Semi Regular Weakly Open Sets in Topological Spaces

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Abstract — This paper considers a new class of sets called semi-regular weakly open (briefly srw-open) sets are introduced and studied in topological spaces. i.e. A subset G of topological space X is said to be semi-regular weakly open if if F \subseteq \sin t(A), whenever F \subseteq A and F is rw-closed set in X. The new class strictly lies between semi-open sets, \( \alpha \)-open sets and g-open sets in topological spaces. Also, as applications, using some properties of srw-open sets and srw-closed sets we investigate srw-interior and srw-closure operators and their properties respectively.

Keywords— srw-closed sets, srw-open sets, srw-neighbourhoods, srw-interior, srw-closure.

I. INTRODUCTION

Levine and Stone [6, 13] introduced generalized open sets, regular open sets in topological spaces respectively, then regular weakly open sets, generalized semi closed sets, generalized \( \alpha \)-closed sets and \( \alpha \)-generalized closed sets semi open sets, regular w-closed sets, pgrw-closed sets and semi-regular weakly closed sets have been introduced and studied by Benchalli S. S. and Wali R. S. [2], Arya S.P. and Nour T.M. [1], Maki et al. [7], Levin [7], Wali R. S. and Mendalgeri [17], Wali R. S. and Chilakkad [18] and Wali R. S. and Mathad [16] respectively.

We introduce and study the semi-regular weakly open (briefly srw-open) sets, semi-regular weakly neighbourhood (briefly srw-nhd) and operators; srw-interior and srw-closure in topological space and obtain some of their properties.

II. PRELIMINARIES

Throughout this paper X and Y represent the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of topological space X, cl(A) and int(A) denote the closure of A and interior of A respectively. Let X\A denotes the complement of A in X. Now, we recall the following definitions.

Definition 2.1 A subset A of a topological space X is called

i) Regular open [14], if \( A = \text{int}(cl(A)) \) and regular closed if \( cl(\text{int}(A)) = A \).

ii) Pre-open [10], if \( A \subseteq \text{int}(cl(A)) \) and pre-closed if \( cl(\text{int}(A)) \subseteq A \).

iii) Semi open [7], if \( A \subseteq cl(\text{int}(A)) \) and semi-closed if \( cl(\text{int}(A)) \subseteq A \).

iv) \( \alpha \)-open [11], if \( A \subseteq \text{int}(cl(\text{int}(A))) \) and \( \alpha \)-closed if \( cl(\text{int}(A)) \subseteq A \).

v) Semi pre open [11], if \( A \subseteq cl(\text{int}(cl(A))) \) and semi pre-closed if \( cl(\text{int}(cl(A))) \subseteq A \).

vi) \( \pi \)-open [19], if A is a finite union of regular open sets.

Definition 2.2 A subset A of a topological space X is called

i) Generalized closed (briefly g-closed) [7], if \( cl(A) \subseteq U \) whenever \( A \subseteq U \) and U is open in X.

ii) Semi-generalized closed (briefly sg-closed) [3], if \( scl(A) \subseteq U \) whenever \( A \subseteq U \) and U is semi open in X.

iii) Generalized semi-closed (briefly gs-closed) [1], if \( scl(A) \subseteq U \) whenever \( A \subseteq U \) and U is open in X.

iv) Generalized \( \alpha \)-closed (briefly g\( \alpha \)-closed) [4], if \( \alpha cl(A) \subseteq U \) whenever \( A \subseteq U \) and U is \( \alpha \)-open in X.

v) Generalized \( \alpha \)-closed (briefly g\( \alpha \)-closed) [9], if \( cl(A) \subseteq U \) whenever \( A \subseteq U \) and U is open in X.

vi) Generalized semi pre-closed (briefly gsp-closed) [5], if \( spcl(A) \subseteq U \) whenever \( A \subseteq U \) and U is open in X.

vii) Regular generalized closed (briefly rg-closed) [12], if \( cl(A) \subseteq U \) whenever \( A \subseteq U \) and U is regular open in X.

viii) Weakly closed (briefly w-closed) [13], if \( cl(A) \subseteq U \) whenever \( A \subseteq U \) and U is semi-open in X.
ix) Regular weakly closed (briefly rw-closed) [2], if
\[ c\ell(A) \subseteq U \] whenever \( A \subseteq U \) and \( U \) is regular semi-open in \( X \).
x) \( \alpha \)-regular weakly closed (briefly \( \alpha \) rw-closed)
[17], if \( \alpha c\ell(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is rw-open set in \( X \).

The complements of above all closed sets are their respective open sets in the same topological space \( X \).

The semi-pre-closure (resp. semi-closure, resp. pre-closure, resp. \( \alpha \) -closure) of a subset \( A \) of \( X \) is the intersection of all semi-pre- closed (resp. semi- closed, resp. pre- closed, resp. \( \alpha \) -closed) sets containing \( A \) and is denoted by \( scp(A) \) (resp. \( scl(A) \), resp. \( pcl(A) \), resp. \( cl(A) \)).

**Definition 2.3** A subset \( A \) of a space \( X \) is said to be semi regular weakly closed (briefly srw-closed) set [16], if \( scl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is rw-open set in \( X \).

We denote the family of all srw-closed sets, srw-open sets, \( \alpha \) rw-open sets and semi-open sets of \( X \) by SRWC(X), SRWO(X), \( \alpha \) RWO(X) and SO(X) respectively.

**Lemma 2.4 i)** For a subset \( A \) of \( X \), \( \alpha rW - c\ell(A) \) and defined as \( \alpha rW - c\ell(A) = \cap F \subseteq X : A \subseteq F \in \alpha rWC(X) \).

**ii)** For a subset \( A \) of \( X \), semi-closure of \( A \) [6] is denoted by \( scl(A) \) and defined as \( scl(A) = \cap F \subseteq X : A \subseteq F \in SC(X) \).

**iii)** For a subset \( A \) of \( X \), gs-closure of \( A \) [1] is denoted by \( gs - c\ell(A) \) and defined as \( gs - c\ell(A) = \cap F \subseteq X : A \subseteq F \in GSC(X) \).

### III. SEMI REGULAR WEAKLY OPEN (BRIEFLY SRW-OPEN) SETS

In this section, we introduce and study srw-open sets in topological space and obtain some of their basic properties.

**Definition 3.1** A subset \( A \) of \( X \) is called Semi Regular Weakly open (briefly srw-open) set, if \( X \cap A \) is srw-closed set in \( X \). The family of all semi regular weakly open sets in \( X \) is denoted as SRWO(X).

**Theorem 3.2** If a subset \( A \) of space \( X \) is \( \alpha rW \)-open, then it is srw-open in \( X \) but not conversely.

**Proof:** Let \( A \) be a \( \alpha rW \)-open set in a space \( X \). Then \( X \cap A \) is a \( \alpha rW \)-closed set. By Theorem 3.2 of [16], \( X \cap A \) is srw-closed. Therefore \( A \) is a srw-open set in \( X \).

The converse of the above Theorem need not be true as shown in example 3.3.

**Example 3.3** Let \( X = \{ a, b, c, d \} \) with topology \( \tau = \{ X, \phi, \{ a \}, \{ b, c \}, \{ a, b, c \} \} \). Then \( \{ a, d \} \) and \( \{ b, c, d \} \) are srw-open sets in \( X \) but it is not \( \alpha rW \)-open sets in \( X \).

**Theorem 3.4** If a subset \( A \) of space \( X \) is semi-open, then it is semi-open in \( X \) but converse is not true.

**Proof:** Let \( A \) be a semi-open set in a space \( X \). Then \( X \cap A \) is a semi-closed set. By Theorem 3.6 of [16], \( X \cap A \) is srw-closed. Therefore \( A \) is a srw-open set in \( X \).

The converse of the above Theorem need not be true as shown in example 3.5.

**Example 3.5** Let \( X = \{ a, b, c, d \} \) with topology \( \tau = \{ X, \phi, \{ a \}, \{ b, c \}, \{ a, b, c \} \} \). Then \( \{ b \} \) and \( \{ c \} \) are srw-open sets in \( X \) but not semi-open sets in \( X \).

**Corollary 3.6** From Levine [7], it is evident that every open set is semi-open set but not conversely. By Theorem 3.4 every semi-open set is srw-open set in \( X \) but not conversely and hence every open set is srw-open set in \( X \).

**Corollary 3.7** From Wali and Prabhavati [17], it is evident that every \( \alpha \) -open set is \( \alpha rW \)-open set but not conversely and hence every \( \alpha \) -open set is srw-open set but not conversely.

**Corollary 3.8** From Stone [14], it is evident that every regular open set is open, but not conversely. By Corollary 3.7, every open set is srw-open set but conversely and hence every regular open set is srw-open set in \( X \).

**Corollary 3.9** From Velicko [15], it is evident that every \( \delta \) -open ( \( \delta \) -open) set is open but not conversely. By Corollary 3.7, every open set is srw-open set but not conversely and hence every \( \delta \) -open ( \( \delta \) -open) set is srw-open set in \( X \).

**Theorem 3.10** If a subset \( A \) of a space \( X \) is srw-open, then it is a gs-open set in \( X \).

**Proof:** Let \( A \) be a srw-open set in \( X \), then \( X \cap A \) is a srw-closed set in \( X \). By Theorem 3.4 of [16], every srw-closed set is gs-closed set in \( X \) i.e. \( X \cap A \) is a gs-closed set in \( X \). Therefore \( A \) is a gs-open set in \( X \).

The converse of the above Theorem need not be true as shown in example 3.11.

**Example 3.11** Let \( X = \{ a, b, c, d \} \) with topology \( \tau = \{ X, \phi, \{ a \}, \{ b, c \}, \{ a, b, c \} \} \). Then \( \{ a, c \} \) and \( \{ a, b \} \) are gs-open sets in \( X \) but not srw-open sets in \( X \).

**Theorem 3.12** If a subset \( A \) of space \( X \) is srw-open, then it is a gs-open set in \( X \).

**Proof:** Let \( A \) be a srw-open set in \( X \), then \( X \cap A \) is a srw-closed set in \( X \). By Theorem 3.10 of [16], every srw-closed set is gs-closed set in \( X \) i.e. \( X \cap A \) is a gs-closed set in \( X \). Therefore \( A \) is a gs-open set in \( X \).

The converse of the above Theorem need not be true as shown in example 3.13.

**Example 3.13** Let \( X = \{ a, b, c, d \} \) with topology \( \tau = \{ X, \phi, \{ a \}, \{ b, c \}, \{ a, b, c \} \} \). Then \( \{ a, b \} \) and \( \{ c, d \} \) are gs-open sets in \( X \) but not srw-open sets in \( X \).
The concepts of g-open, w-open, α g-open and w α-open sets are independent with the concept of srw-open set as shown in the following example 3.14.

**Example 3.14** Let $X=\{a, b, c, d\}$ with topology $\tau=\{X, \emptyset, \{a, b, c\}, \{a, d\}\}$. Then $\{a, d\}$ is a srw-open, however it can be verified that it is not g-open, w-open, g-open and w-open set. Also, the set $\{a, b\}$ and $\{a, c\}$ are g-open, w-open, α g-open and w α-open set but not srw-open set in X.

Thus the above discussion leads to the following implication diagram:

![Implication Diagram](image)

**Remark 3.15** Union and intersection of two srw-open sets need not be srw-open set as shown in the following example 3.16.

**Example 3.16** Let $X=\{a, b, c, d\}$ with topology $\tau=\{X, \emptyset, \{a, b, c\}, \{a, b, c\}\}$. Then $\text{SRWO}(X)=\{X, \emptyset, \{b, c\}, \{a, b, c\}\}$. Let $A=\{b\}$, $B=\{a, d\}$ and $C=\{b, c, d\}$. Here A and B are srw-open sets but $A \cup B = \{a, b, d\}$ is not srw-open. Also B and C are srw-open sets but $B \cap C = \{d\}$ is not srw-open set in X.

**Theorem 3.17** If $A \subseteq X$ is srw-closed, then $\text{srcl}(A) \cap \text{srcl}(A)$ is srw-open set in X.

**Proof:** Let $A \subseteq X$ is srw-closed and let F be a rw-closed set such that $F \subseteq \text{srcl}(A) \setminus A$. Then by Theorem 3.19 of [16], $F=\emptyset$ that implies $F \subseteq \text{srcl}(A) \setminus A$ and Theorem 3.17 $\text{srcl}(A) \setminus A$ is srw-open set in X.

**Theorem 3.18** A subset A of a topological space X is srw-open if and only if $F \subseteq \text{srcl}(A)$ whenever F is rw-closed and $F \subseteq A$.

**Proof:** Let $F \subseteq A$ is srw-closed and let F be a rw-closed set and $F \subseteq A$. Then $X \setminus A \subseteq X \setminus F$ where $X \setminus F$ is rw-open. Since $X \setminus A$ is srw-closed, $\text{srcl}(X \setminus A) \subseteq X \setminus F$ and hence $X \setminus \text{srcl}(A) \subseteq X \setminus F$ that implies $F \subseteq \text{sin}(A)$.

Conversely, suppose $F \subseteq \text{sin}(A)$ whenever $F \subseteq A$, F is rw-closed. To prove: A is srw-open. Suppose, $X \setminus U \subseteq A$ where U is rw-open. Then $X \setminus U \subseteq A$ where $X \setminus U$ is rw-closed. By assumption $X \setminus U \subseteq \text{sin}(A)$ that implies $\text{srcl}(X \setminus A) \subseteq U$. This proves that $X \setminus A$ is srw-closed and hence A is srw-open set in X.

**Theorem 3.19** Every singleton point set in a space X is either srw-open or rw-open in X.

**Proof:** Let $x \in X$ where X is a topological space. To prove: $\{x\}$ is either srw-open or rw-open set in X i.e. to prove that $X \setminus \{x\}$ is either srw-closed or rw-open, which follows from Theorem 3.25 of [16]. The next Theorem shows that all the sets between $\text{sin}(A)$ and A are srw-open whenever A is srw-open.

**Theorem 3.20** If $\text{sin}(A) \subseteq B \subseteq A$ and A is a srw-open set in X, then B is srw-open set in X.

**Proof:** Let $\text{sin}(A) \subseteq B \subseteq A$ and A is a srw-open set. Then $X \setminus A \subseteq X \setminus B \subseteq X \setminus \text{sin}(A)$ that implies $X \setminus A \subseteq X \setminus B \subseteq \text{srcl}(X \setminus A)$, since $X \setminus A$ is srw-open set. By Theorem 3.23 of [16], $X \setminus B$ is srw-closed set. Therefore B is srw-open in X.

**Theorem 3.21** If $A \subseteq X$ is srw-closed, then $\text{srcl}(A) \setminus A$ is srw-open set in X.

**Proof:** Let $A \subseteq X$ is srw-closed set and F be a rw-closed set such that $F \subseteq \text{sin}(A) \setminus A$. By Theorem 3.19 of [16], $F=\emptyset$, so $F \subseteq \text{sin}(\text{srcl}(A) \setminus A)$ By Theorem 3.18 $\text{srcl}(A) \setminus A$ is srw-open set in X. The converse of above Theorem does not hold shown by example 3.22.

**Example 3.22** Let $X=\{a, b, c, d\}$ with topology $\tau=\{X, \emptyset, \{a, b, c\}, \{a, b, c\}\}$. Then $A=\{a, c, d\}$ then $\text{srcl}(A)=\{b, c, d\}$ and $\text{srcl}(A) \setminus A=\{b\}$ is an srw-open set, but A is not an srw-closed set in X.

**Theorem 3.23** If a subset A of X is srw-open in X and if G is rw-open in X with $\text{sin}(A) \cup (X \setminus A) \subseteq G$ then $G=\text{X}$.

**Proof:** Suppose that G is an rw-open set and $\text{sin}(A) \cup (X \setminus A) \subseteq G$. Now $(X \setminus A) \subseteq X \setminus \text{srcl}(A) \cap X \setminus (X \setminus A)$ implies that $(X \setminus G) \subseteq \text{srcl}(X \setminus A) \cap A$ Suppose A is srw-open. Since $X \setminus G$ is rw-open and $X \setminus A$ is srw-closed, then by Theorem 3.19 of [16], $X \setminus G=\emptyset$ and hence $G=X$.

The converse of the above Theorem need not be true in general as shown in example 3.24.
Example 3.24 Let $X=\{a, b, c, d\}$ with topology $\tau=\{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then $\text{SRWO}(X) = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$.

$\text{RWO}(X) = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}, \{c,d\}, \{b,d\}, \{a,c\}, \{a,b,c\}\}$.

Let $A=\{a, b, d\}$ and $B=\{a, b, c, d\}$.

$\text{sin}(A) \cup (X \setminus A) = \{a, d\} \cup \{c\} = \{a, c, d\}$.

So for some srw-open set $G$, such that $\text{sin}(A) \cup (X \setminus A) = \{a, c, d\} \subset G$ gives $G=X$ but $A$ is not srw-open in $X$.

Theorem 3.25 Let $X$ be a topological space and $A, B \subseteq X$. If $B$ is srw-open and $\text{sin}(B) \subseteq A$, then $A \cap B$ is srw-open in $X$.

Proof: Since $B$ is srw-open and $\text{sin}(B) \subseteq A$, then $\text{sin}(B) \subseteq A \cap B \subseteq B$, then by Theorem 3.32 of [16]. $A \cap B$ is srw-open set in $X$.

IV. SEMI REGULAR WEAKLY NEIGHBOURHOODS (BRIEFLY SRW-NHD)

Definition 4.1 Let $(X, \tau)$ be a topological space and let $x \in X$. A subset $N$ is said to be srw-nhd of $x$, if and only if there exists a srw-open set $G$ such that $x \in G \subseteq N$.

Definition 4.2 i) A subset $N$ of $X$ is a srw-nhd of $A \subseteq X$ in topological space $(X, \tau)$, if there exists an srw-open set $G$ such that $A \subseteq G \subseteq N$.

ii) The collection of all srw-nhd of $x \in X$ is called srw-nhd system at $x \in X$ and shall be denoted by srw-N$(x)$.

Theorem 4.3 Every neighborhood $N$ of $x \in X$ is a srw-nhd of $x$.

Proof: Let $N$ be neighborhood of point $x \in X$. To prove that $N$ is a srw-nhd of $x$. By definition of neighborhood, there exists an open set $G$ such that $x \in G \subseteq N$. Hence $N$ is srw-nhd of x.

Remark 4.4 In general, a srw-nhd $N$ of $x$ in $X$, as shown from example 4.5.

Example 4.5 Let $X=\{a, b, c, d\}$ with topology $\tau=\{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then $\text{SRWO}(X) = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$.

The set $\{a, b, d\}$ is srw-nhd of the point $b$, since the srw-open set $\{b\}$ is such that $b \in \{b\} \subset \{a,b,d\}$.

However, the set $\{a, b, d\}$ is not a neighbourhood of the point $b$, since no open set $G$ exists such that $b \in G \subset \{a,b,d\}$.

Theorem 4.6 If a subset $N$ of a space $X$ is srw-open, and then $N$ is a srw-nhd of each of its points.

Proof: Suppose $N$ is srw-open. Let $x \in N$ we claim that $N$ is a srw-nhd of $x$. For $N$ is a srw-open set such that $b \in N \subset N$. Since $x$ an arbitrary point of $N$, it follows that $N$ is a srw-nhd of each of its points.

The converse of the above theorem is not true in general as seen from the following example 4.7.

Example 4.7 Let $X=\{a, b, c, d\}$ with topology $\tau=\{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then $\text{SRWO}(X) = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$.

The set $\{a, b, c\}$ is srw-nhd of the point $a$, since the srw-open set $\{a\}$ is such that $a \in \{a\} \subset \{a\}$. Also the set $\{a, b, c\}$ is a srw-nhd of the point $c$, since the srw-open set $\{c\}$ is such that $c \in \{c\} \subset \{a,c\}$. i.e. $\{a, c\}$ is a srw-nhd of each of its points. However the set $\{a, c\}$ is not a srw-open set in $X$.

Theorem 4.8 Let $X$ be a topological space. If $F$ is a srw-closed subset of $X$ and $x \in (X \setminus A)$, then there exists a srw-nhd $N$ of $x$ such that $N \cap F = \emptyset$.

Proof: Let $F$ be srw-closed subset of $X$ and $x \in (X \setminus F)$. Then $(X \setminus F)$ is an srw-open set of $X$. By Theorem 4.6, $(X \setminus F)$ contains a srw-nhd of each of its points. Hence there exists a srw-nhd $N$ of $x$ such that $N \cap F = \emptyset$.

Theorem 4.9 Let $X$ be a topological space and for each $x \in X$, let srw-$N(x)$ be the collection of all srw-nhds of $x$. Then we have the following results.

i) $\forall x \in X$, srw-$N(x) \neq \emptyset$.

ii) $x \in \text{srw-N}(x) \Rightarrow x \in N$.

iii) $N \in \text{srw-N}(x)$ and $N \subseteq M \Rightarrow M \in \text{srw-N}(x)$.

iv) $N \in \text{srw-N} (x) \Rightarrow \exists M \in \text{srw-N} (y)$ for every $y \in M$.

Proof: i) Since $X$ is an srw-open set, it is a srw-nhd of every $x \in X$. Hence there exists at least one srw-nhd(X) for each $x \in X$. Hence srw-$N(x) \neq \emptyset$ for every $x \in X$.

ii) If $N \in \text{srw-N}(x)$, then $N$ is a srw-nhd of $x$. So, by definition of srw-nhd $x \in X$.

iii) Let $N \in \text{srw-N}(x)$ and $N \subseteq M$, then there is a srw-open set $G$ such that $x \in G \subset N$. Since $N \subseteq M$, $x \in G \subset M$ and so $M$ is a srw-nhd of $x$. Hence $M \in \text{srw-N}(x)$.

iv) If $N \in \text{srw-N}(x)$, then there exists an srw-open set $M$ and is an srw-open set, it is a srw-nhd of each of its points. Therefore $M \in \text{srw-N}(y)$ for $y \in M$.
V. Semi-Regular Weakly Interior Operator

In this section, the notion of srw-interior is defined and some of its basic properties are studied.

**Definition 5.1** Let A be a subset of X. A point \( x \in A \) is said to be srw-interior point of A, if A is a srw-nhd of x. The set of all srw-interior of A and is denoted by srw-int(A).

**Definition 5.2** For a subset A of X, srw-interior of A is defined as srw-int(A) to be the union of all srw-open sets contained in A. In symbolically, \( \text{srw-int}(A) = \bigcup\{G \subset X : G \subseteq A \text{ and } G \text{ is srw-open in } X\} \).

**Theorem 5.3** If A is a subset of X, then \( \text{srw-int}(A) = \bigcup\{G \subset X : G \subseteq A \text{ and } G \text{ is srw-open in } X\} \).

**Proof:** Let A be a subset of X. Let \( x \in \text{srw-int}(A) \) then applying srw.

Hence \( x \in \text{srw-int}(A) \). If A is a subset of X, then applying srw.

\( x \in \text{srw-int}(A) \) and \( G \subseteq A \) and G is srw-open in X.

**Theorem 5.4** Let A and B are subsets of X. Then

i) \( \text{srw-int}(A) = X \) and \( \text{srw-int}(\phi) = \phi \).

ii) \( \text{srw-int}(A) \subset A \).

iii) If \( A \) is any srw-open set contained in \( A \), then \( B \subset \text{srw-int}(A) \).

iv) If \( A \subset B \), then \( \text{srw-int}(A) \subset \text{srw-int}(B) \).

v) \( \text{srw-int}(\text{srw-int}(A)) = \text{srw-int}(A) \).

**Proof:** i) Since X is only srw-open set contained in X. i.e. by definition 5.2, \( \text{srw-int}(A) = \bigcup\{G \subset X : G \text{ is srw-open, } G \subseteq A\} = X \cup \{\text{all srw-open sets}\} = X \). Hence \( \text{srw-int}(X) = X \).

Since \( \phi \) is only srw-open set contained in \( X \). Hence \( \text{srw-int}(\phi) = \phi \).

ii) Let \( x \in \text{srw-int}(A) \) then \( x \) is an srw-interior of A \( \Rightarrow \) A is an srw-nhd of x \( \Rightarrow \) \( x \in A \).

Hence \( x \in \text{srw-int}(A) \Rightarrow x \in A \). Hence \( \text{srw-int}(A) \subset A \).

iii) Let B be any srw-open set such that \( B \subset A \).

\( B \subset A \) then since B is srw-open set contained in A.

x is srw-interior point of A i.e. \( x \in \text{srw-int}(A) \).

Hence \( B \subset \text{srw-int}(A) \).

iv) Let A and B subsets of X such that \( A \subset B \) let \( x \in \text{srw-int}(A) \). Then x is srw-interior point of A and so A is srw-nhd of x. Since \( A \subset B \), B is also srw-nhd of x \( \Rightarrow x \in \text{srw-int}(B) \). Thus we have shown that \( x \in \text{srw-int}(A) \) \( \Rightarrow x \in \text{srw-int}(B) \).

Hence \( x \in \text{srw-int}(A) \subset x \in \text{srw-int}(B) \).

v) Let A be any subset of X. By the definition of srw-interior

\( \text{srw-int}(A) = \{G : G \subset A \text{ and } G \in \text{SRWO}(X)\} \).

if \( G \subset A \) then applying srw-interior on both sides, \( \text{srw-int}(G) \subset \text{srw-int}(A) \) \( \Rightarrow \)

\( G \subset \text{srw-int}(A) \). Since G is srw-open set contained in srw-int(A), i.e.

\( \text{srw-int}(\text{srw-int}(A)) = \text{srw-int}(A) \).

\( \text{srw-int}(\text{srw-int}(A)) \subset \text{srw-int}(\text{srw-int}(A)) \).

\( \text{srw-int}(\text{srw-int}(A)) \subset \text{srw-int}(\text{srw-int}(A)) \).

\( \text{srw-int}(\text{srw-int}(A)) = \text{srw-int}(A) \).

**Theorem 5.7** If a subset A of X is srw-open, then \( \text{srw-int}(A) = A \).

**Proof:** Let A be srw-open subset of X. We know that \( \text{srw-int} (A) \subset A \). Also, A is srw-open set contained in A. From theorem 5.6(iii), A \( \subset \text{srw-int}(A) \).

Hence \( \text{srw-int}(A) = A \).

The converse of Theorem 5.7 need not be true as seen in the following example 5.8.

**Example 5.8** Let X = \{a, b, c, d\} with topology \( \tau = \{X, \{a\}, \{b, c\}\} \) . Let \( \text{SRWO}(X) = \{X, \{a\}, \{b, c\}, \{a, b, c\}\} \).

Let \( A = \{a, b\} \) then \( \text{srw-int}(A) = A \) but A is not a srw-open set in X.

**Theorem 5.9** If A and B are subsets of X, then

i) \( \text{srw-int}(A) \cup \text{srw-int}(B) \subset \text{srw-int}(A \cup B) \).

ii) \( \text{srw-int}(A \cap B) \subset \text{srw-int}(A \cap B) \).

**Proof:** i) We know that \( A \subset (A \cup B) \) and \( B \subset (A \cup B) \).

We have by Theorem 5.6(iii), \( A \subset \text{srw-int}(A \cup B) \) and \( B \subset \text{srw-int}(A \cup B) \).

This implies that \( \text{srw-int}(A) \cup \text{srw-int}(B) \subset \text{srw-int}(A \cup B) \).

ii) We know that \( A \subset (A \cup B) \) and \( B \subset (A \cup B) \).

We have by Theorem 5.6(iii), \( \text{srw-int}(A) \cup \text{srw-int}(B) \subset \text{srw-int}(A \cup B) \).

I.e. \( \text{srw-int}(A \cap B) \subset \text{srw-int}(A) \cap \text{srw-int}(B) \).

**Theorem 5.10** If \( A \subset X \), then

i) \( \text{srw-int}(A) \subset vrw-int(A) \).
ii) $\sin t(A) \subset \text{srw-} \text{int}(A)$.

**Proof:**

i) Let $A$ is subset of $X$. Let $x \in \text{srw-} \text{int}(A) \Rightarrow x \in \bigcup \{G : G$ is srw-open, $G \subset A\}$ then there exist an srw-open set $G$ such that $x \in G \subset A$. Hence $\text{srw-} \text{int}(A) \subset \text{srw-} \text{int}(A)$.

ii) Let $A$ be subset of $X$, let $x \in \text{srw-} \text{int}(A) \Rightarrow x \in \bigcup \{G \subset X : G$ is srw-open, $G \subset A\}$ implies that there exists a semi-open set $G$ such that $x \in G \subset A$ then there exists a srw-open set $G$ such that $x \in G \subset A$. As every semi open set is srw-open set in $X$. Therefore $x \in \text{srw-} \text{int}(A)$.

**Remark 5.11** If $A$ is subset of $X$, then

i) $A \subset \text{srw-} \text{int}(A)$.

ii) $\text{int}(A) \subset \text{srw-} \text{int}(A)$.

iii) $r- \text{int}(A) \subset \text{srw-} \text{int}(A)$.

**Theorem 5.12** If $A$ is subset of $X$, then $\text{srw-} \text{int}(A) \subset \text{gs-} \text{int}(A)$.

**Proof:** Let $A$ be a subset of $X$, let $x \in \text{srw-} \text{int}(A) \Rightarrow x \in \bigcup \{G \subset X : G$ is srw-open, $G \subset A\}$ then there exists srw-open set $G$ such that $x \in G \subset A$ then there exists open set $G$ such that $x \in G \subset A$, as every srw-open set is gs-open set in $X$. Therefore $x \in \bigcup \{G \subset X : G \subset A\}$ is gs-open set in $X$. Therefore $x \in \text{gs-} \text{int}(A)$.

Thus $x \in \text{srw-} \text{int}(A) \Rightarrow x \in \text{gs-} \text{int}(A)$.

Hence $\text{srw-} \text{int}(A) \subset \text{gs-} \text{int}(A)$.

**Remark 5.13** containment relations in the above theorem 5.10 may be proper as seen in the following example 5.12.

**Example 5.14** Let $X=\{a, \ b, \ c, \ d\}$ with topology $\tau=\{X, \phi, \{a, b, c\}\}$. Let $\text{SRWO}(X)=\{X, \phi, \{a, b, c\}\}$. Let $A=\{a, b\}$ and $B=\{a, c\}$, then

i) $\text{srw-} \text{int}(A) \subset \text{srw-} \text{int}(A) \Rightarrow \{a\} \subset \{a, b\}$.

But $\text{srw-} \text{int}(A) \neq \text{srw-} \text{int}(A)$.

vi) $\text{srw-} \text{int}(A) \subset \text{gs-} \text{int}(A)$.

**Remark 5.15** If $A$ is subset of $X$, then $\text{srw-} \text{int}(A) \subset \text{gs-} \text{int}(A)$.

**VI. Semi-Regular Weakly Closure Operator**

Now we introduce the concept of srw-closure in topological spaces by using the notations of srw-closed sets and obtain some of their properties. For any $A \subset X$, it is proved that the complement of srw-interior of srw-closure of the complement of $A$.

**Definition 6.1** For a subset $A$ of $X$, srw-closure of $A$ is defined as srw-cl$(A)$ to be the intersection of all srw-closed sets containing $A$. In symbolically, srw-cl$(A)$ is ${\bigcap \{F \subset X : A \subset F \text{ and } F \in \text{SRWO}(X)\}}$.

**Theorem 6.2** If $A$ and $B$ are subsets of a space $X$. Then

i) srw-cl$(X)=X$ and srw-cl$(\phi)=\phi$.

ii) $A \subset \text{srw-} \text{cl}(A)$.

iii) If $B$ is any srw-closed set containing $A$ then $\text{srw-} \text{cl}(A) \subset B$.

iv) If $A \subset B$ then $\text{srw-} \text{cl}(A) \subset \text{srw-} \text{cl}(B)$.

v) $\text{srw-} \text{cl}(A)=\text{srw-} \text{cl}(\text{srw-} \text{cl}(A))$.

**Proof:**

i) By definition 3.1, $X$ is the only srw-closed set containing $X$. Therefore srw-cl$(X)={\bigcap A \subset \text{srw-} \text{cl}(A)}$.

ii) By definition 6.1, it is obvious that $A \subset \text{srw-} \text{cl}(A)$.

iii) Let $B$ be any srw-closed set containing $A$. Since srw-cl$(A)$ is the intersection of all srw-closed sets containing $A$, srw-cl$(A)$ is contained in every srw-closed set containing $A$. Hence in particular srw-cl$(A) \subset B$.

iv) Let $A$ and $B$ be subsets of $X$ such that $A \subset B$.

By definition 6.1. If $B \subset F \in \text{SRWC}(X)$, then $\text{srw-} \text{cl}(B) \subset F$.

Since $A \subset B$, $A \subset B \subset F \in \text{SRWC}(X)$.

We have $\text{srw-} \text{cl}(A) \subset F$.

Therefore $\text{srw-} \text{cl}(B) \subset \{F : B \subset F \in \text{SRWC}(X)\}$.

v) Let $A$ be any subset of $X$. By definition 6.1. If $A \subset F \in \text{SRWC}(X)$, then $\text{srw-} \text{cl}(A) \subset F$.

Since $F$ is srw-closed set containing srw-cl$(A)$, by (iii) $\text{srw-} \text{cl}(\text{srw-} \text{cl}(A)) \subset F$.

Hence $\text{srw-} \text{cl}(\text{srw-} \text{cl}(A)) \subset$
\[ \{F : A \subseteq F \in \text{SRWC}(X)\} = \text{srw-cl}(A). \] i.e. \( \text{srw-cl}(\text{srw-cl}(A)) = \text{srw-cl}(A) \).

**Remark 6.3** i) \( \text{srw-closure} \) of a set \( A \) is not always \( \text{srw-closed}. \)

ii) If \( A \subseteq X \) is \( \text{srw-closed} \), then \( \text{srw-cl}(A) = A \).

**Proof:** ii) Let \( A \) be \( \text{srw-closed subset of} X \). We know that \( A \subseteq \text{srw-cl}(A) \). Also \( A \subseteq A \) and \( A \) is \( \text{srw-closed}. \) By the theorem 6.2 (iii), \( \text{srw-cl}(A) \subseteq A \). Hence \( \text{srw-cl}(A) = A \). However if \( \text{srw-cl}(A) = A \) then it is not true that \( A \) is \( \text{srw-closed} \) as seen from following example.

**Example 6.4** Let \( X = \{a, b, c, d\} \) with topology \( \tau = \{X, \phi, \{a, b, c\}, \{a, b, c, d\}\} \). Let \( A = \{b\} \) then \( \text{srw-cl}(A) = \{b\} \) but \( A \) is not a \( \text{srw-closed set}. \)

**Theorem 6.5** If \( A \) and \( B \) are subsets of a space \( X \), then

i) \( \text{srw-cl}(A \cap B) \subseteq \text{srw-cl}(A) \cap \text{srw-cl}(B) \).

ii) \( \text{srw-cl}(A) \cup \text{srw-cl}(B) \subseteq \text{srw-cl}(A \cup B) \).

**Proof:** Let \( A \) and \( B \) be subsets of \( X \).

i) Clearly \( A \cap B \subseteq A \) and \( A \cap B \subseteq B \). By Theorem 6.2 (iv), \( \text{srw-cl}(A \cap B) \subseteq \text{srw-cl}(A) \) and \( \text{srw-cl}(A \cap B) \subseteq \text{srw-cl}(B) \). Hence \( \text{srw-cl}(A \cap B) \subseteq \text{srw-cl}(A) \cap \text{srw-cl}(B) \).

ii) Clearly \( A \subseteq (A \cup B) \) and \( B \subseteq (A \cup B) \). By Theorem 6.2 (iv), \( \text{srw-cl}(A) \subseteq \text{srw-cl}(A \cup B) \) and \( \text{srw-cl}(B) \subseteq \text{srw-cl}(A \cup B) \). Hence \( \text{srw-cl}(A) \cup \text{srw-cl}(B) \subseteq \text{srw-cl}(A \cup B) \).

**Theorem 6.6** If \( A \) is a subset of a space \( X \), then

i) \( \text{srw-cl}(A) \subseteq \text{srw-cl}(A) \).

ii) \( \text{srw-cl}(A) \subseteq \text{cl}(A) \).

**Proof:** Let \( A \) be subset of space \( X \).

i) From lemma 2.4 (i), if \( A \subseteq F \in \text{cRWC}(X) \), then \( A \subseteq \overline{F} \). Hence every \( \text{srw-closed} \) subset \( F \) closed. That is \( \text{srw-cl}(A) \subseteq F \).

Therefore \( \text{srw-cl}(A) \subseteq \cap \{F : A \subseteq F \in \text{cRWC}(X)\} = \text{srw-cl}(A) \).

ii) From lemma 2.4 (ii), if \( A \subseteq F \) and \( F \) is semi closed subset of \( X \) then \( A \subseteq \overline{F} \subseteq X \).

Remark 6.7 Containment relations in the above theorem 6.6 may be proper as seen in the following example 6.7.

**Example 6.8** Let \( X = \{a, b, c\} \) with topological space \( \tau = \{X, \phi, \{a, b, c\}, \{a, b, c, d\}\} \). \( \text{SRWC}(X) = \{X, \phi, \{a, d\}, \{b, c\}, \{a, d\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}\} \). Let \( A = \{a\} \) and \( B = \{a, b, d\} \) then \( \text{srw-cl}(A) = \{a\}, \text{srw-cl}(A) = \{a, d\}, \text{cl}(B) = X, \text{cl}(B) = \{a, b, d\} \).

i) \( \text{srw-cl}(\{a\}) \subseteq \text{srw-cl}(\{a\}) \Rightarrow \{a\} \subseteq \{a, d\} \). But \( \text{srw-cl}(A) \neq \text{srw-cl}(A) \).

ii) \( \text{srw-cl}(\{a\}) \subseteq \text{cl}(\{a\}) \Rightarrow \{a\} \subseteq \{a, d\} \). But \( \text{srw-cl}(A) \neq \text{cl}(A) \).

**Remark 6.9** If \( A \) is a subset of a space \( X \), then

i) \( \text{srw-cl}(A) \subseteq \phi - \text{cl}(A) \).

ii) \( \text{srw-cl}(A) \subseteq \text{cl}(A) \).

iii) \( \text{srw-cl}(A) \subseteq \phi - \text{cl}(A) \).

**Theorem 6.10** If \( A \) is a subset of a space \( X \), then \( \text{gs-cl}(A) \subseteq \text{srw-cl}(A) \).

**Proof:** Let \( A \) be a subset of space \( X \). From lemma 2.4 (iii), if \( A \subseteq F \in \text{SRWC}(X) \), then \( A \subseteq \mathcal{GSC}(X) \), because every \( \text{srw-closed} \) subset is \( \text{gs-closed} \). That is \( \text{gs-cl}(A) \subseteq F \). Therefore \( \text{gs-cl}(A) \subseteq \cap \{F : A \subseteq F \in \text{SRWC}(X)\} = \text{srw-cl}(A) \).

**Remark 6.11** If \( A \) is a subset of a space \( X \), then \( \text{gs-cl}(A) \subseteq \text{srw-cl}(A) \).

**Theorem 6.12** Let \( \overline{x} \in X \), then \( \overline{x} \) is \( \text{srw-cl}(A) \) if and only if \( V \cap A = \phi \) for every \( \text{srw-open set} V \) containing \( x \).

**Proof:** Let \( x \in \text{srw-cl}(A) \). Suppose there exists a \( \text{srw-open set} V \) containing \( x \) such that \( V \cap A = \phi \).

Since \( A \subseteq X \setminus V \) and by 6.2 (iv), \( \text{srw-cl}(A) \subseteq X \setminus V \). This implies \( x \in \text{srw-cl}(A) \) which is contradiction.

Conversely, we assume that \( V \cap A = \phi \) for every \( \text{srw-open set} V \) containing \( x \). Suppose \( x \notin \text{srw-cl}(A) \), then by definition 6.2 (i), there exists a \( \text{srw-closed subset} F \) containing \( A \) such that \( x \notin X \). Therefore \( x \in X \setminus F \) and \( X \setminus F \) is an \( \text{srw-open} \). Since \( A \subseteq F \), \( \text{X} \setminus F \cap A = \phi \) which is impossible as \( x \in X \setminus F \) and \( x \in A \). Hence \( x \notin \text{srw-cl}(A) \).

**Theorem 6.13** Let \( A \) be a subset of \( X \). Then

i) \( X \setminus \text{srw-int}(A) = \text{srw-cl}(X \setminus A) \)

ii) \( \text{srw-int}(A) = X \setminus \text{srw-cl}(X \setminus A) \)

iii) \( \text{srw-cl}(A) = X \setminus (\text{srw-int}(X \setminus A)) \)

**Proof:** (i) Let \( x \in X \setminus \text{srw-int}(A) \). Then \( x \notin \text{srw-int}(A) \), i.e. every \( \text{srw-open set} U \)
containing \( x \) is such that \( U \not\subseteq A \). i.e. every srw-open set \( U \) containing \( x \) is such that \( U \cap X \setminus A \neq \emptyset \). By Theorem 6.12, 
\[ x \in \text{srw} - \text{cl}(X \setminus A) \]. Therefore
\[ X \setminus (\text{srw} - \text{int}(A)) \subset \text{srw} - \text{cl}(X \setminus A). \]
Conversely, let \( x \in \text{srw} - \text{cl}(X \setminus A) \). Then by Theorem 6.12, every srw-open set \( U \) containing \( x \) is such that \( U \cap X \setminus A \neq \emptyset \). i.e. every srw-open set \( U \) containing \( x \) is such that \( U \not\subseteq A \) implies that by definition of \( \text{srw-int}(A) \), \( x \in \text{srw} - \text{int}(A) \). i.e. 
\[ x \in X \setminus (\text{srw} - \text{int}(A)) \quad \text{and} \quad \text{srw} - \text{cl}(X \setminus A) \subset X \setminus (\text{srw} - \text{int}(A)). \]
Thus 
\[ X \setminus (\text{srw} - \text{int}(A)) = \text{srw} - \text{cl}(X \setminus A). \]

(i) By taking complements to above (i).

(ii) Follows by replacing \( A \) by \( X \setminus A \) in (i).

VII. CONCLUSION
In this article we have studied most of the basic properties. With the help of these properties we will investigate srw-continuous and irresolute functions in topological spaces and fuzzy topological spaces.

REFERENCES
