A new multidimensional integral transform concerning the multivariable Aleph-function

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ABSTRACT

In the present document, we use the multidimensional integral transform introduced by Chandel et al [2] concerning the multivariable Aleph-function defined by Ayant [1]. Some interesting special cases are also discussed.

KEYWORDS: Aleph-function of several variables, multidimensional integral transform, Multivariable I-function, Aleph-function of two variables, I-function of two variables.

1. Introduction

Chandel et al [2] introduce a new multidimensional integral transform defined by:

\[ R_{\alpha_1, \ldots, \alpha_r}^{(a,b)} \{ \} = \frac{\Gamma(\alpha_1 + \cdots + \alpha_r)\Gamma(1/2 + a - b + \alpha_1 + \cdots + \alpha_r)2^{2a+2\alpha_1+\cdots+2\alpha_r-1}}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_r)\Gamma(2a + 2\alpha_1 + \cdots + 2\alpha_r)\Gamma(1/2 - (a + b + \alpha_1 + \cdots + \alpha_r))} \]

\int_0^\infty \cdots \int_0^\infty (x_1 + \cdots + x_r)^a(1 + x_1 + \cdots + x_r)^{-1/2} \left[ (x_1 + \cdots + x_r)^{1/2} + (1 + x_1 + \cdots + x_r)^{1/2} \right]^{2b} x_1^{\alpha_1-1} \cdots x_r^{\alpha_r-1} \, dx_1 \cdots dx_r \tag{1.1} \]

where \( 0 < Re(a + \alpha_1 + \cdots + \alpha_r) < 1/2 - Re(b), Re(\alpha_i) > 0, i = 1, \cdots, r \)

and give two-dimensional integral transforms concerning the multivariable H-function defined by Srivastava et al [6]. Here in the present document, we extend this work with the multivariable Aleph-function. The Aleph-function of several variables generalize the multivariable I-function defined by Sharma and Ahmad [4], itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occuring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

We have:

\[ \psi(s_1, \cdots, s_r) \prod_{k=1}^r \theta_k(s_k) \xi_k^\theta s_1 \cdots ds_r \]

where \( \xi_k^\theta = \sqrt{-1} \)

ISSN: 2231-5373 http://www.ijmttjournal.org
For more details, see Ayant [1]. The reals numbers $\tau_i$ are positives for $i = 1, \cdots, R$, $\tau_{ij}^{(k)}$ are positives for $i^{(k)} = 1, \cdots, R^{(k)}$. The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as:

$$\frac{1}{2} A_i^{(k)} < \arg z_k$$

where

$$A_i^{(k)} = \sum_{j=1}^{n_k} \alpha_j^{(k)} - \tau_i \sum_{j=n+1}^{p_i} \alpha_{ji}^{(k)} - \tau_i \beta_{ji}^{(k)} + \sum_{j=1}^{n_k} \gamma_j^{(k)} - \tau_i \gamma_{ji}^{(k)}$$

and

$$+ \sum_{j=m_k+1}^{m_k} \delta_j^{(k)} - \tau_i \gamma_{ji}^{(k)} > 0,$n_k$$

with $k = 1, \cdots, r$, $i = 1, \cdots, R$, $i^{(k)} = 1, \cdots, R^{(k)}$ (1.3)

The complex numbers $z_{1}, \cdots, z_{r}$ are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable Aleph-function. We may establish the asymptotic expansion in the following convenient form:

$$N(z_{1}, \cdots, z_{r}) = 0(\left| z_{1}\right|^\alpha_{1}, \cdots, \left| z_{r}\right|^\alpha_{r}) \rightarrow 0$$

$$N(z_{1}, \cdots, z_{r}) = 0(\left| z_{1}\right|^\beta_{1}, \cdots, \left| z_{r}\right|^\beta_{r}) \rightarrow \infty$$

where, with $k = 1, \cdots, r$:

$$\alpha_{k} = \min [Re(d_{j}^{(k)}/\delta_{j}^{(k)})], j = 1, \cdots, m_k$$

and

$$\beta_{k} = \max [Re((c_{j}^{(k)} - 1)/\gamma_{j}^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper:

$$U = p_{1}, q_{1}, \tau_{1}; R; V = m_{1}, n_{1}; \cdots; m_{r}, n_{r}$$

$$W = p_{1}^{(r)}, q_{1}^{(r)}, \tau_{1}^{(r)}; R_{1}^{(r)}, \cdots, p_{i}^{(r)}, q_{i}^{(r)}, \tau_{i}^{(r)}; R_{i}^{(r)}$$

$$A = \{ (a_{j}^{(1)}, \cdots, a_{j}^{(r)}), (\tau_{i}, \alpha_{ji}^{(1)}, \cdots, \alpha_{ji}^{(r)})_{m+1,p_{i}} \}$$

$$B = \{ (\beta_{ji}^{(1)}, \cdots, \beta_{ji}^{(r)})_{m+1,q_{i}} \}$$

$$C = \{ (c_{j}^{(1)}, \gamma_{j}^{(1)}), (c_{j}^{(r)}, \gamma_{j}^{(r)}), (\tau_{i}^{(1)}, \alpha_{ji}^{(1)}), \cdots, (\tau_{i}^{(r)}, \alpha_{ji}^{(r)})_{m+1,p_{i}} \}$$

$$D = \{ (d_{j}^{(1)}, \delta_{j}^{(1)}, (d_{j}^{(r)}, \delta_{j}^{(r)}), (\tau_{i}^{(1)}, \delta_{ji}^{(1)}), \cdots, (\tau_{i}^{(r)}, \delta_{ji}^{(r)})_{m+1,q_{i}} \}$$

The multivariable Aleph-function write:

$$N(z_{1}, \cdots, z_{r}) = N_{W_{U}; V}^{(z_{1}, \cdots, z_{r})}$$

2. Required formulas

$$R_{\tau_{1}, \cdots, \tau_{r}}^{\{1\}} = 1$$

$$R_{\tau_{1}, \cdots, \tau_{r}}^{\{1\}} \left( x_{1} + \cdots + x_{r} \right)^{1/2} + \left( (x_{1} + \cdots + x_{r})^{1/2} + (1 + x_{1} + \cdots + x_{r})^{1/2} \right)^{-2(n_{1} + \cdots + n_{r})}$$
In this section, making an appeal to (2.2) and (2.3), we derive the following results involving the multivariable Aleph-function defined by Ayant [1]

\[
\begin{align*}
\Gamma(1/2 + a - b + \alpha_1 + \cdots + \alpha_r) \\
4^{\xi_1 + \cdots + \xi_r} \Gamma(2a + 2\alpha_1 + \cdots + 2\alpha_r) \Gamma(1/2 - (a + b + \alpha_1 + \cdots + \alpha_r)) \\
\Gamma(2(a + \alpha_1 + \cdots + \alpha_r + \xi_1 + \cdots + \xi_r)) \Gamma(1/2 - (a + b + \alpha_1 + \cdots + \alpha_r) + \eta_1 - \xi_1 + \cdots + \eta_r - \xi_r) \\
\Gamma(1/2 + a - b + \alpha_1 + \xi_1 + \eta_1 + \cdots + \alpha_r + \xi_r + \eta_r) \\
R^{(a,b)}_{\alpha_1,\ldots,\alpha_r} \left\{ x_1^{\lambda_1} \cdots x_r^{\lambda_r} (x_1 + \cdots + x_r)^{\xi_1 + \cdots + \xi_r} \right\} \\
\left[ (x_1 + \cdots + x_r)^{1/2} + (1 + x_1 + \cdots + x_r)^{1/2} \right]^{-2(\eta_1 + \cdots + \eta_r)} \\
\Gamma(1/2 - a - b + \alpha_1 + \cdots + \alpha_r) \\
\Gamma(1/2 + b + \alpha_1 + \lambda_1 + \cdots + \alpha_r + \lambda_r) \\
\Gamma(2(a + \alpha_1 + \cdots + \alpha_r)) \\
\Gamma(1/2 - a - \alpha_1 - \cdots - \alpha_r) \\
\Gamma(2(a + \alpha_1 + \cdots + \alpha_r + \xi_1 + \cdots + \xi_r + \eta_1 + \cdots + \eta_r + \lambda_1 + \cdots + \lambda_r)) \\
= \frac{\Gamma(\alpha_1 + \cdots + \alpha_r) \prod_{i=1}^{r} \Gamma(\alpha_i + \lambda_i) \Gamma(1/2 + a - b + \alpha_1 + \cdots + \alpha_r) 4^{-(\xi_1 + \cdots + \xi_r + \lambda_1 + \cdots + \lambda_r)}}{\Gamma(\alpha_1) \cdots \Gamma(\alpha_r) \Gamma(\alpha_1 + \lambda_1 + \cdots + \alpha_r + \lambda_r) \Gamma(2(a + \alpha_1 + \cdots + \alpha_r)) \Gamma(1/2 - a - \alpha_1 - \cdots - \alpha_r)} \\
\Gamma(1/2 - a + b + \alpha_1 - \alpha_r - \lambda_r - \eta_r - \xi_1 - \cdots - \eta_r - \xi_r) \\
\Gamma(1/2 + a - b + \alpha_1 + \lambda_1 + \xi_1 + \cdots + \alpha_r + \lambda_r + \xi_r + \eta_r) \\
(2.3)
\end{align*}
\]

valid if \(0 < Re(a + \xi_1 + \alpha_1 + \lambda_1 + \cdots + \xi_r + \alpha_r + \lambda_r) < Re(1/2 - b + \eta_1 + \cdots + \eta_r)\)

\(Re(\alpha_i) > 0, Re(\lambda_i) > 0, i = 1, \cdots, r\)

3. Main integrals

In this section, making an appeal to (2.2) and (2.3), we derive the following results involving the multivariable Aleph-function defined by Ayant [1]

\[
\begin{align*}
R^{(a,b)}_{\alpha_1,\ldots,\alpha_r} \left\{ \begin{array}{c}
\mathcal{N}^{0,n,\mathcal{V}}_{U;W} \\
\vdots \\
\mathcal{N}^{0,n+2,\mathcal{V}}_{U;W}
\end{array} \right\} \\
\frac{\Gamma(1/2 + a - b + \alpha_1 + \cdots + \alpha_r)}{\Gamma(2a + 2\alpha_1 + \cdots + 2\alpha_r) \Gamma(1/2 - a - b - \alpha_1 - \cdots - \alpha_r)} \\
A : C \\
\ldots \\
B : D \\
\begin{pmatrix}
4^{-\xi_1} z_1 \\
\vdots \\
4^{-\xi_r} z_r
\end{pmatrix} \\
(1-2a-2\alpha_1 - \cdots - 2\alpha_r; 2\xi_1, \cdots, 2\xi_r, 1/2 + a + b + \alpha_1 + \cdots + \alpha_r; \eta_1 - \xi_1, \cdots, \eta_r - \xi_r, A : C) \\
\ldots \\
(1/2-a+b-\alpha_1 - \cdots - \alpha_r; \xi_1 + \eta_1, \cdots, \xi_r + \eta_r, B : D)
\end{align*}
\]
where \( U_{21} = p_i + 2; q_i + 1; \tau_i; R \)

Provided that

a) \( 0 < Re(a + \alpha_1 + \cdots + \alpha_r) < 1/2 - Re(b), \, Re(\alpha_i) > 0, \, i = 1, \cdots, r \)

b) \[ \left| \arg z_k \right| < \frac{1}{2} A_i^{(k)} \pi, \] where \( A_i^{(k)} \) is given in (1.3)

\[
R_{\alpha_1, \cdots, \alpha_r}^{(a, b)} \left\{ \begin{array}{c}
N_{U_{21}}^{0, n-2; V} \\
\cdot \\
\cdot \\
N_{U_{21}}^{0, n-2; V} \\
\cdot \\
\cdot \\
4^{-\zeta_1 - \lambda_1 - \lambda_r} \left| z_1 \right| \left( 1 - a - b - \alpha_1 - \cdots - \alpha_r, \eta_1 + \lambda_1 - \zeta_1, \cdots, \eta_r + \lambda_r - \zeta_r \right) \\
4^{-\zeta_r - \lambda_r - \lambda_1} \left| z_r \right| \left( 1/2 + a + b + \alpha_1 + \cdots + \alpha_r, \eta_1 - \zeta_1 - \lambda_1, \cdots, \eta_r - \zeta_r - \lambda_r \right) \\
\cdot \\
\cdot \\
A : C \\
\cdots \\
B : D \\
\end{array} \right\} 
\]

\[
\frac{\Gamma(\alpha_1 + \cdots + \alpha_r) \prod_{i=1}^r \Gamma(\alpha_i + \lambda_i) \Gamma(1/2 + a - b + \alpha_1 + \cdots + \alpha_r)}{\Gamma(\alpha_1) \cdots \Gamma(\alpha_r) \Gamma(\alpha_1 + \lambda_1 + \cdots + \alpha_r + \lambda_r) \Gamma(2(a + \alpha_1 + \cdots + \alpha_r)) \Gamma(1/2 - a - b - \alpha_1 - \cdots - \alpha_r)} 
\]

\[
N_{U_{21}}^{0, n-2; V} \\
\cdot \\
\cdot \\
4^{-\zeta_1 - \lambda_1 - \lambda_r} \left| z_1 \right| \left( 1 - a - b - \alpha_1 - \cdots - \alpha_r, \eta_1 + \lambda_1 - \zeta_1, \cdots, \eta_r + \lambda_r - \zeta_r \right) \\
4^{-\zeta_r - \lambda_r - \lambda_1} \left| z_r \right| \left( 1/2 + a + b + \alpha_1 + \cdots + \alpha_r, \eta_1 - \zeta_1 - \lambda_1, \cdots, \eta_r - \zeta_r - \lambda_r \right) \\
\cdot \\
\cdot \\
A : C \\
\cdots \\
B : D \\
\end{array} \right\} 
\]

where \( U_{21} = p_i + 2; q_i + 1; \tau_i; R \)

Provided that

a) \( 0 < Re(a + \alpha_1 + \cdots + \alpha_r) < 1/2 - Re(b), \, Re(\alpha_i) > 0, \, Re(\lambda_i) > 0, \, i = 1, \cdots, r \)

b) \[ \left| \arg z_k \right| < \frac{1}{2} A_i^{(k)} \pi, \] where \( A_i^{(k)} \) is given in (1.3)
4. Special cases

a) For \( \lambda_i = 0, i = 1, \ldots, r \), (3.2) reduces to (3.1)

\[
R_{(a,b)}^{(\alpha_1, \ldots, \alpha_r)} \left\{ \begin{array}{c}
N_{U_1;:W}^{0,n;V} \\
N_{U;:V}^{0,n;W}
\end{array} \right. \begin{align*}
&z_1(x_1 + \cdots + x_r)^{\zeta_1} \left[ (x_1 + \cdots + x_r)^{1/2} + (1 + x_1 + \cdots + x_r)^{1/2} \right]^{-2\zeta_1} \\
&\vdots \\
&z_r(x_1 + \cdots + x_r)^{\zeta_r} \left[ (x_1 + \cdots + x_r)^{1/2} + (1 + x_1 + \cdots + x_r)^{1/2} \right]^{-2\zeta_r}
\end{align*}
\]

A : C

\[
\frac{\Gamma(1/2 + a - b + \alpha_1 + \cdots + \alpha_r)}{\Gamma(2a + 2\alpha_1 + \cdots + 2\alpha_r)\Gamma(1/2 - a - b - \alpha_1 - \cdots - \alpha_r)}
\left( \begin{array}{c}
4^{-\zeta_1} z_1 \\
\vdots \\
4^{-\zeta_r} z_r
\end{array} \right)
\]

(4.1)

\[
(1-2a-2\alpha_1 - \cdots - 2\alpha_r; 2\zeta_1, \ldots, 2\zeta_r), A : C
\]

\[
(1/2-a+b-\alpha_1 - \cdots - \alpha_r; 2\zeta_1, \ldots, 2\zeta_r), B : D
\]

where \( U_{11} = p_i + 1; q_i + 1; \tau_i; R \)

Provided that

a) \( 0 < Re(a + \alpha_1 + \cdots + \alpha_r) < 1/2 - Re(b), Re(\alpha_i) > 0, i = 1, \ldots, r \)

b) \( \left| \frac{arg z_k}{4\zeta} \right| < \frac{1}{2} A_i^{(k)} \pi \), where \( A_i^{(k)} \) is given in (1.3)

5. Multivariable I-function

If \( \tau_i, \tau_i^{(1)}, \ldots, \tau_i^{(r)} \rightarrow 1 \), the Aleph-function of several variables degenerate to the I-function of several variables. The two formulas have been derived in this section for multivariable I-functions defined by Sharma et al [3].
\[
\begin{aligned}
A : C \\
\ldots \\
B : D \\
\end{aligned}
\left\{ \begin{array}{l}
\frac{\Gamma(1/2 + a - b + \alpha_1 + \cdots + \alpha_r)}{\Gamma(2a + 2\alpha_1 + \cdots + 2\alpha_r)\Gamma(1/2 - a - b - \alpha_1 - \cdots - \alpha_r)} \int_{U_{21}:W}^{0,n+2;V} \left( \begin{array}{c}
4^{-\zeta_1 z_1} \\
\cdot \\
4^{-\zeta_r z_r}
\end{array} \right) dU_{21;W}
\end{array} \right.
\end{aligned}
\]

\[
(1-2a-2\alpha_1 - \cdots - 2\alpha_r; 2\zeta_1, \cdots, 2\zeta_r), (1/2 + a + b + \alpha_1 + \cdots + \alpha_r; \eta_1 - \zeta_1, \cdots, \eta_r - \zeta_r), A : C
\]
\[
\ldots
\]

\[
(1/2 - a - b - \alpha_1 - \cdots - \alpha_r; \zeta_1 + \eta_1, \cdots, \zeta_r + \eta_r), B : D
\]

(5.1)

under the same notations and conditions that (3.1)

\[
\begin{aligned}
R_{(\alpha_1, \ldots, \alpha_r)}^{(\alpha, b)} \\
\int_{U:W}^{0,n;V} \\
\left( \begin{array}{l}
z_1 x_1^{\lambda_1} (x_1 + \cdots + x_r)^{\zeta_1} \left[ (x_1 + \cdots + x_r)^{1/2} + (1 + x_1 + \cdots + x_r)^{1/2} \right]^{-2\eta_1} \\
\cdot \\
z_r x_r^{\lambda_r} (x_1 + \cdots + x_r)^{\zeta_r} \left[ (x_1 + \cdots + x_r)^{1/2} + (1 + x_1 + \cdots + x_r)^{1/2} \right]^{-2\eta_r}
\end{array} \right)
\end{aligned}
\]

\[
\begin{aligned}
A : C \\
\ldots \\
B : D
\end{aligned}
\left\{ \begin{array}{l}
= \frac{\Gamma(\alpha_1 + \cdots + \alpha_r) \prod_{i=1}^{r} \Gamma(\alpha_i + \lambda_i)\Gamma(1/2 + a - b + \alpha_1 + \cdots + \alpha_r)}{\Gamma(\alpha_1) \cdots \Gamma(\alpha_r)\Gamma(\alpha_1 + \lambda_1 + \cdots + \alpha_r + \lambda_r)\Gamma(2(a + \alpha_1 + \cdots + \alpha_r))\Gamma(1/2 - a - b - \alpha_1 - \cdots - \alpha_r)}
\end{array} \right.
\end{aligned}
\]

\[
\int_{U_{21}:W}^{0,n+2;V} \left( \begin{array}{c}
4^{-\zeta_1 - \lambda_1 z_1} \\
\cdot \\
4^{-\zeta_r - \lambda_r z_r}
\end{array} \right) (1-2a-2\alpha_1 - \cdots - 2\alpha_r; 2(\zeta_1 + \lambda_1), \cdots, 2(\zeta_r + \lambda_r)),
\]

\[
(1/2 - a - b - \alpha_1 - \cdots - \alpha_r; \zeta_1 + \eta_1 + \lambda_1, \cdots, \zeta_r + \eta_r + \lambda_r)
\]

\[
(1/2 + a + b + \alpha_1 + \cdots + \alpha_r; \eta_1 - \zeta_1 - \lambda_1, \cdots, \eta_r - \zeta_r - \lambda_r), A : C
\]

\[
\ldots
\]

\[
B : D
\]

(5.2)
6. Aleph-function of two variables

If \( r = 2 \), we obtain the Aleph-function of two variables defined by K. Sharma [5], and we have the following two relations.

\[
R_{\alpha_1, \alpha_2}^{(a, b)} \left\{ \begin{array}{c}
N_{U: W}^{0, n: V} \\
\left( z_1(x_1 + x_2)^{\zeta_1} \left[ (x_1 + x_2)^{1/2} + (1 + x_1 + x_2)^{1/2} \right]^{-2\eta_1} \right) \\
\vdots \\
\vdots \\
z_2(x_1 + x_2)^{\zeta_2} \left[ (x_1 + x_2)^{1/2} + (1 + x_1 + x_2)^{1/2} \right]^{-2\eta_2} \end{array} \right| \begin{array}{c}
A : C \\
\vdots \\
B : D 
\end{array}
\]

\[
= \frac{\Gamma(1/2 + a - b + \alpha_1 + \alpha_2)}{\Gamma(2a + 2\alpha_1 + 2\alpha_2)\Gamma(1/2 - a - b - \alpha_1 - \alpha_2)} N_{U: W}^{0, n+2: V} \left( \begin{array}{c}
4^{-\zeta_1} z_1 \\
\vdots \\
4^{-\zeta_2} z_2 
\end{array} \right)
\]

\[
(1/2 - a - 2\alpha_1 - 2\alpha_2; 2\zeta_1, 2\zeta_2), (1/2 + a + b + \alpha_1 + \alpha_2; \eta_1 - \zeta_1, \eta_2 - \zeta_2), A : C \\
\vdots \\
(1/2 - a + b - \alpha_2; \zeta_1 + \eta_1, \zeta_2 + \eta_2), B : D
\]

under the same notations and conditions that (3.1) with \( r = 2 \).
under the same conditions and notations that (3.2) with $r = 2$

7. I-function of two variables

If $\tau_1, \tau_1', \tau_1'' \rightarrow 1$, then the Aleph-function of two variables degenerate in the I-function of two variables defined by Sharma et al. [4] and we obtain the same formulas with the I-function of two variables.

\[
\begin{align*}
R_{(\alpha_1, \alpha_2)}^{(a,b)} \left\{ \begin{array}{c}
\frac{1}{2} + a + b + \alpha_1 + \alpha_2; \eta_1 - \zeta_1 - \lambda_1, \eta_2 - \zeta_2 - \lambda_2, A : C \\
\quad \vdots \\
\quad B : D \\
\end{array} \right. \\
&= \left. \frac{\Gamma(1/2 + a - b + \alpha_1 + \alpha_2)}{\Gamma(2a + 2\alpha_1 + 2\alpha_2)\Gamma(1/2 - a - b - \alpha_1 - \alpha_2)} \right. \\
&\quad \frac{1}{U^W:V^W} \begin{pmatrix}
z_1(x_1 + x_2)^{\zeta_1} \left[ (x_1 + x_2)^{1/2} + (1 + x_1 + x_2)^{1/2} \right]^{-2\eta_1} \\
\quad \vdots \\
\quad B : D \\
\end{pmatrix}
\end{align*}
\]

(1/2 - a + b - 2\alpha_1; 2\zeta_1, 2\zeta_2), (1/2 + a + b + \alpha_1 + \alpha_2; \eta_1 - \zeta_1, \eta_2 - \zeta_2), A : C \\
\quad \vdots \\
(1/2 - a + b - \alpha_2; \zeta_1 + \eta_1, \zeta_2 + \eta_2), B : D

under the same notations and conditions that (3.1) with $r = 2$
\[
\frac{\Gamma(\alpha_1 + \alpha_2) \prod_{i=1}^{2} \Gamma(\alpha_i + \lambda_i) \Gamma(1/2 + a - b + \alpha_1 + \alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_1 + \alpha_2 + \lambda_2)\Gamma(2(a + \alpha_1 + \alpha_2))\Gamma(1/2 - a - b - \alpha_1 - \alpha_2)}
\]

\[
\int_{U_{21}:W}^{0,n+2:V} \left. \left( \begin{array}{c}
4^{-\zeta_1 - \lambda_1} z_1 \\
\vdots \\
4^{-\zeta_2 - \lambda_2} z_2 \\
(1/2-a+b-\alpha_1 - \alpha_2; \zeta_1 + \eta_1 + \lambda_1, \zeta_2 + \eta_2 + \lambda_2)
\end{array} \right) \right| \\
(1/2+a+b+\alpha_1 + \alpha_2; \eta_1 - \zeta_1 - \lambda_1, \eta_2 - \zeta_2 - \lambda_2), A : C \\
\vdots \\
B : D \\
(7.2)
\]

under the same conditions and notations that (3.2) with \( r = 2 \)

8. Conclusion

In this paper we have evaluated two multidimensional integral transform concerning the multivariable Aleph-function. The formulas established in this paper is of very general nature as it contains multivariable Aleph-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

REFERENCES


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