Lehmer 3 –Mean Labeling of Some New Disconnected Graphs

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Abstract :- A graph G=(V,E) with P vertices and q edges is called Lehmer -3 mean graph ,if it is possible to label vertices x ЄV with distinct label f(x) from 1,2,3,………..q+1 in such a way that when each edge e=uv is labeled with f(e=uv)=\( \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \) (or) \( \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \), then the edge labels are distinct. In this case f is called Lehmer -3 mean labeling of G. In this paper we investigate Lehmer -3 mean labeling of some standard graphs

Keywords: - Graph, Path, Cycle, Comb, Crown, Ladder.

I .INTRODUCTION

A graph considered here are finite undirected and simple. The vertex set and edge set of a graph are denoted by V(G) and E(G) respectively. For detailed survey Gallian survey [1] is refered and standard terminologies and notations are followed from Harary [2]. We will find the brief summary of definitions and informations necessary for the present investigation.

Definition 1.1
A graph G=(V,E) with P vertices and q edges is called Lehmer -3 mean graph, if it is possible to label vertices x ЄV with distinct label f(x) from 1,2,3,………..q+1 in such a way that when each edge e=uv is labeled with
\[ f(e=uv)=\frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \] (or) \[ \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \], then the edge labels are distinct. In this case f is called Lehmer -3 mean labeling of G.

Definition 1.2
A path Pn is obtained by joining u1 to the consecutive vertices ui+1 for 1≤i≤n

Definition 1.3
Comb is a graph obtained by joining a single pendant edge to each vertex of a path

Definition 1.4
A closed path is called a cycle of G.

Definition 1.5
Crown is a graph obtained by joining a single pendant edge to each vertex of a cycle

Definition 1.6
A product graph Pm x Pn is called a planar grid Pm x Pn is called a ladder.

Definition 1.7
PnʘK1,2 is a graph obtained by attaching K1,2 to each vertex of Pn

Definition 1.8
PnʘK1,3 is a graph obtained from the path attaching K1,3 to each of its vertices

Definition 1.9
Pnʘ K3 is a graph connected by a complete graph K3 in its each vertex

II. Main Results

Theorem:2.1

\( (C_nʘk_1)ʘ P_m \) is a Lehmer-3 mean graph.
Proof:
Let $C_n \mathbin{ʘ} K_1$ be the crown with $u_1, u_2, \ldots, u_n, v_1$ as the cycle and $v_i$ be the pendant vertices adjacent to $u_i$, $1 \leq i \leq n$. Let $w_1, w_2, \ldots, w_m$ be the path.

Let $G$ be the graph obtained by the union of $(C_n \mathbin{ʘ} K_1) \mathbin{ʘ} P_m$.

Define a function $f: V(G) \to \{1, 2, 3, \ldots, q+1\}$ by

- $f(u_i) = i$; $1 \leq i \leq n$
- $f(v_i) = n+i$; $1 \leq i \leq n$
- $f(w_j) = 2n+j$; $1 \leq j \leq m$

Then the distinct edge labels are

- $f(u_i u_{i+1}) = i$; $1 \leq i \leq n$
- $f(u_i v_i) = n+1$; $1 \leq i \leq n$
- $f(w_j w_{j+1}) = 2n+j$; $1 \leq j \leq m-1$

Thus $(C_n \mathbin{ʘ} K_1) \mathbin{ʘ} P_m$ forms Lehmer-3 mean graph.

Example: 2.2

The Lehmer-3 mean graph of $(C_3 \mathbin{ʘ} K_1) \mathbin{ʘ} P_4$ is given below.

![Diagram](image)

Theorem: 2.3

$(C_n \mathbin{ʘ} K_1) \mathbin{ʘ} (P_m \mathbin{ʘ} K_1)$ is a Lehmer-3 mean graph.

Proof:
Let $G$ be the given graph.
\( C_n \odot K_1 \) is the crown with \( u_1, u_2, \ldots, u_n, u_1 \) as the cycle and \( v_i \) be the pendent vertices adjacent to \( u_i \), \( 1 \leq i \leq n \).

Let \( P_m \odot K_1 \) be the comb with \( x_1, x_2, \ldots, x_m \) as the vertices and \( y_j \) be the pendent vertices adjacent to \( x_j \), \( 1 \leq j \leq m \).

Define a function \( f : V(G) \to \{1, 2, \ldots, q+1\} \) by
\[
\begin{align*}
  f(u_i) & = i & 1 \leq i \leq n \\
  f(v_i) & = n+i & 1 \leq i \leq n \\
  f(x_j) & = 2n+(2j-1) & 1 \leq j \leq m \\
  f(y_j) & = 2n+(2j) & 1 \leq j \leq m
\end{align*}
\]

Then the distinct edge labeling are
\[
\begin{align*}
  f(u_iu_{i+1}) & = i & 1 \leq i \leq n \\
  f(u_iv_i) & = n+1 & 1 \leq i \leq n \\
  f(x_jx_{j+1}) & = 2n+2j & 1 \leq j \leq m-1 \\
  f(x_jy_j) & = 2n+(2j-1) & 1 \leq j \leq m
\end{align*}
\]

Hence \( G \) forms a Lehmer-3 mean graph

**Example: 2.4**

\((C_n \odot K_1) \cup (P_m \odot K_1)\) is a Lehmer-3 mean graph.

**Theorem: 2.5**

\((C_n \odot K_1) \cup C_m\) is a Lehmer -3 mean graph.

**Proof:**

Let \( G \) be a graph obtained by \((C_n \odot K_1) \cup C_m\).
Let $C_nAK_1$ be the crown with $u_1, u_2, \ldots, u_n$ as a cycle and $v_i$ be the pendent vertices adjacent to $u_i, 1 \leq i \leq n$.

Let $C_m$ be a cycle with $w_j, 1 \leq j \leq m$ as the vertices.

Define a function $f: V(G) \rightarrow \{1, 2, \ldots, q+1\}$ by

$$f(u_i) = 2i - 1; \quad 1 \leq i \leq n$$

$$f(v_i) = 2i; \quad 1 \leq i \leq n$$

$$f(w_j) = 2n + j; \quad 1 \leq j \leq m$$

Thus the edge labels are distinct.

Hence $(C_n \cup K_1) \cup C_m$ is a Lehmer -3 mean graph.

**Example: 2.6**

$(C_5 \cup K_1) \cup C_6$ is a Lehmer-3 mean graph.

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**Theorem: 2.7**

$(C_n \cup K_1) \cup (P_m \cup K_{1,2})$ be a Lehmer -3 mean graph.

**Proof:**

Let $G$ be a graph obtained from the union of $(C_n \cup K_1) \cup (P_m \cup K_{1,2})$.

Let $C_n \cup K_1$ be a crown with vertices $u_1, u_2, \ldots, u_n; v_1, v_2, \ldots, v_n$.

Let $P_m \cup K_{1,2}$ is a path attaching $K_{1,2}$ with vertices $w_1, w_2, \ldots, w_m; x_1, x_2, \ldots, x_m$.

Define a function $f: V(G) \rightarrow \{1, 2, \ldots, q+1\}$ by

$$f(u_i) = 2i - 1; \quad 1 \leq i \leq n$$
Thus we obtained distinct edge labelings.

Hence \((C_n \odot K_1) \cup (P_m \odot K_{1,2})\) is a Lehmer-3 mean graph.

**Example: 2.8**

\((C_4 \odot K_1) \cup (P_4 \odot K_{1,2})\) is a Lehmer-3 mean graph.

**Theorem 2.9**

\((C_n \odot K_1) \cup (P_m \odot K_{1,3})\) be a Lehmer-3 mean graph.

**Proof:**

Let \(G\) be a graph obtained from \((C_n \odot K_1) \cup (P_m \odot K_{1,3})\).

The vertices of \(C_n \odot K_1\) be \(u_1, u_2, \ldots, u_n; \ v_1, v_2, \ldots, v_n\).

\(P_m \odot K_{1,3}\) be a graph with vertices \(x_1, x_2, \ldots, x_m; y_1, y_2, \ldots, y_m; z_1, z_2, \ldots, z_m\).

Define a function \(f: V(G) \rightarrow \{1, 2, \ldots, q+1\}\) by

\[
\begin{align*}
  f(u_i) &= 2i-1 & & 1 \leq i \leq n \\
  f(v_i) &= 2i & & 1 \leq i \leq n \\
  f(w_j) &= 2n + (4j-3) & & 1 \leq j \leq m \\
  f(x_j) &= 2n + (4j-2) & & 1 \leq j \leq m
\end{align*}
\]
\[ f(y_j) = 2n + (4j - 1) \quad ; \quad 1 \leq j \leq m \]
\[ f(z_i) = 2n + (4j) \quad ; \quad 1 \leq j \leq m \]

Thus we get distinct edge labels.

Thus \((C_n \cup K_1) \cup (P_m \cup K_{1,3})\) is a Lehmer -3 mean graph.

**Example:** 2.10

The Lehmer -3 mean labeling of \((C_n \cup K_1) \cup (P_m \cup K_{1,3})\) is given below

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**Theorem:** 2.11

\((C_n \cup K_1) \cup (P_m \cup K_3)\) is a Lehmer -3 mean graph

**Proof:**

Let \(G\) be a graph obtained by \((C_n \cup K_1) \cup (P_m \cup K_3)\)

Let \((C_n \cup K_1)\) be a graph with vertices \(u_1, u_2, \ldots, u_n; v_1, v_2, \ldots\) respectively

Let \((P_m \cup K_3)\) be a graph obtained by joining \(P_m\) with \(K_3\) with vertices \(w_j, x_j, y_j, 1 \leq j \leq m\) respectively

Define a function \(f: V(G) \to \{1, 2, \ldots, q+1\}\) defined by

\[ f(u_i) = 2i - 1 \quad ; \quad 1 \leq i \leq n \]
\[ f(v_i) = 2i \quad ; \quad 1 \leq i \leq n \]
\[ f(w_j) = 2n + (4j - 3) \quad ; \quad 1 \leq j \leq m \]
\[ f(x_j) = 2n + (4j - 2) \quad ; \quad 1 \leq j \leq m \]
\[ f(y_j) = 2n + (4j - 1) \quad ; \quad 1 \leq j \leq m \]

Thus we get the distinct edge labeling

Hence \((C_n \cup K_1) \cup (P_m \cup K_3)\) is a Lehmer - 3 mean graph
Example: 2.12

The labeling pattern of \((C_4 \circ K_1) \cup (P_4 \circ K_3)\) is given below

![Graph Image]

**Theorem: 2.13**

\((C_n \circ k_1) \cup L_m\) is a Lehmer -3 mean graph.

**Proof:**

Let \(G\) be a graph obtained by \((C_n \circ k_1) \cup L_m\).

Let \((C_n \circ k_1)\) be a graph with vertices \(u_1, u_2, \ldots, u_n; v_1, v_2, \ldots, v_n\).

Let \(L_m\) be a ladder with \(m\) vertices as \(w_1, w_2, \ldots, w_m; x_1, x_2, \ldots, x_m\).

Define a function \(f: V(G) \rightarrow \{1, 2, \ldots, q+1\}\) defined by

\[
\begin{align*}
  f(u_i) & = 2i - 1 & & 1 \leq i \leq n \\
  f(v_i) & = 2i & & 1 \leq i \leq n \\
  f(w_1) & = 2n + 1 \\
  f(w_j) & = 2n + (3j - 3) & & 2 \leq j \leq m \\
  f(x_1) & = 2n + 2 \\
  f(x_j) & = 2n + (3j - 2) & & 2 \leq j \leq m
\end{align*}
\]

Thus the distinct edge labels are obtained.

Hence \((C_n \circ k_1) \cup L_m\) is a Lehmer -3 mean graph.
Example: 2.14
The Lehmer -3 mean labeling of \((C_4 \odot k_1) \cup L_4\) is given below.

![Diagram showing the labeling of \((C_4 \odot k_1) \cup L_4\).](image)

**Theorem: 2.15**

\((C_n \odot K_{1,2}) \cup P_m\) is a Lehmer -3 mean graph.

**Proof:**
Let \(G\) be a graph obtained by \((C_n \odot K_{1,2}) \cup P_m\). Let \((C_n \odot K_{1,2})\) be a graph with vertices \(u_1, u_2, \ldots, u_n; v_1, v_2, \ldots, v_n\) and \(w_1, w_2, \ldots, w_n\) respectively. Let \(P_m\) be a path with \(m\) vertices \(x_1, x_2, \ldots, x_m\). Define a function \(f: V(G) \to \{1, 2, \ldots, q+1\}\) by

\[
\begin{align*}
    f(u_i) &= 3i - 2 \quad ; \quad 1 \leq i \leq n \\
    f(v_i) &= 3i - 1 \quad ; \quad 1 \leq i \leq n \\
    f(w_i) &= 3i \quad ; \quad 1 \leq i \leq n \\
    f(x_j) &= 3n + j \quad ; \quad 1 \leq j \leq m
\end{align*}
\]

The edge labelings are distinct. Hence \((C_n \odot K_{1,2}) \cup P_m\) is a Lehmer -3 mean graph.

**Example: 2.16**

\((C_4 \odot K_{1,2}) \cup P_5\) is a Lehmer -3 mean graph.
Theorem 2.17

\((C_n \odot K_{1,2}) \cup (P_m \odot K_1)\) is a Lehmer-3 mean graph

Proof:

Let \(G\) be a graph obtained by \((C_n \odot K_{1,2}) \cup (P_m \odot K_1)\)

Let \((C_n \odot K_{1,2})\) be a graph with vertices \(u_1, u_2, \ldots, u_n; v_1, v_2, \ldots, v_n; w_1, w_2, \ldots, w_n\) respectively.

Let \((P_m \odot K_1)\) be a comb with vertices \(x_1, x_2, \ldots, x_m\) and \(y_1, y_2, \ldots, y_m\) respectively.

Define a function \(f: V(G) \rightarrow \{1, 2, \ldots, q+1\}\) defined by:

\[
\begin{align*}
    f(u_i) &= 3i - 2 & & 1 \leq i \leq n \\
    f(v_i) &= 3i - 1 & & 1 \leq i \leq n \\
    f(w_i) &= 3i & & 1 \leq i \leq n \\
    f(x_j) &= 3n + (2j - 1) & & 1 \leq j \leq m \\
    f(y_j) &= 3n + 2j & & 1 \leq j \leq m
\end{align*}
\]

Thus the edge labelings are distinct.

Hence \((C_n \odot K_{1,2}) \cup (P_m \odot K_1)\) is a Lehmer-3 mean graph.

Example 2.18

\((C_6 \odot K_{1,2}) \cup (P_4 \odot K_1)\) is a Lehmer-3 mean graph
**Theorem 2.19**

(CᵣʘK₁,₂) ∪ (PₘʘK₁,₂) be a Lehmer -3 mean graph.

**Proof:**

Let G be a graph obtained from the union of (CᵣʘK₁,₂) and (PₘʘK₁,₂).

Let CᵣʘK₁,₂ be a graph with vertices u₁, u₂, ....., uᵣ; v₁, v₂, ......., vᵣ and w₁, w₂, ......., wᵣ.

Let (PₘʘK₁,₂) be a graph with vertices x₁, y₁, z₁; 1 ≤ j ≤ m.

Define a function f: V(G) → {1, 2, ......., q+1} by

\[
\begin{align*}
    f(u_i) &= 3i-2 &; 1 ≤ i ≤ n \\
    f(v_i) &= 3i-1 &; 1 ≤ i ≤ n \\
    f(w_i) &= 3i &; 1 ≤ i ≤ n \\
    f(x_j) &= 3n+(3j-2) &; 1 ≤ j ≤ m \\
    f(y_j) &= 3n+(3j-1) &; 1 ≤ j ≤ m \\
    f(z_j) &= 3n+3j &; 1 ≤ j ≤ m.
\end{align*}
\]

we obtain distinct edge labelings.

Thus (CᵣʘK₁,₂) ∪ (PₘʘK₁,₂) is a Lehmer -3 mean graph.

**Example 2.20**

(C₅ʘK₁,₂) ∪ (P₄ʘK₁,₂) is a Lehmer -3 mean graph.
**Theorem: 2.21**

\((C_n \circ K_{1,2}) \cup (P_m \circ K_{1,3})\) is a Lehmer-3 mean graph

**Proof:**

Let \((C_n \circ K_{1,2}) \cup (P_m \circ K_{1,3})\) be a graph obtained from the union of \((C_n \circ K_{1,2})\) and \((P_m \circ K_{1,3})\).

Let \(u_1, u_2, \ldots, u_n; v_1, v_2, \ldots, v_n; \ w_1, w_2, \ldots, w_n; \) be the vertices of \((C_n \circ K_{1,2})\) and let \(x_1, x_2, \ldots, x_m; y_1, y_2, \ldots, y_m; \)
\(z_1, z_2, \ldots, z_m; t_1, t_2, \ldots, t_m\) be the vertices of \((P_m \circ K_{1,3})\).

Define a function \(f: V(G) \rightarrow \{1, 2, \ldots, q+1\}\) defined by

\[
\begin{align*}
f(u_i) &= 3i - 2 & \text{if } 1 \leq i \leq n, \\
f(v_i) &= 3i - 1 & \text{if } 1 \leq i \leq n, \\
f(w_i) &= 3i & \text{if } 1 \leq i \leq n, \\
f(x_j) &= 3n + (4j - 3) & \text{if } 1 \leq j \leq m, \\
f(y_j) &= 3n + (4j - 2) & \text{if } 1 \leq j \leq m, \\
f(z_j) &= 3n + (4j - 1) & \text{if } 1 \leq j \leq m, \\
f(t_j) &= 3n + (4j) & \text{if } 1 \leq j \leq m.
\end{align*}
\]

Thus we obtain distinct edge labels.

Hence \((C_n \circ K_{1,2}) \cup (P_m \circ K_{1,3})\) is a Lehmer-3 mean graph.

**Example: 2.22**

\((C_5 \circ K_{1,2}) \cup (P_4 \circ K_{1,3})\) is a Lehmer-3 mean graph.
Theorem: 2.23

(CⁿʘK₁,₂) U(PₘʘK₃) is a Lehmer- 3 mean graph

Proof:

Let (CⁿʘK₁,₂) be a graph with vertices u₁,u₂,….uₙ, v₁,v₂,…….,vₙ, w₁,w₂,…….,wₙ respectively

Let (PₘʘK₃) be a graph with vertices xᵢ,yᵢ,zᵢ; 1≤j≤m respectively

Let G be a graph obtained from the union of (CⁿʘK₁,₂) and (PₘʘK₃)

Define a function f: V(G)→ {1,2,……..q+1} defined by

f(uᵢ)= 3i-2 ; 1≤i≤n
f(vᵢ)=3i-1 ; 1≤i≤n
f(wᵢ)=3i ; 1≤i≤n
f(xᵢ)=3n+(4j-3); 1≤j≤m
f(yᵢ)=3n+(4j-2); 1≤j≤m
f(zᵢ)=3n+(4j-1); 1≤j≤m

Thus we get distinct edge labeling

Hence (CⁿʘK₁,₂) U(PₘʘK₃) is a Lehmer- 3 mean graph.

Example: 2.24

(CⁿʘK₁,₂) U(PₘʘK₃) is a Lehmer- 3 mean graph
Theorem 2.25

$(C_n \ʘ K_{1,2}) \cup C_m$ is a Lehmer 3 mean graph.

Proof:

Let $G$ be a graph obtained from the union of $(C_n \ʘ K_{1,2})$ and $C_m$.

Let $(C_n \ʘ K_{1,2})$ be a graph with vertices $u_1, u_2, \ldots, u_n; v_1, v_2, \ldots, v_n$ and $w_1, w_2, \ldots, w_n$ respectively.

Let $C_m$ be a path with $m$ vertices $x_1, x_2, \ldots, x_m$.

Define a function $f: V(G) \rightarrow \{1, 2, \ldots, q+1\}$ by

$f(u_i) = 3i - 2$ ; $1 \leq i \leq n$

$f(v_i) = 3i - 1$ ; $1 \leq i \leq n$

$f(w_i) = 3i$ ; $1 \leq i \leq n$

$f(x_j) = 3n + j$ ; $1 \leq j \leq m$

Thus we obtain distinct edge labels.

Hence $(C_n \ʘ K_{1,2}) \cup C_m$ is a Lehmer 3 mean graph.
Example: 2.26

Lehmer-3 mean labeling of \((C_4 \oplus K_{1,2}) \cup C_5\) is given below

![Graph Diagram]

REFERENCES