,c} then (1,2)-j-int (A) ∩ (1,2)-j-int (B) = {a,b,c} So (1,2)-j-int (A) ∩ (1,2)-j-int (B) ≠ (1,2)-j-int (A ∩ B).

4.6 Example 

(1,2)-j-int(A) ∩ (1,2)-j-int(B) ≠ (1,2)-j-int (A ∪ B) 

Let X={a,b,c}, τ₁={φ, X, {a}} then (1,2)-j-int (A) ≠ (1,2)-j-int (B). If we take A={a} and B={b} then (1,2)-j-int (A) = τ₂={φ, X, {a,b}} So (1,2)-j-int (A) ∩ (1,2)-j-int (B) = {a} ≠ (1,2)-j-int (X). Hence (1,2)-j-int (A) ∩ (1,2)-j-int (B) ≠ (1,2)-j-int (A ∪ B).

4.7 Lemma 

For any subset A we have 

(i) 1-int (A) ⊆ (1,2)-j-int (A), but (1,2)-j-int (A) ≠ 1-int (A). 

(ii) 2-int (A) ⊆ (1,2)-j-int (A), but (1,2)-j-int (A) ≠ 2-int (A).

Proof 

Follows from the fact that every 1-open set is (1,2)-j-open.

The converse of the above lemma is not true which is shown in the following example:

4.8 Example 

Let X={a,b,c}, τ₁={φ, X, {a}} then (1,2)-j-int (X) = τ₂={φ, X, {a,b,c}}. If we take A={a} and B = {b} then (1,2)-j-int (A) = τ₁={φ, X, {a}}. Hence (1,2)-j-int (A) ≠ (1,2)-j-int (A) ≠ 1-int (A).

5 (1,2)-j-closed sets 

5.1 Definition 

Let A is said to be (1,2)-j-closed if int₄ (Pcl₂(A)) ⊆ A.

5.2 Definition 

Let (X,τ₁,τ₂) be a bitopological space, A ⊆ X, A is said to be (2,1)-j-closed set if int₂ (Pcl₁(A)) ⊆ A.

5.3 Definition
Let \((X, \tau_1, \tau_2)\) be a bitopological space, \(A \subset X\), \(A\) is said to be 1-j-closed set if \(\text{int}_1(Pcl_1(A)) \subset A\).

5.4 Definition

Let \((X, \tau_1, \tau_2)\) be a bitopological space, \(A \subset X\), \(A\) is said to be 2-j-closed set if \(A \subset \text{int}_2(Pcl_2(A)) \subset A\).

5.5 Theorem

For any subset \(A\) and \(B\) of a bitopological space \(X\), the following statements are true:

(i) The \((1,2)\)-j-cl\((A)\) is the smallest \((1,2)\)-j-closed set containing \(A\).

(ii) \(A\) is an \((1,2)\)-j-closed set iff \(A= (1,2)\)-j-cl\((A)\).

(iii) \((1,2)\)-j-cl\(\{\phi\} = \phi\).

(iv) \((1,2)\)-j-cl\(X = X\).

(v) \(A \supseteq (1,2)\)-j-cl\((A)\).

(vi) If \(A \subseteq B\), then \((1,2)\)-j-cl\((A) \subseteq (1,2)\)-j-cl\((B)\).

(vii) \((1,2)\)-j-cl\((A \cap B) \subseteq (1,2)\)-j-cl\((A) \cap (1,2)\)-j-cl\((B)\).

(viii) \((1,2)\)-j-cl\((A) \cup (1,2)\)-j-cl\((B) \subseteq (1,2)\)-j-cl\((A \cup B)\).

Proof

(i) \((1,2)\)-j-cl\((A)\) is intersection of all \((1,2)\)-j-closed set containing \(A\) so \((1,2)\)-j-cl\((A)\) is the smallest \((1,2)\)-j-closed set containing \(A\).

(ii) If \(A\) is \((1,2)\)-j-closed set, then \(A\) itself is the smallest \((1,2)\)-j-closed set containing \(A\). Hence \((1,2)\)-j-cl\((A) = A\).

Conversely,

If \((1,2)\)-j-cl\((A) = A\). Since \((1,2)\)-j-cl\((A)\) is \((1,2)\)-j-closed and so \(A\) is also \((1,2)\)-j-closed.

(v) Since \((1,2)\)-j-cl\((A)\) is the smallest \((1,2)\)-j-closed set containing \(A\), \(A \subseteq (1,2)\)-j-cl\((A)\).

(vi) Let \(A \subset B\), and by the pervious theorem we have \(B \subset (1,2)\)-j-cl\((B)\). Since \(A \subset B\), we have \(A \subset (1,2)\)-j-cl\((B)\) but \((1,2)\)-j-cl\((B)\) is \((1,2)\)-j-closed set. Thus \((1,2)\)-j-cl\((B)\) is \((1,2)\)-j-closed set containing \(A\). Since \((1,2)\)-j-cl\((A)\) is the smallest \((1,2)\)-j-closed set containing \(A\), we have \((1,2)\)-j-cl\((A) \subseteq (1,2)\)-j-cl\((B)\) so, \(A \subseteq B \Rightarrow (1,2)\)-j-cl\((A) \subseteq (1,2)\)-j-cl\((B)\).

(vii) \(A \cap B \subset A \Rightarrow (1,2)\)-j-cl\((A \cap B) \subset (1,2)\)-j-cl\((A) \cap (1,2)\)-j-cl\((B)\).

Hence \((1,2)\)-j-cl\((A \cap B) \subseteq (1,2)\)-j-cl\((A) \cap (1,2)\)-j-cl\((B)\).

5.2 Example

\((1,2)\)-j-cl\((A) \cap (1,2)\)-j-cl\((B) \neq (1,2)\)-j-cl\((A \cap B)\)

Let \(X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a, b\}, \{a, c\}\}, \tau_2 = \{\phi, X, \{a, b\}\}\) then \((1,2)\)-j-cl\((X) = \{a, b, c\}, (1,2)\)-j-cl\((X) = \{a, b, c\}\), (1,2)-j-cl\((X) = \{a, b, c\}\). If we take \(A = \{a, b\}\) and \(B = \{b, c\}\) then \((1,2)\)-j-cl\((A) = X\) and \((1,2)\)-j-cl\((B) = X\). So \((1,2)\)-j-cl\((A) \cap (1,2)\)-j-cl\((B) = \{a, b\}, (1,2)\)-j-cl\((A \cap B) = (1,2)\)-j-cl\((\{b\}) = \{b\}\). Hence \((1,2)\)-j-cl\((A) \cap (1,2)\)-j-cl\((B) \neq (1,2)\)-j-cl\((A \cap B)\).

5.3 Example

\((1,2)\)-j-cl\((A) \cup (1,2)\)-j-cl\((B) \neq (1,2)\)-j-cl\((A \cup B)\)

Let \(X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a, b\}, \{a, c\}\}, \tau_2 = \{\phi, X, \{a, b\}\}\) then \((1,2)\)-j-cl\((X) = \{a, b, c\}, (1,2)\)-j-cl\((X) = \{a, b, c\}\), (1,2)-j-cl\((X) = \{a, b, c\}\). If we take \(A = \{a\}\) and \(B = \{b\}\) then \((1,2)\)-j-cl\((A) = \{a\}\) and \((1,2)\)-j-cl\((B) = \{b\}\) so, \((1,2)\)-j-cl\((A) \cup (1,2)\)-j-cl\((B) = \{a, b\}, (1,2)\)-j-cl\((A \cup B) = (1,2)\)-j-cl\((\{a, b\}) = X\). Hence \((1,2)\)-j-cl\((A) \cup (1,2)\)-j-cl\((B) \neq (1,2)\)-j-cl\((A \cup B)\).

5.4 Proposition

A subset \(A\) is \((1,2)\)-j-closed iff \(Pcl_1 \text{int}_2(A) \subset A\).

Proof

Suppose that \(A\) is \((1,2)\)-j-closed set in a bitopological space \(X\), the \(X \setminus A\) is \((1,2)\)-j-open. Hence \(X \setminus A \subset \text{int}_1Pcl_2(X \setminus A)\). But \(Pcl_2(X \setminus A) = X \setminus \text{Pint}_2(A)\), so \(X \setminus A \subset \text{int}_1(Pcl_2(X \setminus A) \subset \text{int}_2(A)\). Again \(\text{Pint}_1(X \setminus \text{int}_2(A)) = X \setminus Pcl_1 \text{int}_2(A)\). Therefore, we get \(X \setminus A \subset X \setminus Pcl_1 \text{int}_2(A)\). Taking complement of both sides, we obtain \(Pcl_1 \text{int}_2(A) \subset A\).

Conversely,

Suppose the \(Pcl_1 \text{int}_2(A) \subset A\), then by taking complement of both sides we obtain \(X \setminus A \subset Pcl_1 \text{int}_2(X \setminus A)\) which implies that \(X \setminus A\) is \((1,2)\)-j-open. Hence \(A\) is \((1,2)\)-j-closed.
5.5 Proposition

Every 1-closed subset of a bitopological space \((X,\tau_1, \tau_2)\) is (1,2)-j-closed

**Proof**

Let \(A\) be 1-closed subset of bitopological space \((X,\tau_1, \tau_2)\). Then \(A=\text{Pcl}_1(A)\).

Hence \(\text{Pcl}_1\text{int}_2(A) \subseteq A\), so \(A\) is (1,2)-j-closed.

5.6 Proposition

Every 2-closed subset of a bitopological space \((X,\tau_1, \tau_2)\) is (2,1)-j-closed

**Proof**

Let \(A\) be 2-closed subset of bitopological space \((X,\tau_1, \tau_2)\). Then \(A=\text{Pcl}_2(A)\).

Hence \(\text{Pcl}_1\text{int}_2(A) \subseteq A\), so \(A\) is (2,1)-j-closed.

References


