On Neutrosophic Semi-Open sets in Neutrosophic Topological Spaces

P. Iswarya#1, Dr. K. Bageerathi*2

#Assistant Professor, Department of Mathematics, Govindammal Aditanar College for Women, Tiruchendur, (T N), INDIA

*Assistant Professor, Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur, (T N), INDIA

Abstract - The purpose of this paper is to define the product related neutrosophic topological space and proved some theorems based on this. We introduce the concept of neutrosophic semi-open sets and neutrosophic semi-closed sets in neutrosophic topological spaces and derive some of their characterizations. Finally, we analyze neutrosophic semi-interior and neutrosophic semi-closure operators also.

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INTRODUCTION

Theory of Fuzzy sets [17], Theory of Intuitionistic fuzzy sets [2], Theory of Neutrosophic sets [9] and the theory of Interval Neutrosophic sets [11] can be considered as tools for dealing with uncertainties. However, all of these theories have their own difficulties which are pointed out in [9]. In 1965, Zadeh [17] introduced fuzzy set theory as a mathematical tool for dealing with uncertainties where each element had a degree of membership. The Intuitionistic fuzzy set was introduced by Atanassov [2] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. The Neutrosophic set was introduced by Smarandache [9] and explained, neutrosophic set is a generalization of Intuitionistic fuzzy set. In 2012, Salama, Alblowi [15], introduced the concept of Neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of Intuitionistic fuzzy topological space and a Neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element.

This paper consists of six sections. The section I consists of the basic definitions and some properties which are used in the later sections. The section II, we define product related neutrosophic topological space and proved some theorem related to this definition. The section III deals with the definition of neutrosophic semi-open set in neutrosophic topological spaces and its various properties. The section IV deals with the definition of neutrosophic semi-closed set in neutrosophic topological spaces and its various properties. The section V and VI are dealt with the concepts of neutrosophic semi-interior and neutrosophic semi-closure operators.

I. PRELIMINARIES

In this section, we give the basic definitions for neutrosophic sets and its operations.

Definition 1.1 [15] Let X be a non-empty fixed set. A neutrosophic set [ NS for short ] A is an object having the form A = { ( x, μA(x), σA(x), γA(x)) : x ∈ X } where μA(x), σA(x) and γA(x) which represents the degree of membership function, the degree
indeterminacy and the degree of non-membership function respectively of each element \( x \in X \) to the set \( A \).

**Remark 1.2** [15] A neutrosophic set \( A = \{ (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X \} \) can be identified to an ordered triple \( \langle \mu_A, \sigma_A, \gamma_A \rangle \) in \( 0, 1 \ast \) on \( X \).

**Remark 1.3** [15] For the sake of simplicity, we shall use the symbol \( A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle \) for the neutrosophic set \( A = \{ (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X \} \).

**Example 1.4** [15] Every IFS \( A \) is a non-empty set in \( X \) is obviously an NS on \( X \) having the form

\[
A = \{ (x, \mu_A(x), 1 - \mu_A(x) + \gamma_A(x), \gamma_A(x)) : x \in X \}.
\]

Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the NS \( 0_N \) and \( 1_N \) in \( X \) as follows:

- **Definition 1.6** [15] Let \( A = \langle \mu_A, \sigma_A, \gamma_A \rangle \) be a NS on \( X \), then the complement of the set \( A \) \( \complement A \) (for short) may be defined as three kinds of complements:
  - (C1) \( \complement A = \{ (x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \gamma_A(x)) : x \in X \} \)
  - (C2) \( \complement A = \{ (x, \gamma_A(x), \sigma_A(x), \mu_A(x)) : x \in X \} \)
  - (C3) \( \complement A = \{ (x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)) : x \in X \} \)

Proposition 1.7 [15] For any neutrosophic set \( A \), then the following conditions are holds:
1. \( 0_N \subseteq A, 0_N \subseteq 0_N \)
2. \( A \subseteq 1_N, 1_N \subseteq 1_N \)

**Definition 1.8** [15] Let \( X \) be a non-empty set, and \( A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle, B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle \) are NSs. Then

1. \( A \cap B \) may be defined as:
   - (i) \( A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x) \rangle \)
   - (ii) \( A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x) \rangle \)

2. \( A \cup B \) may be defined as:
   - (i) \( A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x) \rangle \)
   - (ii) \( A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x) \rangle \)

We can easily generalize the operations of intersection and union in Definition 1.8 to arbitrary family of NSs follows as:

**Definition 1.9** [15] Let \( \{ A_j : j \in J \} \) be an arbitrary family of NSs in \( X \), then
1. \( \cap A_j \) may be defined as:
   - (i) \( \cap A_j = \langle x, \bigwedge_{j \in J} \mu_{A_j}(x), \bigwedge_{j \in J} \sigma_{A_j}(x), \bigwedge_{j \in J} \gamma_{A_j}(x) \rangle \)
   - (ii) \( \cap A_j = \langle x, \bigwedge_{j \in J} \mu_{A_j}(x), \bigwedge_{j \in J} \sigma_{A_j}(x), \bigwedge_{j \in J} \gamma_{A_j}(x) \rangle \)

2. \( \cup A_j \) may be defined as:
   - (i) \( \cup A_j = \langle x, \bigvee_{j \in J} \mu_{A_j}(x), \bigvee_{j \in J} \sigma_{A_j}(x), \bigvee_{j \in J} \gamma_{A_j}(x) \rangle \)
   - (ii) \( \cup A_j = \langle x, \bigvee_{j \in J} \mu_{A_j}(x), \bigvee_{j \in J} \sigma_{A_j}(x), \bigvee_{j \in J} \gamma_{A_j}(x) \rangle \)

Proposition 1.10 [15] For all \( A \) and \( B \) are two neutrosophic sets then the following conditions are true:
1. \( C(A \cap B) = C(A) \cup C(B) \)
2. \( C(A \cup B) = C(A) \cap C(B) \).

Here we extend the concepts of fuzzy topological space [5] and intuitionistic fuzzy topological space [6,7] to the case of neutrosophic sets.

**Definition 1.11** [15] A neutrosophic topology \( \{ NT \} \) on \( X \) is a family \( \tau \) of neutrosophic subsets in \( X \) satisfying the following axioms:
1. \( 0_N \subseteq 1_N, 1_N \subseteq \tau \)
2. \( G_1 \cap G_2 \in \tau \) for any \( G_1, G_2 \in \tau \)
3. \( \bigcup \{ G_i : i \in I \} \subseteq \tau \)
In this case the pair \((X, \tau)\) is called a neutrosophic topological space [NTS for short]. The elements of \(\tau\) are called neutrosophic open sets [NOS for short]. A neutrosophic set \(F\) is closed if and only if \(C(F)\) is neutrosophic open.

**Example 1.12** [15] Any fuzzy topological space \((X, \tau_0)\) in the sense of Chang is obviously a NTS in the form \(\tau = \{ A : \mu_A \in \tau_0 \}\) wherever we identify a fuzzy set in \(X\) whose membership function is \(\mu_A\) with its counterpart.

**Remark 1.13** [15] Neutrosophic topological spaces are very natural generalizations of fuzzy topological spaces allowing more general functions to be members of fuzzy topology.

**Example 1.14** [15] Let \(X = \{ x \}\) and \(A = \{ (x, 0.5, 0.5, 0.4) : x \in X \}\)
\(B = \{ (x, 0.4, 0.6, 0.8) : x \in X \}\)
\(D = \{ (x, 0.5, 0.6, 0.4) : x \in X \}\)
\(C = \{ (x, 0.4, 0.5, 0.8) : x \in X \}\)
Then the family \(\tau = \{ 0_X, A, B, C, D, 1_X \}\) of NSs in \(X\) is neutrosophic topology on \(X\).

**Definition 1.15** [15] The complement of \(A [C(A)\) for short] of NOS is called a neutrosophic closed set \(NCS\) for short in \(X\).

Now, we define neutrosophic closure and neutrosophic interior operations in neutrosophic topological spaces:

**Definition 1.16** [15] Let \((X, \tau)\) be NTS and \(A = \{ (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) : x \in X \}\) be a NS in \(X\). Then the neutrosophic closure and neutrosophic interior of \(A\) are defined by
\(NCl(A) = \bigcap \{ K : K\ is \ a \ NCS\ in\ X\ and\ A \subseteq K \}\)
\(NInt(A) = \bigcup \{ G : G\ is\ a\ NOS\ in\ X\ and\ G \subseteq A \}\).

It can also be shown that \(NCl(A)\) is NCS and \(NInt(A)\) is a NOS in \(X\).

a) \(A\) is NOS if and only if \(A = NInt(A)\).

b) \(A\) is NCS if and only if \(A = NCl(A)\).

**Proposition 1.17** [15] For any neutrosophic set \(A\) in \((X, \tau)\) we have:
(a) \(NCl\ (C(A)) = C\ (NCl\ (A))\),
(b) \(NInt\ (C(A)) = C\ (NCl\ (A))\).

**Proposition 1.18** [15] Let \((X, \tau)\) be a NTS and \(A, B\) be two neutrosophic sets in \(X\). Then the following properties are holds:
(a) \(NInt\ (A) \subseteq A\),
(b) \(A \subseteq NCl\ (A)\).

(c) \(A \subseteq B \Rightarrow NInt\ (A) \subseteq NInt\ (B)\),
(d) \(A \subseteq B \Rightarrow NCl\ (A) \subseteq NCl\ (B)\),
(e) \(NInt\ (NInt\ (A)) = NInt\ (A)\),
(f) \(NCl\ (NCl\ (A)) = NCl\ (A)\),
(g) \(NInt\ (A \cap B) = NInt\ (A) \cap NInt\ (B)\),
(h) \(NCl\ (A \cup B) = NCl\ (A) \cup NCl\ (B)\),
(i) \(NInt\ (0_X) = 0_X\),
(j) \(NInt\ (1_X) = 1_X\),
(k) \(NCl\ (0_X) = 0_X\),
(l) \(NCl\ (1_X) = 1_X\),
(m) \(A \subseteq B \Rightarrow C (B) \subseteq C (A)\),
(n) \(NCl\ (A \cap B) \subseteq NCl\ (A) \cap NCl\ (B)\),
(o) \(NInt\ (A \cup B) \supseteq NInt\ (A) \cup NInt\ (B)\).

**II. PRODUCT RELATED NEUTROSOPHIC TOPOLOGICAL SPACES**

In this section, we define some basic and important results which are very useful in later sections. In order topology, the product of the closure is equal to the closure of the product and product of the interior is equal to the interior of the product. But this result is not true in neutrosophic topological space. For this reason, we define the product related neutrosophic topological space. Using this definition, we prove the above mentioned result.

**Definition 2.1** A subfamily \(\beta\) of NTS \((X, \tau)\) is called a base for \(\tau\) if each NS of \(\tau\) is a union of some members of \(\beta\).

**Definition 2.2** Let \(X, Y\) be nonempty neutrosophic sets and \(A = \{ (x, \mu_A(x), \sigma_A(x), \gamma_A(x)) \}, B = \{ (y, \mu_B(y), \sigma_B(y), \gamma_B(y)) \}\) NSs of \(X\) and \(Y\) respectively. Then \(A \times B\) is a NS of \(X \times Y\) is defined by
\(\text{(P}_1\text{)} (A \times B) (x, y) = (x, y), \min (\mu_A(x), \mu_B(y)), \min (\sigma_A(x), \sigma_B(y)), \max (\gamma_A(x), \gamma_B(y))\)
\(\text{(P}_2\text{)} (A \times B) (x, y) = (x, y), \min (\mu_A(x), \mu_B(y)), \max (\sigma_A(x), \sigma_B(y)), \min (\gamma_A(x), \gamma_B(y))\)

Notice that
\(\text{(CP}_1\text{)} C ((A \times B) (x, y)) = (x, y), \max (\mu_A(x), \mu_B(y)), \max (\sigma_A(x), \sigma_B(y)), \min (\gamma_A(x), \gamma_B(y))\)
\(\text{(CP}_2\text{)} C ((A \times B) (x, y)) = (x, y), \max (\mu_A(x), \mu_B(y)), \min (\sigma_A(x), \sigma_B(y)), \min (\gamma_A(x), \gamma_B(y))\)

**Lemma 2.3** If \(A\) is the NS of \(X\) and \(B\) is the NS of \(Y\), then
(i) \((A \times 1_X) \cap (1_Y \times B) = A \times B\),
(ii) \((A \times 1_X) \cup (1_Y \times B) = C (C (A) \times C (B))\),
(iii) \(C (A \times B) = (C (A) \times 1_X) \cup (1_Y \times C (B))\).
Proof: Let \( A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle, B = \langle y, \mu_B(y), \sigma_B(y), \gamma_B(y) \rangle \).

(i) Since \( A \times 1_N = \langle x, \min (\mu_A, 1_N), \min (\sigma_A, 1_N), \min (\gamma_A, 1_N) \rangle \),
max \( (\gamma_A, 0_N) \rangle = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle = A \) and
similarly \( 1_N \times B = \langle y, \min (\mu_B, 1_N), \min (\sigma_B, 1_N), \min (\gamma_B, 1_N) \rangle = B \), we have \((A \times 1_N) \cap (1_N \times B) = A \cap B \).

(ii) Similarly to (i).

(iii) Obvious by putting A, B instead of \( C(A), C(B) \) in (ii).

Definition 2.4 Let X and Y be two nonempty neutrosophic sets and \( f : X \rightarrow Y \) be a neutrosophic function. (i) If \( B = \{ (y, \mu_B(y), \sigma_B(y), \gamma_B(y)) : y \in Y \} \) is a NS in Y, then the pre image of B under \( f \) is denoted and defined by \( f^{-1}(B) = \{ (x, f^{-1}(\mu_B(x)), f^{-1}(\sigma_B(x)), f^{-1}(\gamma_B(x))) : x \in X \} \).

(ii) If \( A = \{ (x, \alpha_A(x), \delta_A(x), \lambda_A(x)) : x \in X \} \) is a NS in X, then the image of A under \( f \) is denoted and defined by \( f(A) = \{ (y, f(\alpha_A(x)), f(\delta_A(x)), f(\lambda_A(x))) : y \in Y \} \) where \( f(\lambda_A) = C(f(\alpha_A(x))) \).

In (i), (ii), since \( \mu_B, \sigma_B, \gamma_B, \alpha_A, \delta_A, \lambda_A \) are neutrosophic sets, we explain that \( f^{-1}(\mu_B(x)) = \mu(f(x)), f^{-1}(\sigma_B(x)) = \sigma(f(x)) \), and \( f(\alpha_A) = \sup \alpha_A(x) \) if \( x \in f^{-1}(y) \) .

Definition 2.5 Let \((X, \tau) \) and \((Y, \sigma)\) be NTSSs. The neutrosophic product topological space \([NTPTS \) for short\] \( ) \) of \((X, \tau) \) and \((Y, \sigma)\) is the cartesian product \( X \times Y \) of \( X \) and \( Y \) taken together with the \( NT \) \( \xi \) of \( X \times Y \) which is generated by the family \( \{P^{-1}_1(A), P^{-1}_2(B) : A \in \tau, B \in \sigma \} \).

Remark 2.6 In the above definition, since \( P^{-1}_1(A) = A_1 \times 1_N \) and \( P^{-1}_2(B) = 1_N \times B \) and \( A_1 \times 1_N \cap 1_N \times B = A_1 \times B \), the family \( \beta = \{ A_1 \times B_1 : A_1 \in \tau, B_1 \in \sigma \} \) forms a base for \( NTPTS \xi \) of \( X \times Y \).
Theorem 2.14 If A and B are NSs of NSTs X and Y respectively, then
(i) NCI (A × NCI (B) ⊃ NCI (A × B)) ,
(ii) NInt (A × NInt (B) ⊆ NInt (A × B) .

Proof : (i) Since A ⊆ NCI (A) and B ⊆ NCI (B), hence A × B ⊆ NCI (A × NCI (B)). This implies that NCI (A × B) ⊆ NCI (A × NCI (B)) and from Lemma 2.13, NCI (A × B) ⊆ NCI (A × NCI (B)). (ii) follows from (i) and the fact that NInt (C (A)) = C (NCI (A)).

Definition 2.15 Let (X, τ), (Y, σ) be NSTs and A ∈ τ, B ∈ σ. We say that (X, τ) is neutrosophic product related to (Y, σ) if for any NSs C of X and D of Y, whenever C (A) ⊈ C and (B) ⊈ D ⇒ C (A) × NCI (A) ⊇ 1 × C (B) ⊇ C × D , there exist A ∈ τ, B ∈ σ such that C (A) ⊇ C or C (B) ⊇ D and C (A) × NCI (A) ⊇ 1 × C (B) = C (A) × 1 × 1 × C (B).

Lemma 2.16 For NSs A, B of NSTs X and Y respectively, we have
(i) ∩ {A_i, B_j} = min (∩ A_i, ∩ B_j) ;
   ∪ {A_i, B_j} = max (∪ A_i, ∪ B_j).
(ii) ∩ {A_i, 1_j} = (∩ A_i) × 1_N ;
   ∪ {A_i, 1_j} = (∪ A_i) × 1_N.
(iii) ∩ {1_i × B_j} = 1_N × (∩ B_j) ;
   ∪ {1_i × B_j} = 1_N × (∪ B_j).

Proof : Obvious.

Theorem 2.17 Let (X, τ) and (Y, σ) be NSTs such that X is neutrosophic product related to Y. Then for NSs A of X and B of Y, we have
(i) NCI (A × B) = NCI (A) × NCI (B) ,
(ii) NInt (A × B) = NInt (A) × NInt (B).

Proof : (i) Since NCI (A × B) ⊆ NCI (A × NCI (B) (By Theorem 2.14) it is sufficient to show that NCI (A × B) ⊃ NCI (A) × NCI (B). Let A_i ∈ τ and B_j ∈ σ. Then NCI (A × B) = (x, y) ∩ C (A_i × B_j) :
   C (A_i × B_j) ⊇ A_i × B_j = C (A_i) × B_j ⊇ C (A_i × B_j) :
   C (A_i) × NCI (A) ⊇ A_i × 1_N .
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III. NEUTROSOPIHC SEMI-OPEN SETS IN NEUTROSOPIHC TOPOLOGICAL SPACES

In this section, the concepts of the neutrosophic semi-open set is introduced and also discussed their characterizations.

Definition 3.1 Let A be NS of a NST X. Then A is said to be neutrosophic semi-open [written NSO] set of X if there exists a neutrosophic open set NO such that NO ⊆ A ⊆ NCI (NO).

The following theorem is the characterization of NSO set in NST.

Theorem 3.2 A subset A in a NST X is NSO set if and only if A ⊆ NCI (NInt (A)).

Proof : Sufficiency: Let A ⊆ NCI (NInt (A)). Then for NO = NInt (A), we have NO ⊆ A ⊆ NCI (NO). Necessity: Let A be NSO set in X. Then NO ⊆ A ⊆ NCI (NO) for some neutrosophic open set NO. But NO ⊆ NInt (A) and thus NCI (NO) ⊆ NCI (NInt (A)). Hence A ⊆ NCI (NO) ⊆ NCI (NInt (A)).

Theorem 3.3 Let (X, τ) be a NST. Then union of two NSO sets is a NSO set in the NST X.

Proof : Let A and B are NSO sets in X. Then A ⊆ NCI (NInt (A)) and B ⊆ NCI (NInt (B)). Therefore A ∪ B ⊆ NCI (NInt (A)) ∪ NCI (NInt (B)) = NCI (NInt (A) ∪ NInt (B)) ⊆ NCI (NInt (A ∪ B)) [By Proposition 1.18 (a)] . Hence A ∪ B is NSO set in X.

Theorem 3.4 Let (X, τ) be a NST. If \{A_a\}_a∈Δ is a collection of NSO sets in a NST X. Then \_a∈Δ A_a is NSO set in X.

Proof : For each \( a ∈ Δ \), we have a neutrosophic open set NO_a such that NO_a ⊆ A_a ⊆ NCI (NO_a). Then \_a∈Δ NO_a ⊆ \_a∈Δ A_a ⊆ \_a∈Δ NCI (NO_a) ⊆ NCI (\_a∈Δ NO_a) . Hence let NO = \_a∈Δ NO_a .
**Remark 3.5** The intersection of any two NSO sets need not be a NSO set in X as shown by the following example.

**Example 3.6** Let \( X = \{ a, b \} \) and 
\[
A = \langle (0.3, 0.5, 0.4), (0.6, 0.2, 0.5) \rangle \\
B = \langle (0.2, 0.6, 0.7), (0.5, 0.3, 0.1) \rangle \\
C = \langle (0.3, 0.6, 0.4), (0.6, 0.3, 0.1) \rangle \\
D = \langle (0.2, 0.5, 0.7), (0.5, 0.2, 0.5) \rangle.
\]
Then \( \tau = \{ 0_N, A, B, C, D, 1_N \} \) is NTS on X. Now, we define the two NSO sets as follows:

\[ A_1 = \langle (0.4, 0.6, 0.4), (0.8, 0.3, 0.4) \rangle \] and 
\[ A_2 = \langle (1, 0.9, 0.2), (0.5, 0.7, 0.0) \rangle. \] Here \( NInt(A_1) = A, \ NC\{NInt(A_1)\} = 1_N \) and \( NInt(A_2) = B, \ NC\{NInt(A_2)\} = 1_N. \) But \( A_1 \cap A_2 = \langle (0.4, 0.6, 0.4), (0.5, 0.3, 0.4) \rangle \) is not a NSO set in X.

**Theorem 3.7** Let \( A \) be NSO set in the NTS X and suppose \( A \subseteq B \subseteq NCI(A) \). Then \( B \) is NSO set in X.

**Proof:** There exists a neutrosophic open set NO such that \( NO \subseteq A \subseteq NCI(NO) \). Then \( NO \subseteq B \). But \( NCI(A) \subseteq NCI(NO) \) and thus \( B \subseteq NCI(NO) \). Hence \( NO \subseteq B \subseteq NCI(NO) \) and \( B \) is NSO set in X.

**Theorem 3.8** Every neutrosophic open set in the NTS X is NSO set in X.

**Proof:** Let \( A \) be neutrosophic open set in NTS X. Then \( A = NInt(A) \). Also \( NInt(A) \subseteq NCI(NInt(A)) \). This implies that \( A \subseteq NCI(NInt(A)) \). Hence by Theorem 3.2, \( A \) is NSO set in X.

**Remark 3.9** The converse of the above theorem need not be true as shown by the following example.

**Example 3.10** Let \( X = \{ a, b, c \} \) with \( \tau = \{ 0_N, A, B, 1_N \} \). Some of the NSO sets are 
\[
A = \langle (0.4, 0.5, 0.2), (0.3, 0.2, 0.1), (0.9, 0.6, 0.8) \rangle \\
B = \langle (0.2, 0.4, 0.5), (0.1, 0.1, 0.2), (0.6, 0.5, 0.8) \rangle \\
C = \langle (0.5, 0.6, 0.1), (0.4, 0.3, 0.1), (0.9, 0.8, 0.5) \rangle \\
D = \langle (0.3, 0.5, 0.4), (0.1, 0.6, 0.2), (0.7, 0.5, 0.8) \rangle \\
E = \langle (0.5, 0.6, 0.1), (0.4, 0.6, 0.1), (0.9, 0.8, 0.5) \rangle \\
F = \langle (0.3, 0.5, 0.4), (0.1, 0.3, 0.2), (0.7, 0.5, 0.8) \rangle \\
G = \langle (0.4, 0.5, 0.2), (0.3, 0.6, 0.1), (0.9, 0.6, 0.8) \rangle \\
H = \langle (0.3, 0.5, 0.4), (0.1, 0.2, 0.2), (0.7, 0.5, 0.8) \rangle \\
I = \langle (0.4, 0.5, 0.2), (0.3, 0.3, 0.1), (0.9, 0.6, 0.8) \rangle \\
J = \langle (0.3, 0.5, 0.4), (0.1, 0.2, 0.2), (0.7, 0.5, 0.8) \rangle.
\]

Here \( C, D, E, F, G, H, I \) and \( J \) are NSO sets but are not neutrosophic open sets.

**Proposition 3.11** If \( X \) and \( Y \) are NTS such that \( X \) is neutrosophic product related to \( Y \). Then the neutrosophic product \( A \times B \) of a neutrosophic semi-open set \( A \) of \( X \) and a neutrosophic semi-open set \( B \) of \( Y \) is a neutrosophic semi-open set of the neutrosophic product topological space \( X \times Y \).

**Proof:** Let \( O_1 \subseteq A \subseteq NCI(O_1) \) and \( O_2 \subseteq B \subseteq NCI(O_2) \) where \( O_1 \) and \( O_2 \) are neutrosophic open sets in \( X \) and \( Y \) respectively. Then, \( O_1 \times O_2 \subseteq A \times B \subseteq NCI(O_1 \times O_2) \). By Theorem 2.17 (i) , \( NCI(O_1) \times NCI(O_2) = NCI(O_1 \times O_2) \). Therefore \( O_1 \times O_2 \subseteq A \times B \subseteq NCI(O_1 \times O_2) \). Hence by Theorem 3.1, \( A \times B \) is neutrosophic semi-open set in \( X \times Y \).

**IV. NEUTROSOPHIC SEMI-CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES**

In this section, the neutrosophic semi-closed set is introduced and studied their properties.

**Definition 4.1** Let \( A \) be NS of a NTS X. Then \( A \) is said to be neutrosophic semi-closed [written NSC] set of \( X \) if there exists a neutrosophic closed set \( NC \) such that \( NInt(NC \subseteq A \subseteq NC) \).

**Theorem 4.2** A subset \( A \) in a NTS X is NSC set if and only if \( NInt(NC(A)) \subseteq A \).

**Proof:** Sufficiency: Let \( NInt(NCI(A)) \subseteq A \). Then for \( NC = NCI(A) \), we have \( NInt(NC) \subseteq A \subseteq NC \). Necessity: Let \( A \) be NSC set in X. Then \( NInt(NC) \subseteq A \subseteq NC \) for some neutrosophic closed set \( NC \). But \( NCI(A) \subseteq NC \) and thus \( NInt(NCI(A)) \subseteq NInt(NC) \). Hence \( NInt(NCI(A)) \subseteq NInt(NC) \subseteq A \).

**Proposition 4.3** Let \( (X, \tau) \) be a NTS and \( A \) be a neutrosophic subset of \( X \). Then \( A \) is NSC set if and only if \( C(A) \) is NSO set in X.

**Proof:** Let \( A \) be a neutrosophic semi-closed subset of \( X \). Then by Theorem 4.2 , \( NInt(NCI(A)) \subseteq A \). Taking complement on both sides, \( C(A) \subseteq C(NInt(NCI(A))) = NCI(C(NCI(A))) \). By using Proposition 1.17 (b), \( C(A) \subseteq NInt(NC(A)) \). By Theorem 3.2, \( C(A) \) is neutrosophic semi-open. Conversely let \( C(A) \) is neutrosophic semi-open. By Theorem 3.2,
C (A) ∈ NCI (NInt (C (A))). Taking complement on both sides, A ⊃ C (NCI (NInt (C (A))) = NInt (C (NInt (C (A)))). By using Proposition 1.17 (b), A ⊃ NInt (NCI (A)). By Theorem 4.2, A is neutrosophic semi-closed set.

Theorem 4.4 Let (X, τ) be a NTS. Then intersection of two NSC sets is a NSC set in the NTS X.

Proof : Let A and B are NSC sets in X. Then NInt (NCI (A)) ⊆ A and NInt (NCI (B)) ⊆ B. Therefore A∩B ⊃ NInt (NCI (A)) ∩ NInt (NCI (B)) = NInt (NCI (A) ∩ NCNI (B)) ⊃ NInt (NCI (A) ∩ NCNI (B)) [By Proposition 1.17 (n)]. Hence A∩B is NSC set in X.

Theorem 4.5 Let {Aα}α∈A be a collection of NSC sets in a NTS X. Then ∩α∈A Aα is NSC set in X.

Proof : For each α∈A, we have a neutrosophic closed set NCα such that NInt (NCα) ⊆ Aα ⊆ NCα. Then NInt (∩α∈A NCα) ⊆ ∩α∈A NInt (NCα) ⊆ ∩α∈A Aα ⊆ ∩α∈A NCα. Hence let NC = ∩α∈A NCα.

Remark 4.6 The union of any two NSC sets need not be a NSC set in X as shown by the following example.

Example 4.7 Let X = {a} and A = (1, 0.5, 0.7), B = (0, 0.9, 0.2), C = (1, 0.9, 0.2), D = (0, 0.5, 0.7). Then τ = {0N, A, B, C, D, 1N} is NTS on X. Now, we define the two NSC sets as follows:

A1 = (0.4, 0.5, 1) and A2 = (0.2, 0.0, 0.8). Here NC1 (A1) = (0.7, 0.5, 1), NInt (NC1 (A1)) = 0N and NC2 (A2) = (0.2, 0.1, 0). NInt (NC2 (A2)) = 0N. But A1 ∪ A2 = (0.4, 0.5, 0.8) is not a NSC set in X.

Theorem 4.8 Let A be NSC set in the NTS X and suppose NInt (A) ⊆ B ⊆ A. Then B is NSC set in X.

Proof : There exists a neutrosophic closed set NC such that NInt (NC) ⊆ A ⊆ NC. Then B ⊆ NC. But NInt (NC) ⊆ NInt (A) and thus NInt (NC) ⊆ B. Hence NInt (NC) ⊆ B ⊆ NC and B is NSC set in X.

Theorem 4.9 Every neutrosophic closed set in the NTS X is NSC set in X.

Proof : Let A be neutrosophic closed set in NTS X. Then A = NC1 (A). Also NInt (NC1 (A)) ⊆ NC1 (A). This implies that NInt (NC1 (A)) ⊆ A. Hence by Theorem 4.2, A is NSC set in X.

Remark 4.10 The converse of the above theorem need not be true as shown by the following example.

Example 4.11 Let X = {a, b, c} with τ = {0N, A, B, 1N} and C (τ) = {1N, C, D, 0N} where A = ((0.5, 0.6, 0.3), (0.1, 0.7, 0.9), (1, 0.6, 0.4)), B = ((0, 0.4, 0.7), (0.1, 0.6, 0.9), (0.5, 0.5, 0.8)), C = ((0.3, 0.4, 0.5), (0.9, 0.3, 0.1), (0.4, 0.4, 1)), D = ((0.7, 0.6, 0), (0.9, 0.4, 0.1), (0.8, 0.5, 0.5)). E = ((0.2, 0.4, 0.9), (0, 0.2, 0.9), (0.3, 0.2, 1)). Here the NSC sets are C, D and E. Also E is NSC set but is not neutrosophic closed set.

Proposition 4.12 If X and Y are neutrosophic spaces such that X is neutrosophic product related to Y. Then the neutrosophic product X × Y of a neutrosophic semi-closed set A of X and a neutrosophic semi-closed set B of Y is a neutrosophic semi-closed set of the neutrosophic product topological space X × Y.

Proof : Let NInt (C1) ⊆ A ⊆ C1 and NInt (C2) ⊆ B ⊆ C2 where C1 and C2 are neutrosophic closed sets in X and Y respectively. Then NInt (C1) × NInt (C2) ⊆ A × B ⊆ C1 × C2. By Theorem 2.17 (ii), NInt (C1) × NInt (C2) = NInt (C1 × C2). Therefore NInt (C1 × C2) ⊆ A × B ⊆ C1 × C2. Hence by Theorem 4.1, A × B is neutrosophic semi-closed set in X × Y.

V. NEUTROSOPHIC SEMI-INTERIOR IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section, we introduce the neutrosophic semi-interior operator and their properties in neutrosophic topological space.

Definition 5.1 Let (X, τ) be a NTS. Then for a neutrosophic subset A of X, the neutrosophic semi-interior of A [ NS Int (A) for short] is the union of all neutrosophic semi-open sets of X contained in A.

That is, NS Int (A) = ∪ {G : G is a NSO set in X and G ⊆ A}. 
Proposition 5.2 Let \((X, \tau)\) be a NTS. Then for any neutrosophic subsets \(A\) and \(B\) of a NTS \(X\) we have
(i) \(NS\ Int (A) \subseteq A\)
(ii) \(A\) is NSO set in \(X \iff NS\ Int (A) = A\)
(iii) \(NS\ Int (NS\ Int (A)) = NS\ Int (A)\)
(iv) If \(A \subseteq B\) then \(NS\ Int (A) \subseteq NS\ Int (B)\)
Proof : (i) follows from Definition 5.1.
Let \(A\) be NSO set in \(X\). Then \(A \subseteq NS\ Int (A)\). By using (i) we get \(A = NS\ Int (A)\). Conversely assume that \(A = NS\ Int (A)\). By using Definition 5.1, \(A\) is NSO set in \(X\). Thus (ii) is proved.
By using (ii), \(NS\ Int (NS\ Int (A)) = NS\ Int (A)\). This proves (iii).
Since \(A \subseteq B\), by using (i), \(NS\ Int (A) \subseteq A \subseteq B\). That is \(NS\ Int (A) \subseteq B\). By (iii), \(NS\ Int (NS\ Int (A)) \subseteq NS\ Int (B)\). Thus \(NS\ Int (A) \subseteq NS\ Int (B)\). This proves (iv).

Theorem 5.3 Let \((X, \tau)\) be a NTS. Then for any neutrosophic subset \(A\) and \(B\) of a NTS, we have
(i) \(NS\ Int (A \cap B) = NS\ Int (A) \cap NS\ Int (B)\)
(ii) \(NS\ Int (A \cup B) \supseteq NS\ Int (A) \cup NS\ Int (B)\)
Proof : Since \(A \cap B \subseteq A\) and \(A \cap B \subseteq B\), by using Proposition 5.2 (iv), \(NS\ Int (A \cap B) \subseteq NS\ Int (A) \cap NS\ Int (B)\). This implies that \(NS\ Int (A \cap B) \subseteq NS\ Int (A) \cap NS\ Int (B) \iff \ldots\). Using Proposition 5.2 (i), \(NS\ Int (A) \subseteq A\) and \(NS\ Int (B) \subseteq B\). This implies that \(NS\ Int (A) \cap NS\ Int (B) \subseteq A \cap B\). Now applying Proposition 5.2 (iv), \(NS\ Int ((NS\ Int (A) \cap NS\ Int (B)) \subseteq NS\ Int (A) \cap B)\). By (1), \(NS\ Int (NS\ Int (A)) \cap NS\ Int (NS\ Int (B)) \subseteq NS\ Int (A \cap B)\). By Proposition 5.2 (iii), \(NS\ Int (A) \cap NS\ Int (B) \subseteq NS\ Int (A \cap B) \iff \ldots\). From (1) and (2), \(NS\ Int (A \cap B) = NS\ Int (A) \cap NS\ Int (B)\). This implies (i).
Since \(A \subseteq A \cup B\) and \(B \subseteq A \cup B\), by using Proposition 5.2 (iv), \(NS\ Int (A) \subseteq NS\ Int (A \cup B)\) and \(NS\ Int (B) \subseteq NS\ Int (A \cup B)\). This implies that \(NS\ Int (A) \cup NS\ Int (B) \subseteq NS\ Int (A \cup B)\). Hence (ii).
The following example shows that the equality need not be hold in Theorem 5.3 (ii).

Example 5.4 Let \(X = \{ a, b, c \}\) and \(\tau = \{ 0_N, A, B, C, D, 1_N \}\) where
\[
A = \langle (0.4, 0.7, 0.1), (0.5, 0.6, 0.2), (0.9, 0.7, 0.3) \rangle, \\
B = \langle (0.4, 0.6, 0.1), (0.7, 0.7, 0.2), (0.9, 0.5, 0.1) \rangle, \\
C = \langle (0.4, 0.7, 0.1), (0.7, 0.7, 0.2), (0.9, 0.7, 0.1) \rangle, \\
D = \langle (0.4, 0.6, 0.1), (0.5, 0.6, 0.2), (0.9, 0.5, 0.3) \rangle.
\]
Then \((X, \tau)\) is a NTS. Consider the NSs are
\[
E = \langle (0.7, 0.6, 0.1), (0.7, 0.6, 0.1), (0.9, 0.5, 0) \rangle \\
\text{and } F = \langle (0.4, 0.6, 0.1), (0.5, 0.7, 0.2), (1, 0.7, 0.1) \rangle.
\]
Then \(NS\ Int (E) = D\) and \(NS\ Int (F) = D\). This implies that \(NS\ Int (E) \cup NS\ Int (F) = D\). Now,
\[
E \cup F = \langle (0.7, 0.6, 0.1), (0.7, 0.7, 0.1), (1, 0.7, 0) \rangle,
\]
it follows that \(NS\ Int (E \cup F) = B\). Then \(NS\ Int (E \cup F) \subseteq NS\ Int (E) \cup NS\ Int (F)\).

VI. NEUTROSOPHIC SEMI-CLOSURE IN NEUTROSOPHIC TOPOLOGICAL SPACES

In this section, we introduce the concept of neutrosophic semi-closure operators in a NTS.

Definition 6.1 Let \((X, \tau)\) be a NTS. Then for a neutrosophic subset \(A\) of \(X\), the neutrosophic semi-closure of \(A\) [\(NS\ Cl (A)\)] for short is the intersection of all neutrosophic semi-closed sets of \(X\) contained in \(A\). That is, \(NS\ Cl (A) = \cap \{ K : K\text{ is a NSC set in } X \text{ and } K \supseteq A \}\).

Proposition 6.2 Let \((X, \tau)\) be a NTS. Then for any neutrosophic subsets \(A\) of \(X\),
(i) \(C (NS\ Cl (A)) = NS\ Cl (C (A))\),
(ii) \(C (NS\ Cl (A)) = NS\ Cl (C (A))\).
Proof : By using Definition 5.1, \(NS\ Cl (A) = \cup \{ G : G\text{ is a NSO set in } X \text{ and } G \subseteq A \}\). Taking complement on both sides, \(C (NS\ Cl (A)) = C (\cup \{ G : G\text{ is a NSO set in } X \text{ and } G \subseteq A \}) = \cap \{ C (G) : C (G)\text{ is a NSC set in } X \text{ and } C (A) \subseteq C (G) \}.\) Replacing \(C (G)\) by \(K\), we get \(C (NS\ Cl (A)) = \cap \{ K : K\text{ is a NSC set in } X \text{ and } K \supseteq C (A) \}\). By Definition 6.1, \(C (NS\ Cl (A)) = NS\ Cl (C (A))\). This proves (i).
By using (i), \(C (NS\ Cl (C (A))) = NS\ Cl (C (C (A))) = NS\ Cl (A)\). Taking complement on both sides, we get \(NS\ Cl (C (A)) = C (NS\ Cl (A))\). Hence proved (ii).

Proposition 6.3 Let \((X, \tau)\) be a NTS. Then for any neutrosophic subsets \(A\) and \(B\) of a NTS \(X\) we have
(i) \(A \subseteq NS\ Cl (A)\)
(ii) \(A\) is NSC set in \(X \iff NS\ Cl (A) = A\)
(iii) \(NS\ Cl (NS\ Cl (A)) = NS\ Cl (A)\)
(iv) If \(A \subseteq B\) then \(NS\ Cl (A) \subseteq NS\ Cl (B)\)
Proof : (i) follows from Definition 6.1.
Let \(A\) be NSC set in \(X\). By using Proposition 4.3, \(C (A)\) is NSO set in \(X\). By Proposition 6.2 (ii), \(NS\ Cl (C (A)) = C (A) \subseteq C (NS\ Cl (A)) = C (A) \subseteq NS\ Cl (A) = A\). Thus proved (ii).
By using (ii), \(NS\ Cl (NS\ Cl (A)) = NS\ Cl (A)\). This proves (iii).
Since \(A \subseteq B\), \(C (B) \subseteq C (A)\). By using Proposition 6.2 (iv), \(NS\ Cl (C (B)) \subseteq NS\ Cl (C (A))\). Taking complement on both sides, \(C (NS\ Cl (C (B))) \subseteq C (NS\ Cl (C (A)))\). By Proposition 6.2 (ii), \(NS\ Cl (A) \subseteq NS\ Cl (B)\). This proves (iv).
Proposition 6.4 Let $A$ be a neutrosophic set in a NTS $X$. Then $NInt (A) \subseteq NS \ Int (A) \subseteq A \subseteq NS \ Cl (A) \subseteq NCi (A)$.

Proof: It follows from the definitions of corresponding operators.

Proposition 6.5 Let $(X, \tau)$ be a NTS. Then for a neutrosophic subset $A$ and $B$ of a NTS $X$, we have

(i) $NS \ Cl (A \cup B) = NS \ Cl (A) \cup NS \ Cl (B)$

(ii) $NS \ Cl (A \cap B) \subseteq NS \ Cl (A) \cap NS \ Cl (B)$.

Proof: Since $NS \ Cl (A \cup B) = NS \ Cl (C (C (A \cup B)))$, by using Proposition 6.2 (i), $NS \ Cl (A \cup B) = C (NS \ Cl (C (A \cup B))) = C (NS \ Cl (A) \cup NS \ Cl (B))$. Then using Proposition 6.3 (iv), $NS \ Cl (A \cap B) \subseteq NS \ Cl (A) \cap NS \ Cl (B)$. This implies that $NS \ Cl (A \cap B) \subseteq NS \ Cl (A) \cap NS \ Cl (B)$. This proves (ii).

The following example shows that the equality need not hold in Proposition 6.5 (ii).

Example 6.6 Let $X = \{ a, b, c \}$ with $\tau = \{ 0_a, A, B, C, D, I_x \}$ and $\tau = \{ 1_x, E, F, G, H, 0_b \}$ where $A = ((0.5, 0.6, 0.1), (0.6, 0.7, 0.1), (0.9, 0.5, 0.2))$

$B = ((0.4, 0.5, 0.2), (0.8, 0.6, 0.3), (0.9, 0.7, 0.3))$

$C = ((0.4, 0.5, 0.2), (0.6, 0.6, 0.3), (0.9, 0.5, 0.3))$

$D = ((0.5, 0.6, 0.1), (0.8, 0.7, 0.1), (0.9, 0.7, 0.2))$

$E = ((0.1, 0.4, 0.5), (0.1, 0.3, 0.6), (0.2, 0.5, 0.9))$

$F = ((0.2, 0.5, 0.4), (0.3, 0.4, 0.8), (0.3, 0.3, 0.9))$

$G = ((0.2, 0.5, 0.4), (0.3, 0.4, 0.8), (0.3, 0.3, 0.9))$

$H = ((0.1, 0.4, 0.5), (0.1, 0.3, 0.8), (0.2, 0.3, 0.9))$. Then $(X, \tau)$ is a NTS. Consider the NSs are $I = ((0.1, 0.2, 0.5), (0.2, 0.3, 0.7), (0.3, 0.3, 1))$ and $J = ((0.2, 0.4, 0.8), (0.1, 0.2, 0.8), (0.2, 0.5, 0.9))$. Then $NS \ Cl (I) = G$ and $NS \ Cl (J) = G$.

This implies that $NS \ Cl (I) \cap NS \ Cl (J) = G$. Now $I = 1 \cap J = ((0.1, 0.2, 0.8), (0.1, 0.2, 0.8), (0.2, 0.3, 1))$, it follows that $NS \ Cl (I \cap J) = H$. Then $NS \ Cl (I) \cap NS \ Cl (J) \subseteq NS \ Cl (I \cap J)$.

Theorem 6.7 If $A$ and $B$ are NSs of NTSS X and Y respectively, then

(i) $NS \ Cl (A \times B) \supseteq NS \ Cl (A \times B) \subseteq NS \ Cl (A \times B)$

(ii) $NS \ Int (A \times B) \subseteq NS \ Int (A \times B)$.

Proof: (i) Since $A \subseteq NS \ Cl (A)$ and $B \subseteq NS \ Cl (B)$, hence $A \times B \subseteq NS \ Cl (A \times B)$. This implies that $NS \ Cl (A \times B) \subseteq NS \ Cl (A \times B)$ and From Proposition 4.12, $NS \ Cl (A \times B) \subseteq NS \ Cl (A) \times NS \ Cl (B)$.

(ii) follows from (i) and the fact that $NS \ Cl (C (A)) = C (NS \ Cl (A))$.

Lemma 6.8 For NSs $A_i$‘s and $B_j$‘s of NTSSs X and Y respectively, we have

(i) $\cap \{ A_i, B_j \} = \min (\cap A_i, \cap B_j)$;

(ii) $\cup \{ A_i, B_j \} = \max (\cup A_i, \cup B_j)$.

Theorem 6.9 Let $(X, \tau)$ and $(Y, \sigma)$ be NTSSs such that $X$ is neutrosophic product related to $Y$. Then for NSs $X$ and $Y$, we have

(i) $NS \ Cl (A \times B) \supseteq NS \ Cl (A) \times NS \ Cl (B)$

(ii) $NS \ Int (A \times B) \subseteq NS \ Int (A) \times NS \ Int (B)$.

Proof: (i) Since $NS \ Cl (A \times B) \subseteq NS \ Cl (A) \times NS \ Cl (B)$ (By Theorem 6.7 (i)) it is sufficient to show that $NS \ Cl (A \times B) \supseteq NS \ Cl (A) \times NS \ Cl (B)$. Let $A_i \in \tau$ and $B_j \in \sigma$. Then $NS \ Cl (A \times B) = \{(x, y), \cap \{(A_i \times B_j) : C ((A_i \times B_j)) \} \} \times \{(x, y), \cup \{(A_i \times B_j) : C ((A_i \times B_j)) \} \}$.

Let $A_i \times B_j \subseteq A \times B$, then $\cap \{ A_i \times B_j \} \subseteq \cap A \times B$ and $\cup \{ A_i \times B_j \} \subseteq \cup A \times B$. Then $\cap \{ A_i \times B_j \} \subseteq \cap A \times B$ and $\cup \{ A_i \times B_j \} \subseteq \cup A \times B$.

(ii) follows from (i).
(i) $\text{NS CI}(A) \supseteq A \cup \text{NS CI} (\text{NS Int}(A))$,
(ii) $\text{NS Int}(A) \subseteq A \cap \text{NS Int} (\text{NS CI} (A))$,
(iii) $\text{NInt} (\text{NS CI} (A)) \subseteq \text{NInt} (\text{NCI} (A))$,
(iv) $\text{NInt} (\text{NS CI} (A)) \supseteq \text{NInt} (\text{NS CI} (\text{NS Int} (A)))$.

Proof: By Proposition 6.3 (i), $A \subseteq \text{NS CI} (A)$ ----- (1). Again using Proposition 5.2 (i), $\text{NS Int} (A) \subseteq A$. Then $\text{NS CI} (\text{NS Int} (A)) \subseteq \text{NS CI} (A)$ ----- (2). By (1) & (2) we have, $A \cup \text{NS CI} (\text{NS Int} (A)) \subseteq \text{NS CI} (A)$. This proves (i).

By Proposition 5.2 (ii), $\text{NS Int} (A) \subseteq A$ ----- (1). Again using proposition 6.3 (i), $A \subseteq \text{NS CI} (A)$. Then $\text{NS Int} (A) \subseteq \text{NS Int} (\text{NS CI} (A))$ ----- (2). From (1) & (2), we have $\text{NS Int} (A) \subseteq A \cap \text{NS Int} (\text{NS CI} (A))$. This proves (ii).

By Proposition 6.4, $\text{NS CI} (A) \subseteq \text{NCl} (A)$. We get $\text{NInt} (\text{NS CI} (A)) \subseteq \text{NInt} (\text{NCl} (A))$. Hence (iii).

By (i), $\text{NS CI} (A) \supseteq A \cup \text{NS CI} (\text{NS Int} (A))$. We have $\text{NInt} (\text{NS CI} (A)) \supseteq \text{NInt} (A \cup \text{NS CI} (\text{NS Int} (A)))$. Since $\text{NInt} (A \cup B) \supseteq \text{NInt} (A) \cup \text{NInt} (B)$, $\text{NInt} (\text{NS CI} (A) \supseteq \text{NInt} (A) \cup \text{NInt} (\text{NS CI} (\text{NS Int} (A)))) \supseteq \text{NInt} (\text{NS CI} (\text{NS Int} (A)))$. Hence (iv).

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