k-cordial labeling of some prisms.

M.V. Modha#, K.K. Kanani*1

* Research scholar, R.K. University, Rajkot-360020, Gujarat, India.

Abstract — We discuss here k-cordial labeling of prism for all odd k. We prove that prisms $P_m \times C_n$, $P_m \times C_{n+1}$, $P_m \times C_{n+1}$ are k-cordial for all odd k and $m \geq 2$. In addition to this we prove that all the Prisms $P_m \times C_{n}$ are k-cordial for all odd k, $m \geq 2$ and $m \neq tk$; $t \geq 1$.

Keywords—Abelian Group; k-cordial Labeling; Prism.

I. INTRODUCTION

Throughout this work, by a graph we mean finite, connected, undirected, simple graph G = (V(G), E(G)) of order |V(G)| and size |E(G)|.

Definition 1.1 A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s) if the domain of the mapping is the set of vertices(edges) then the labeling is called a vertex labeling(an edge labeling.)

Definition 1.2 Let $<A,*>$ be any Abelian group. A graph is said to be A-cordial if there is a mapping $f: V(G) \rightarrow A$ which satisfies the following two conditions when the edge $e=uv$ is labeled as $f(u)*f(v)$

(i) $|f(v)-f(u)| \leq 1$, for all $a,b \in A$;
(ii) $|f(u)-f(v)| \leq 1$, for all $a,b \in A$.

Where,

$v_l(a)$=the number of vertices with label a;
$v_l(b)$=the number of vertices with label b;
$e_l(a)$=the number of edges with label a;
$e_l(b)$=the number of edges with label b.

Definition 1.3 The prism $P_m \times C_n$ is obtained by taking the cartesian product of path $P_m$ with cycle $C_n$.

Seoud and Abdel Maqusoud[5] proved that the prism $C_n \times P_m$ is cordial except for the case $C_{4k+2} \times P_2$ for $m \geq 2$.

For any undefined term in graph theory we rely upon Clark and Holton [2].

II. MAIN RESULTS

Theorem 2.1 All the Prisms $P_m \times C_n$ are k-cordial for all odd k and $n = k; k + 1; k + 3$, $m \geq 2$.

Proof We divide the proof in three cases.

Case I: For $n = k$, we prove that prism $G = P_m \times C_k$ is k-cordial, $m \geq 2$.

Let $G = P_m \times C_k$ be the prism. Let $v_1, v_2, ..., v_{mk}$ be the vertices of the prism. Let $v_1, v_2, ..., v_k$ be vertices of the outer most cycle. Start the labeling pattern from the vertices $v_1, v_2, ..., v_k$ arranged in the clockwise direction. Let $v_{k+1}$ be the vertex of first inner cycle which is adjacent to $v_k$. Next label the vertices $v_{k+2}, v_{k+3}, ..., v_{2k}$ in the clockwise direction. Continue this pattern up to the last inner cycle. Let $v_{(m-1)jk}$ be the vertex of the last inner cycle which is adjacent to $v_{(m-1)jk}$. Finally label the vertices $v_{(m-1)jk+1}, v_{(m-1)jk+2}, v_{(m-1)jk+3}, ..., v_{mk}$ of the last inner cycle in the clockwise direction.

Note that $|V(G)| = mk$ and $|E(G)| = (2m - 1)k$.

In this case we define k-cordial labeling $f: V(G) \rightarrow Z_k$ as follows.

$f(v_i) = p_i$, where $(2i - 1) \equiv p_i (mod k)$, for $i = 1, 2, ..., mk$.

Case II: For $n = k + 1$, we prove that prism $G = P_m \times C_{k+1}$ is k-cordial, $m \geq 2$.

Let $G = P_m \times C_{k+1}$ be the prism. Let $v_1, v_2, ..., v_{mk+1}$ be the vertices of the prism. Let $v_1, v_2, ..., v_k$ be vertices of the outer most cycle. Start the labeling pattern from the vertices $v_1, v_2, ..., v_k, v_{k+1}$ arranged in the clockwise direction. Let $v_{k+2}$ be the vertex of first inner cycle which is adjacent to $v_{k+1}$. Next label the vertices $v_{k+2}, v_{k+3}, ..., v_{2k}$ in the clockwise direction. Continue this pattern up to the last inner cycle. Let $v_{(m-1)jk}$ be the vertex of the last inner cycle which is adjacent to $v_{(m-1)jk}$. Finally label the vertices $v_{(m-1)jk+1}, v_{(m-1)jk+2}, v_{(m-1)jk+3}, ..., v_{mk+1}$ of the last inner cycle in the clockwise direction.

Note that $|V(G)| = mk(k+1)$ and $|E(G)| = (2m - 1)(k+1)$.

In this case we define k-cordial labeling $f: V(G) \rightarrow Z_k$ as follows.

$f(v_i) = p_i$, where $(2i - 1) \equiv p_i (mod k)$, for $i = 1, 2, ..., mk+1$.

Case III: For $n = k + 3$, we prove that prism $G = P_m \times C_{k+3}$ is k-cordial, $m \geq 2$.

Let $G = P_m \times C_{k+3}$ be the prism. Let $v_1, v_2, ..., v_{mk+3}$ be the vertices of the prism. Let $v_1, v_2, v_3, ..., v_{k+2}, v_{k+3}$ be vertices of the outer most cycle. Start the labeling pattern from $v_1, v_2, v_3, ..., v_{k+2}, v_{k+3}$ arranged in the clockwise direction. Let $v_{k+4}$ be the vertex of the first inner cycle which is adjacent to $v_{k+3}$. Next label the vertices $v_{k+4}, v_{k+5}, v_{k+6}, ..., v_{2k+5}, v_{2k+6}$ in the clockwise
direction. Continue this pattern up to the last inner cycle. Let \( v_{(m-1)k+3m-2} \) be the vertex of the last inner cycle which is adjacent to \( v_{(m-1)k+3m-3} \). Finally label the vertices,

\[
v_{(m-1)k+3m-2} V_{(m-1)k+3m-1} V_{(m-1)k+3m} V_{(m-1)k+3m+1} \ldots V_{m(k+3)-1} V_{m(k+3)}
\]

of the last inner cycle in the clockwise direction.

Note that \(|V(G)| = m(k+3)\) and \(|E(G)| = (2m - 1)(k + 3)\).

In this case we define \( k \)-cordial labeling \( f : V(G) \rightarrow \mathbb{Z}_k \) as follows.

\[ f(v_i) = p_i, \quad \text{where} \quad (2i - 1) \equiv p_i \pmod{k}, \quad \text{for} \quad i = 1, 2, ..., m(k + 3). \]

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for \( k \)-cordial labeling.

Illustration 2.2(a): The prism \( P_3 \times C_6 \) and its 9-cordial labeling is shown in Fig 2.1.

![Fig 2.1: 9-Cordial labeling of prism \( P_3 \times C_6 \).](image)

Illustration 2.2(b): The prism \( P_3 \times C_6 \) and its 5-cordial labeling is shown in Fig 2.2.

![Fig 2.2: 5-Cordial labeling of prism \( P_3 \times C_6 \).](image)

Illustration 2.2(c): The prism \( P_3 \times C_{10} \) and its 7-cordial labeling is shown in Fig 2.3.

![Fig 2.3: 7-Cordial labeling of prism \( P_3 \times C_{10} \).](image)

**Theorem 2.3** All the Prisms \( P_3 \times C_n \) are \( k \)-cordial for all odd \( k \) and \( n = 2k - 1, \ m \geq 2 \) and \( m \neq tk, t \geq 1 \).

Proof: For \( n = 2k - 1 \).

Let \( G = P_3 \times C_{2k-1} \) be the prism. Let \( v_p, v_2, v_3, \ldots, v_{m(2k-1)} \) be the vertices of the prism. Let \( v_1, v_2, v_3, \ldots, v_{2k-1} \) be the vertices of outer most cycle. Start the labeling pattern from \( v_1, v_2, v_3, \ldots, v_{2k-1} \) arranged in the clockwise direction. Let \( v_{2k-1} \) be the vertex of first inner cycle which is adjacent to \( v_{2k-1} \). Next label \( v_{2k}, v_{2k+1}, v_{2k+2}, \ldots, v_{2(2k-1)} \) vertices in the clockwise direction. Continue this pattern up to last inner cycle. Let \( v_{(m-1)(2k-1)+1} \) be the vertex of the last inner cycle which is adjacent to \( v_{(m-1)(2k-1)} \). Finally label the vertices \( v_{(m-1)(2k-1)+1}, v_{(m-1)(2k-1)+2}, \ldots, v_{(m-1)(2k-1)+2(m-1)(2k-1)+1} \) of the last inner cycle in the clockwise direction.

We note that \(|V(G)| = m(2k-1)\) and \(|E(G)| = (2m-1)(2k-1)\).

**Case I:** \( m \neq (tk+1) \)

In this case we define \( k \)-cordial labelling \( f : V(G) \rightarrow \mathbb{Z}_k \) as follows.

\[ f(v_i) = p_i, \quad \text{where} \quad (2i - 1) \equiv p_i \pmod{k}, \quad \text{for} \quad i = 1, 2, ..., m(2k-1). \]

**Case II:** \( m = (tk+1) \)

In this case we define \( k \)-cordial labeling \( f : V(G) \rightarrow \mathbb{Z}_k \) as follows.

\[ f(v_i) = p_i, \quad \text{where} \quad (2i - 1) \equiv p_i \pmod{k}, \quad \text{for} \quad i = 1, 2, ..., (m(2k-1)-1), \]

\[ f(v_{m(2k-1)}) = k - 1, \quad \text{where} \quad i = (m(2k-1)). \]

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for \( k \)-cordial labeling.
III. CONCLUDING REMARKS

Here we have contributed two general results to the theory of k-cordial labeling. To derive similar results for other graph families is an open area of research.

References