Some $V_4$-cordial families with its balanced $V_4$-cordial labeling

V. J. Kaneria$^{1}$, Jaydev R. Teraiya$^{2}$

$^{1}$Assistant Professor, Department of Mathematics, Saurashtra University, Rajkot, Gujarat, India
$^{2}$Assistant Professor, Department of Mathematics, Marwadi University, Rajkot, Gujarat, India

Abstract: In this paper we discussed about balanced $V_4$-cordial labeling. We proved that $G^*$, $P_n \times G$ and $G^*$ are balanced $V_4$-cordial graphs, when $G$ is a balanced $V_4$-cordial graph.

Keywords: $V_4$-cordial graph, balanced $V_4$-cordial labeling, Star of a graph $G$ and the complete star of a graph $G$.

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I. INTRODUCTION

Labeled graph have many diversified applications. The cordial labeling introduced by Cahit [1] is a weaker version of graceful and harmonious labeling. We follow Harary [2] for the basic notation and terminology of graph theory. Gallian [3] provide vast amount of literature on survey of different types of graph labeling. Also, he proved that the complete graph is cordial if and only if.

After this, many researchers have studied cordial graph and similar type graph labeling. $V_4$-cordial labeling was introduced by Riskin [4] in 2013.

A cordial graph $G$ with a cordial labelling $f$ is called a balanced cordial graph if $|v_f(0) - v_f(1)| = 0$. Kaneria, Patadiya and Teraiya [5] proved that $P_n \times C_4$, $C_n \times C_4$ is balanced cordial. Also, Kaneria, Teraiya and Patadiya [6] proved that $P(t, C_4)$ is a balanced cordial if $t$ is odd and it is vertex balanced cordial if $t$ is even, where $n \in \mathbb{N}$. $C(t, C_4)$ is a balanced cordial if $t \equiv 0 \pmod{4}$ and it is vertex balanced cordial if $t \equiv 1, 3 \pmod{4}$ where $n \in \mathbb{N}$ and $C_4^n$ is a balanced cordial graph. $V \subseteq \mathbb{N}$.

Let $V_4 = \{e, a, b, c\}$ be the Klein four group with the binary operation $*$. A $V_4$-cordial graph $G$ with a $V_4$-cordial labeling $f$ is said to be a balanced $V_4$-cordial graph if $|v_f(p) - v_f(q)| + |e_f(p) - e_f(q)| \leq 2$ for $p, q \in V_4$. Let $G$ be a $V_4$-cordial graph with a $V_4$-cordial labeling $f$ on $G$. Define $g: V(G) \rightarrow V_4$ by $f(u) = g(u)$, when $f(u) = 0$ and $f(x), f(y), f(z) \in \{a, b, c\}$. Observed that $g$ is also a $V_4$-cordial labeling on $G$. Also, observed that $g$ is a balanced $V_4$-cordial labeling, when $f$ is a balanced $V_4$-cordial labeling for $G$.

Star of graph $G$ is denoted by $G^*$ and it obtain by $|V(G)| + 1$ copies of $G$ say $G^{(0)}, G^{(1)}, G^{(2)}, \ldots, G^{(p)}$, where $V(G) = \{v_1, v_2, \ldots, v_p\}$. It is obtained by joining each vertex of $G^{(i)}$ with the corresponding vertex $v_i$ of $G^{(i)}$ for $i = 1, 2, \ldots, p$. We call $G^{(0)}$ as central copy of $G$. It is obvious that $K_1^* = K_1, K_2^* = P_2$ and $K_3^* = P_3 \times P_3$.

In this paper we have obtain a balanced $V_4$-cordial labeling for $G^*$, $G^*$ and $P_n \times G$, where $G$ is balanced $V_4$-cordial graph.

II. Main Results

Theorem -2.1

If $G$ is a balanced $V_4$-cordial graph, then so is $G^*$.

Proof: Let $V(G) = \{v_1, v_2, \ldots, v_p\}$ and $q = |E(G)|$. Let $f: V(G) \rightarrow V_4$ be a balanced cordial labeling for $G$. It is obvious that $p, q \equiv 0 \pmod{4}$ and $e_f(s) = \frac{p}{4}$. Let $H = G^*$ and $V(H) = \bigcup_{i=0}^{p} V(G^{(i)}) = \{v_1^{(i)}, v_2^{(i)}, \ldots, v_p^{(i)}\}$ for all $i$.

Note that $V(H) = p(p + 1)$ and $|E(H)| = (p + 1)q + p$.

Define $g: V(H) \rightarrow V_4$ as follows. For any $v_j^{(i)} \in V(H)$,
Note that above defined labeling function $g$ on $G$ is also balanced $V_4$-cordial labeling.

Define $h : V(P_n \times G) \rightarrow V_4$ as follows.

For any $v_j^{(i)} \in V(P_n \times G)$

$$h(v_j^{(i)}) = \begin{cases} f(v_j^{(i)}) & \text{when } i \text{ is odd} \\ g(v_j^{(i)}) & \text{when } i \text{ is even} \end{cases}$$

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$a_i = \frac{p_i q_i + 1}{2}$

$p_i q_i$ is the number of edges between the $i$th copy of $G$ and the $j$th copy of $G$.

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Note that above defined labeling function $g$ on $G$ is also balanced $V_4$-cordial labeling.
It is observed that $v_0(0) = (p + 1)v_f(0) = \frac{(p+1)p}{4}$, $v_0(a) = v_0(b) = v_0(c) = \frac{p}{4}$. Moreover $e_{V_4}(0) = v_f(0) + \frac{pq}{4} + \frac{pp}{4} = \frac{1}{4}[pq + q + p^2 = Q4 = ega = egb = egc]$. Therefore, $H$ is a balanced $V_4$-cordial graph.

i.e. $G^*$ is balanced $V_4$-cordial.

References


