Properties of supra N-open sets

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Abstract: The purpose of this paper is to introduce and study some of the properties of supra N-open sets via supra N-derived, supra N-border, supra N-frontier and supra N-exterior. Also we introduce some separation axioms using supra N-open sets.

Mathematics subject classification: 54A10, 54A05, 54D10.

Keywords: supra N-interior, supra N-closure, supra N-derived, supra N-frontier, supra N-border, supra N-exterior, supra N-T\textsubscript{0}, supra N-T\textsubscript{1}, supra N-T\textsubscript{2}.

1. INTRODUCTION

In 1983, A.S.Mashhour et al \cite{4} introduced supra topological spaces and studied, s-continuous functions and s*-continuous functions. The Authors have introduced the notion of supra N-closed set\cite{6} in supra topological spaces.

In this paper, we bring out some of the concept of supra N-derived, supra N-border, supra N-frontier and supra N-exterior of a set and study their properties. And also brings out some of the separation axioms by using supra N-open sets.

2. PRELIMINARIES

\textbf{Definition 2.1.} A subfamily \( \mu \) of \( X \) is said to be supra topology on \( X \) if

\begin{enumerate}
\item \( X, \emptyset \in \mu \)
\item If \( A_i \in \mu, \forall i \in J \) then \( \bigcup A_i \in \mu \)
\end{enumerate}

\( (X, \mu) \) is called supra topological space.

The element of \( \mu \) are called supra open sets in \( (X, \mu) \) and the complement of supra open set is called supra closed sets and it is denoted by \( \mu^c \).

\textbf{Definition 2.3.} Let \( (X, \tau) \) be a topological space and \( \mu \) be a supra topology on \( X \). We call \( \mu \) a supra topology associated with \( \tau \), if \( \tau \subseteq \mu \).

\textbf{Definition 2.4.} A subset \( A \) of a space \( X \) is called

\begin{enumerate}
\item supra semi-open set\cite{3}, if \( A \subseteq \text{cl}^\mu(\text{int}^\mu(A)) \).
\item supra \( \alpha \)-open set\cite{2}, if \( A \subseteq \text{int}^\mu(\text{cl}^\mu(\text{int}^\mu(A))) \).
\end{enumerate}

\textbf{Definition 3.1.} Let \( A \) be subset of a supra topological space \( X \). An element \( x \in X \) is said to be a supra N-limit point of \( A \), if every supra N-open set \( U \) in \( X \) containing \( x \), such that \( U \cap (A \setminus \{x\}) \neq \emptyset \).

\textbf{Definition 3.2.} The set of all supra N-limit points of \( A \) is called supra N-derived set of \( A \). It is denoted by \( D^\mu_N(A) \).

\textbf{Theorem 3.3.} Let \( A, B \) be subsets of a supra topological space \( X \), then

\begin{enumerate}
\item \( D^\mu_N(A) \subseteq D^\mu(A) \).
\end{enumerate}
Example 3.4

Let $A = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, X\}$. super N-open sets are \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}. suprA N-closed sets are \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}. Let $A = \{a\}$, $B = \{b, c\}$, $C = \{a, b\}$. Hence $D_N^\mu(A) \subseteq D_N^\mu(B)$.

(i) If $A \subseteq B$, then $D_N^\mu(A) \subseteq D_N^\mu(B)$.

(ii) If $A \subseteq B$ then $D_N^\mu(A) \subseteq D_N^\mu(A \cap B)$.

(iii) $D_N^\mu(A \cap B) \subseteq D_N^\mu(A) \cap D_N^\mu(B)$.

(iv) $D_N^\mu(A) \cap A = D_N^\mu(A) \cap (X \setminus Nint^\mu(A))$.

(v) $D_N^\mu(A \cup D_N^\mu(A)) \subseteq A \cup D_N^\mu(A)$.

Proof

(i) It is obvious, since every super N-open set is super N-open set.

(ii) Let $x \in D_N^\mu(A)$, then $x$ is super N-limit point of $A$. Then every neighbourhood of $x$ contains a point of $A$ different from $x$. Since $A \subseteq B$, every neighbourhood of $x$ contains a point of $B$ different from $x$. Hence $x \in D_N^\mu(B)$. Therefore $D_N^\mu(A) \subseteq D_N^\mu(B)$.

(iii) Since $D_N^\mu(A \cap B) \subseteq D_N^\mu(A)$ and $D_N^\mu(A \cap B) \subseteq D_N^\mu(B)$, proof follows from (ii).

(iv) If $x \in D_N^\mu(A) \setminus A$ and $U$ is a super N-open set containing $x$, then $U \cap (D_N^\mu(A) \setminus \{x\}) \neq \emptyset$. Let $y \in U \cap (D_N^\mu(A) \setminus \{x\})$. Then, since $y \in D_N^\mu(A)$ and $y \in U$, so $U \cap (A - \{y\}) \neq \emptyset$. Let $z \in U \cap (A - \{y\})$. Then, $z \neq x$ for $z \in A$ and $x \notin A$. Hence $U \cap (A - \{x\}) \neq \emptyset$. Therefore $x \in D_N^\mu(A)$.

(v) Let $x \in D_N^\mu(A \cup D_N^\mu(A))$. If $x \in A$, the result is obvious. So let $x \in D_N^\mu(A \cup D_N^\mu(A)) \setminus A$, then for super N-open set $U$ containing $x$, such that $U \cap (A \cup D_N^\mu(A) \setminus \{x\}) \neq \emptyset$. Thus $U \cap (A - \{x\}) \neq \emptyset$. Hence from (iv) $x \in D_N^\mu(A)$. Hence $x \in A \cup D_N^\mu(A)$. Therefore $D_N^\mu(A \cup D_N^\mu(A)) \subseteq A \cup D_N^\mu(A)$. The proof of the above theorem is shown by the following example.

Example 4.3

Let $\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, X\}$. super N-open sets are \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}. super N-closed sets are \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}. Let $A = \{a\}$, $B = \{b, c\}$, $C = \{a, b\}$, $D = \{a, c\}$. Hence $D_N^\mu(A) \subseteq D_N^\mu(B)$.

(i) If $A \subseteq B$, then $D_N^\mu(A) \subseteq D_N^\mu(B)$.

(ii) If $A \subseteq B$ then $D_N^\mu(A) \subseteq D_N^\mu(A \cap B)$.

(iii) $D_N^\mu(A \cap B) \subseteq D_N^\mu(A) \cap D_N^\mu(B)$.

(iv) $D_N^\mu(A) \cap A = D_N^\mu(A) \cap (X \setminus Nint^\mu(A))$.

(v) $D_N^\mu(A \cup D_N^\mu(A)) \subseteq A \cup D_N^\mu(A)$.

4. SUPRA N-BORDER

Definition 4.1 For a subset $A$ of $X$, supra N-Border of $A$ is defined as $Bd_N^\mu(A) = A - Nint^\mu(A)$.

Theorem 4.2 For a subset $A$ of a supra topological space $X$, the following statement hold.

(i) $Bd_N^\mu(A) \subseteq Bd^\mu(A)$, where $Bd^\mu(A)$ denotes supra border of $A$.

(ii) $Nint^\mu(A) \cup Bd_N^\mu(A) \subseteq A$.

(iii) $Nint^\mu(A) \cap Bd_N^\mu(A) = \emptyset$.

(iv) $A$ is supra N-open iff $Bd_N^\mu(A) = \emptyset$.

(v) $Bd_N^\mu(A \cup D_N^\mu(A)) \subseteq A \cup D_N^\mu(A)$.

(iv) $Bd_N^\mu(A) = A - Nint^\mu(A)$ iff $A$ is supra N-open.

(v) $Bd_N^\mu(A) - Nint^\mu(A) = 0$ iff $A$ is supra N-open.

Proof

(i) Obvious, since every super open set is supra N-open.

(ii) $Nint^\mu(A) \cup Bd_N^\mu(A) = Nint^\mu(A) \cup (A - Nint^\mu(A)) = A$.

(iii) $Nint^\mu(A) \cap Bd_N^\mu(A) = Nint^\mu(A) \cap (A - Nint^\mu(A)) = \emptyset$.

(iv) $Bd_N^\mu(A) = A - Nint^\mu(A)$ iff $A$ is supra N-open.

(v) $Bd_N^\mu(A) - Nint^\mu(A) = 0$ iff $A$ is supra N-open.

(vi) If $x \in Nint^\mu(Bd_N^\mu(A))$, then $x \in Bd_N^\mu(A)$. On the other hand, since $Bd_N^\mu(A) \subseteq A$, $x \in Nint^\mu(Bd_N^\mu(A)) \subseteq Nint^\mu(A)$. Hence $x \in Nint^\mu(A) \cap Bd_N^\mu(A)$, which contradicts (iii). Hence $Nint^\mu(Bd_N^\mu(A)) = \emptyset$.

(vii) $Bd_N^\mu(Bd_N^\mu(A)) = Bd_N^\mu(A - Nint^\mu(A)) = (A - Nint^\mu(A)) - Nint^\mu(A - Nint^\mu(A)) = A - Nint^\mu(A) = Bd_N^\mu(A)$.

(viii) $Bd_N^\mu(A) = A - Nint^\mu(A) = A - (X - Nint^\mu(A)) = A - Nint^\mu(A)$.

(ix) $Bd_N^\mu(A) = A - Nint^\mu(A) = A - (A \setminus D_N^\mu(X - A)) = D_N^\mu(X - A)$.

The proof of the above theorem is shown by the following example.

Example 4.3

Let $\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}\}$. super N-open sets are \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}. supra N-closed sets are \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}. Let $A = \{a\}$, $B = \{b, c\}$, $C = \{a, b\}$, $D = \{a, c\}$. Hence $Bd_N^\mu(A) \subseteq Bd^\mu(A)$.

(i) If $A \subseteq B$, then $Bd_N^\mu(A) \subseteq Bd^\mu(B)$.

(ii) If $A \subseteq B$ then $Bd_N^\mu(A) \subseteq Bd^\mu(A \cap B)$.

(iii) $Bd_N^\mu(A \cap B) \subseteq Bd^\mu(A) \cap Bd^\mu(B)$.

(iv) $Bd_N^\mu(A) \cap A = Bd_N^\mu(A) \cap (X \setminus Nint^\mu(A))$.

(v) $Bd_N^\mu(A \cup D_N^\mu(A)) \subseteq A \cup D_N^\mu(A)$.

Let $A = \{a\}$, $Bd_N^\mu(A) = \{a\}$. Hence $Bd_N^\mu(A) \subseteq Bd^\mu(A)$.
5. SUPRA N-FRONTIER

**Definition 5.1** For a subset A of a supra topological space X, Fr_n(A) = Ncl_n(A)−Nint_n(A) is said to be supra N-frontier of A.

**Theorem 5.2** For a subset A of a supra topological space X, the following statements hold:

(i) Fr_{n+1}(A) ⊆ Fr_{n}(A), where Fr_{n}(A) denotes supra frontier of A.

(ii) Ncl_{n}(A) = Nint_{n}(A) ∪ Fr_{n}(A).

(iii) Nint_{n}(A) ∩ Fr_{n}(A) = ∅.

(iv) Bd_{n}(A) ⊆ Fr_{n}(A).

(v) Fr_{n}(A) = Ncl_{n}(A) ∩ Ncl_{n}(X−A).

(vi) Fr_{n}(A) is supra N-closed.

(vii) Fr_{n}(Fr_{n}(A)) = Fr_{n}(A).

(viii) Fr_{n}(X−A) = Fr_{n}(X−A).

(ix) A ⊆ Fr_{n}(A).

The proof of the above theorem is shown by the following example.

**Example 5.3** Let X = {a, b, c} and τ = {X, ∅, {a, b}, {a, c}, {b, c}, {a, b, c}}. N-open set are {X, ∅, {a, b}, {a, c}, {b, c}, {a, b, c}}. N-closed set are {X, ∅, {a, c}, {b, c}, {a, b, c}}.

(i) Let A = {a, b}, Fr_{n}(A) = X and Fr_{n}(A) = {c}. Therefore Fr_{n}(A) ⊆ Fr_{n}(A).

(ii) Let A = {a, b}, Ncl_{n}(A) = {a, c}. Nint_{n}(A) = {a} and Fr_{n}(A) = {c}. Hence Ncl_{n}(A) = Nint_{n}(A) ∪ Fr_{n}(A).

(iii) Let A = {a, b}, Nint_{n}(A) = {a} and Fr_{n}(A) = {c}. Hence Nint_{n}(A) ⊆ Fr_{n}(A).

(iv) Let A = {b, c}, Bd_{n}(A) = {b, c} and Fr_{n}(A) = {c}. Hence Bd_{n}(A) ⊆ Fr_{n}(A).

(v) Let A = {b, c}, X = {X, ∅, {a, b}, {a, c}, {b, c}, {a, b, c}}.

(vi) Let A = {b, c}, Fr_{n}(A) = Fr_{n}(A) = Ncl_{n}(X−A).

(vii) Let A = {b, c}, Fr_{n}(Fr_{n}(A)) = Fr_{n}(A) = Ncl_{n}(X−A).

(viii) Let A = {b, c}, Fr_{n}(Fr_{n}(A)) = Fr_{n}(A) = X.

6. SUPRA N-EXTERIOR

**Definition 6.1** For a subset A of a supra topological space X, Ext_{n}(A) = Nint_{n}(X−A) is said to be supra N-exterior of A.

**Theorem 6.2** For a subset A of a supra topological space X, the following statements holds.

(i) Ext_{n}(A) ⊆ Ext_{n}(A), where Ext_{n}(A) denotes supra exterior of A.

(ii) Ext_{n}(A) is supra N-open.

(iii) Ext_{n}(A) = Nint_{n}(X−A) = X − Ncl_{n}(A).

(iv) Ext_{n}(A) = Ext_{n}(A) = Ext_{n}(A).

(v) Ext_{n}(A) ⊆ Ext_{n}(A) ⊆ Ext_{n}(A).

(vi) Ext_{n}(A) ⊆ Ext_{n}(A) ⊆ Ext_{n}(A).

(vii) Ext_{n}(A) ⊆ Ext_{n}(A) ⊆ Ext_{n}(A).
Example 6.3 The proof of the above theorem is also seen from following example.

Proof
(i) \( \text{Ext}^h(A) \leq \text{Ext}^{h+}(A) \), Since every supra open set is supra \( N \)-open set.
(ii) \( \text{Nint}^{h+}(\text{Ext}^{h+}(A)) = \text{Nint}^{h+}(\text{Nint}(X-A)) = \text{Nint}(X-A) = \text{Ext}^{h+}(A) \).

Remark 7.6

Example 7.7 Let \( A = \{a\} \) and \( B = \{b\} \). \( \text{Ext}^{h+}(\{a\}) = \{b\} \).

\( \text{Ext}^{h+}(\{b\}) = \{a\} \).

\( \text{Ext}^{h+}(\{a, b\}) = \{c\} \).

7. SOME SEPARATION AXIOMS.

Definition 7.1 A space \((X, \tau)\) is said to be supra \( N \)-\( T_0 \) if for \( x, y \in X \) such that \( x \neq y \), there exist a supra \( N \)-open set \( U \) of \( X \) containing \( x \) but not \( y \) (or) a supra \( N \)-open set \( V \) of \( X \) containing \( y \) but not \( x \).

Definition 7.2 A space \((X, \tau)\) is said to be supra \( N \)-\( T_1 \), if for \( x, y \in X \) such that \( x \neq y \), there exist a supra \( N \)-open set \( U \) of \( X \) containing \( x \) but not \( y \) and a supra \( N \)-open set \( V \) of \( X \) containing \( y \) but not \( x \).

Definition 7.3 A space \((X, \tau)\) is said to be supra \( N \)-\( T_2 \), if for \( x, y \in X \) such that \( x \neq y \), there exist disjoint supra \( N \)-open set \( U \) and \( V \) such that \( x \in U \) and \( y \in V \).

Remark 7.4 Every supra \( N \)-\( T_1 \) is supra \( N \)-\( T_0 \), converse need not be true. It is shown by the following example.

Example 7.5 Let \( X = \{a, b, c\} \), \( \tau = \{X, \varphi, \{a, b\}, \{a, c\}\} \). supra \( N \)-open sets are \( \{X, \varphi, \{a, b\}, \{a, c\}\} \). Let \( x = a \) and \( y = b \). Then \( X \) is supra \( N \)-\( T_0 \) space but not supra \( N \)-\( T_1 \) space, since there is no supra \( N \)-open set containing one point but not the other.

Remark 7.6 Every supra \( N \)-\( T_2 \) is supra \( N \)-\( T_0 \), converse need not be true. It is shown by the following example.

Example 7.7 Let \( X = \{a, b, c, d\} \), \( \tau = \{X, \varphi, \{a, b\}, \{a, c\}, \{a, d\}\} \). supra \( N \)-open sets are \( \{X, \varphi, \{a, b\}, \{a, c\}, \{a, d\}\} \). Let \( x = c \) and \( y = d \). Then \( X \) is supra \( N \)-\( T_0 \) space but not supra \( N \)-\( T_1 \) space, since there is no disjoint supra \( N \)-open sets containing \( c \) and \( d \) of \( X \).

Theorem 7.8 Let \( f(X, \tau) \rightarrow (Y, \sigma) \) be a supra \( N \)-irresolute, injective map. If \( Y \) is supra \( N \)-\( T_1 \), then \( X \) is supra \( N \)-\( T_1 \).

Proof Let \( Y \) be supra \( N \)-\( T_1 \) space. Let \( x, y \in X \), such that \( x \neq y \), since \( f \) is injective map, then \( f(x) \neq f(y) \in Y \). Since \( Y \) is supra \( N \)-\( T_1 \) space, there exist supra \( N \)-open set \( U \) and \( V \) in \( Y \) such that \( f(x) \in U \) and \( f(y) \in V \). Since \( f \) is supra \( N \)-irresolute, then \( f^{-1}(U) \) and \( f^{-1}(V) \) are supra \( N \)-open sets in \( X \) but not \( x \) or \( y \).
N-open sets in X. Then \( x \in f^{-1}(U) \), \( x \notin f^{-1}(V) \) and \( y \in f^{-1}(V) \), \( y \notin f^{-1}(U) \). Hence X is supra N-T₁ space.

**Theorem 7.9** Let \( f:(X, \tau) \rightarrow (Y, \sigma) \) be a supra N- irresolute, injective map. If Y is supra N-T₂, then X is supra N-T₂.

**Proof** Let Y be supra N-T₂ space. Let \( x,y \in X \), such that \( x \neq y \). since \( f \) is injective map, then \( f(x) \neq f(y) \in Y \). Since Y is supra N-T₂ space, there exist supra N-open set U and V in Y such that \( f(x) \in U \), \( f(y) \in V \). Since \( f \) is supra N- irresolute, then \( f^{-1}(U) \) and \( f^{-1}(V) \) are supra N-open sets in X. Then \( x \in f^{-1}(U) \), \( y \in f^{-1}(V) \). Hence X is supra N-T₂ space.

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