Intuitionistic Anti-Fuzzy HX Ring

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Abstract - In this paper, we define the notion of intuitionistic anti fuzzy HX subring of a HX ring and some of their related properties are investigated. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic anti-fuzzy HX subring and discuss some of its properties. We discuss the concept of an image, pre-image of an intuitionistic anti-fuzzy HX subring and homomorphic, anti homomorphic properties of an intuitionistic anti-fuzzy HX subring are discussed.

Keywords - Intuitionistic fuzzy set, fuzzy HX ring, intuitionistic anti-fuzzy HX subring, homomorphism and anti homomorphism of an intuitionistic anti-fuzzy HX subring, image and pre-image of an intuitionistic anti-fuzzy HX subring.

I. INTRODUCTION

In 1965, Zadeh [12] introduced the concept of fuzzy subset μ of a set X as a function from X into the closed unit interval [0, 1] and studied their properties. Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague and it is applied to logic, set theory, group theory, ring theory, real analysis, measure theory etc. In 1967, Rosenfeld [11] defined the idea of fuzzy subgroups and gave some of its properties. Li Hong Xing [5] introduced the concept of HX group. In 1982 Wang-jin Liu[7] introduced the concept of fuzzy ring and fuzzy ideal. With the successful upgrade of algebraic structure of group many researchers considered the algebraic structure of some other algebraic systems in which ring was considered as first. In 1988, Professor Li Hong Xing [6] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [2,3] gave the structures of HX ring on a class of ring. R.Muthuraj et.al[10], introduced the concept of fuzzy HX ring. In this paper we define a new algebraic structure of an intuitionistic anti-fuzzy HX subring of a HX ring and investigate some related properties. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic anti-fuzzy HX subring and discuss some of its properties. Also we introduce the image and pre-image of an intuitionistic anti-fuzzy HX subring and discuss some of its properties.

2 PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, R = (R, +, *) is a Ring. e is the additive identity element of R and xy, we mean x.y

2.1 Definition [3]

Let R be a ring. In 2^R : {φ}, a non-empty set S ⊆ 2^R : {φ} with two binary operation ‘*’ and ‘.’ is said to be a HX ring on R if S is a ring with respect to the algebraic operation defined by

i. A + B = {a + b / a ∈ A and b ∈ B}, which its null element is denoted by 0, and the negative element of A is denoted by – A.
ii. AB = {ab / a ∈ A and b ∈ B},
iii. A ( B + C ) = AB + AC and (B + C) A = BA + CA.

3.1 Intuitionistic anti-fuzzy HX subring of a HX Ring

In this section we define the concept of an intuitionistic anti fuzzy HX subring of a HX ring and discuss some related results.

3.1.1 Definition

Let R be a ring. Let H = { (x, μ(x), η(x)) / x ∈ R} be an intuitionistic fuzzy set defined on a ring R, where μ : R → [0,1], η : R → [0,1] such that 0 ≤ μ(x) + η(x) ≤ 1. Let B ⊆ 2^R – {φ} be a HX ring. An intuitionistic fuzzy subset λ^H = { (A, λ^H(A), λ^H(A)) / A ∈ B} and 0 ≤ λ^H(A) + λ^H(A) ≤ 1} of B is called an intuitionistic fuzzy HX subring on B or an intuitionistic fuzzy subring induced by H if the following conditions are satisfied. For all A, B ∈ B,

i. λ^H ( A – B ) ≥ min { λ^H (A), λ^H (B) },
ii. λ^H ( AB ) ≥ min { λ^H (A), λ^H (B) },
iii. λ^H (A–B) ≤ max { λ^H (A), λ^H (B) },
iv. λ^H (AB) ≤ max { λ^H (A), λ^H (B) },

where λ^H (A) = max { μ(x) / for all x ∈ A ⊆ R} and λ^H (A) = min{η(x) / for all x ∈ A ⊆ R}.

3.1.2 Definition

Let R be a ring. Let H = { (x, μ(x), η(x)) / x ∈ R} be an intuitionistic fuzzy set defined on a ring R, where μ : R → [0,1], η : R → [0,1] such that 0 ≤ μ(x) + η(x) ≤ 1. Let B ⊆ 2^R – {φ} be a HX ring. An intuitionistic fuzzy subset λ^H = { (A, λ^H(A), λ^H(A)) / A ∈ B} and 0 ≤ λ^H(A) + λ^H(A) ≤ 1} of B is called an intuitionistic anti-fuzzy HX subring on B or an intuitionistic anti-fuzzy subring induced by H if the following conditions are satisfied. For all A, B ∈ B,

i. λ^H ( A – B ) ≤ max { λ^H (A), λ^H (B) },
ii. λ^H ( AB ) ≤ max { λ^H (A), λ^H (B) },

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\[ \lambda_{\eta}(A-B) \geq \min \{ \lambda_{\eta}(A), \lambda_{\eta}(B) \}, \]

iv. \[ \lambda_{\eta}(AB) \geq \min \{ \lambda_{\eta}(A), \lambda_{\eta}(B) \}. \]

where \( \lambda_{\mu}(A) = \min \{ \mu(x) / \text{for all } x \in A \subseteq R \} \) and \( \lambda_{\eta}(A) = \max \{ \eta(x) / \text{for all } x \in A \subseteq R \}. \)

3.1.3 Remark

For an intuitionistic anti-fuzzy HX subring \( \lambda_{\mu} \) of a HX ring \( \mathfrak{R} \), the following result is obvious.

For all \( A, B \in \mathfrak{R} \):

i. \( \lambda_{\mu}(A) \leq \lambda_{\mu}(0) \) and \( \lambda_{\mu}(A) = \lambda_{\mu}(-A) \),

ii. \( \lambda_{\mu}(A-B) = 0 \) implies that \( \lambda_{\mu}(A) = \lambda_{\mu}(B) \).

iii. \( \lambda_{\eta}(A) \geq \lambda_{\eta}(0) \) and \( \lambda_{\eta}(A) = \lambda_{\eta}(-A) \),

iv. \( \lambda_{\eta}(A-B) = 0 \) implies that \( \lambda_{\epsilon}(A) = \lambda_{\epsilon}(B) \).

3.1.4 Theorem

If \( H \) is an intuitionistic anti-fuzzy subring of a ring \( R \), then the intuitionistic fuzzy subring \( \lambda_{\mu} \) of an intuitionistic anti-fuzzy HX subring of a HX ring \( \mathfrak{R} \).

Proof

Let \( H \) be an intuitionistic anti-fuzzy subring of \( R \).

i. \[ \max \{ \lambda_{\mu}(A), \lambda_{\mu}(B) \} = \max \{ \min \{ \mu(x) / \text{for all } x \in A \subseteq R \}, \min \{ \mu(y) / \text{for all } y \in B \subseteq R \} \} \]

\[ = \max \{ \mu(x_0), \mu(y_0) \} \]

\[ = \min \{ \mu(x-y) / \text{for all } x-y \in A-B \subseteq R \} \]

\[ \geq \lambda_{\mu}(A-B) \]

ii. \[ \max \{ \lambda_{\mu}(A), \lambda_{\mu}(B) \} = \max \{ \min \{ \mu(x) / \text{for all } x \in A \subseteq R \}, \min \{ \mu(y) / \text{for all } y \in B \subseteq R \} \} \]

\[ = \max \{ \mu(x_0), \mu(y_0) \} \]

\[ \geq \min \{ \mu(x-y) / \text{for all } x-y \in A-B \subseteq R \} \]

\[ \lambda_{\mu}(A-B) \leq \min \{ \lambda_{\mu}(A), \lambda_{\mu}(B) \}. \]

iii. \[ \min \{ \lambda_{\eta}(A), \lambda_{\eta}(B) \} = \min \{ \max \{ \mu(x) / \text{for all } x \in A \subseteq R \}, \max \{ \mu(y) / \text{for all } y \in B \subseteq R \} \} \]

\[ = \min \{ \mu(x_0), \mu(y_0) \} \]

\[ \leq \max \{ \eta(x-y) / \text{for all } x-y \in A-B \subseteq R \} \]

\[ \lambda_{\eta}(A-B) \geq \min \{ \lambda_{\eta}(A), \lambda_{\eta}(B) \}. \]

iv. \[ \min \{ \lambda_{\eta}(A), \lambda_{\eta}(B) \} = \min \{ \max \{ \mu(x) / \text{for all } x \in A \subseteq R \}, \max \{ \mu(y) / \text{for all } y \in B \subseteq R \} \} \]

\[ = \min \{ \mu(x_0), \mu(y_0) \} \]

\[ \leq \eta(x_0, y_0) \]

\[ \leq \max \{ \eta(x-y) / \text{for all } x-y \in A-B \subseteq R \} \]

\[ \leq \lambda_{\eta}(A-B) \]

\[ \lambda_{\eta}(A-B) \geq \min \{ \lambda_{\eta}(A), \lambda_{\eta}(B) \}. \]

Hence, \( \lambda_{\mu} \) is an intuitionistic anti-fuzzy HX subring of a HX ring \( \mathfrak{R} \).

3.1.5 Theorem

Let \( G \) and \( H \) be any two intuitionistic fuzzy sets on \( R \). Let \( \gamma_{G} \) and \( \lambda_{H} \) be any two intuitionistic anti-fuzzy HX subrings of a HX ring \( \mathfrak{R} \), then their intersection, \( \gamma_{G} \cap \lambda_{H} \) is also an intuitionistic anti-fuzzy HX subring of a HX ring \( \mathfrak{R} \).

Proof

Let \( G = \{ (x, \alpha(x), \beta(x)) / x \in \mathbb{R} \} \) and \( H = \{ (x, \mu(x), \eta(x)) / x \in \mathbb{R} \} \) be any two intuitionistic fuzzy sets defined on a ring \( R \). Then, \( \gamma_{G} \cap \lambda_{H} = \{ (\lambda_{\eta}(A), \lambda_{\mu}(A)) / A \in \mathfrak{R} \} \) and \( \gamma_{G} \cap \lambda_{H} = \{ (\lambda_{\eta}(A), \lambda_{\mu}(A)) / A \in \mathfrak{R} \} \) be any two intuitionistic anti-fuzzy HX subrings of a HX ring \( \mathfrak{R} \).

\[ \gamma_{G} \cap \lambda_{H} = \{ (\lambda_{\eta}(A), \lambda_{\mu}(A)) / A \in \mathfrak{R} \} \]

i. \( \gamma_{G} \cap \lambda_{H} = \{ (\lambda_{\eta}(A), \lambda_{\mu}(A)) / A \in \mathfrak{R} \} \)

\[ \leq \min \{ \gamma_{G}(A), \gamma_{H}(A) \}, \max \{ \lambda_{G}(A), \lambda_{H}(A) \} \]

\[ = \max \{ \min \{ \gamma_{G}(A), \gamma_{H}(A) \}, \max \{ \lambda_{G}(A), \lambda_{H}(A) \} \} \]

\[ = \max \{ \gamma_{G}(A), \lambda_{H}(A) \}, \max \{ \gamma_{H}(A), \lambda_{G}(A) \} \]

\[ = \max \{ \gamma_{G}(A), \lambda_{H}(A) \}, \max \{ \gamma_{H}(A), \lambda_{G}(A) \} \]

\[ \gamma_{G} \cap \lambda_{H} = \{ (\lambda_{\eta}(A), \lambda_{\mu}(A)) / A \in \mathfrak{R} \} \]

ii. \( \gamma_{G} \cap \lambda_{H} = \{ (\lambda_{\eta}(A), \lambda_{\mu}(A)) / A \in \mathfrak{R} \} \)

\[ \leq \min \{ \gamma_{G}(A), \gamma_{H}(A) \}, \max \{ \lambda_{G}(A), \lambda_{H}(A) \} \]

\[ = \max \{ \min \{ \gamma_{G}(A), \gamma_{H}(A) \}, \max \{ \lambda_{G}(A), \lambda_{H}(A) \} \} \]

\[ = \max \{ \gamma_{G}(A), \lambda_{H}(A) \}, \max \{ \gamma_{H}(A), \lambda_{G}(A) \} \]

\[ \gamma_{G} \cap \lambda_{H} = \{ (\lambda_{\eta}(A), \lambda_{\mu}(A)) / A \in \mathfrak{R} \} \]

iii. \( \gamma_{G} \cap \lambda_{H} = \{ (\lambda_{\eta}(A), \lambda_{\mu}(A)) / A \in \mathfrak{R} \} \)

\[ \geq \min \{ \gamma_{G}(A), \gamma_{H}(A) \}, \min \{ \lambda_{G}(A), \lambda_{H}(A) \} \]

\[ = \min \{ \gamma_{G}(A), \min \{ \lambda_{G}(A), \lambda_{H}(A) \} \}, \min \{ \gamma_{H}(A), \min \{ \lambda_{G}(A), \lambda_{H}(A) \} \} \]

\[ = \min \{ \gamma_{G}(A), \lambda_{H}(A) \}, \min \{ \gamma_{H}(A), \lambda_{G}(A) \} \]

\[ \gamma_{G} \cap \lambda_{H} = \{ (\lambda_{\eta}(A), \lambda_{\mu}(A)) / A \in \mathfrak{R} \} \]

Hence, \( \gamma_{G} \cap \lambda_{H} \) is an intuitionistic anti-fuzzy HX subring of a HX ring \( \mathfrak{R} \).

3.1.6 Theorem

Let \( G \) and \( H \) be any two intuitionistic fuzzy sets on \( R \). Let \( \gamma_{G} \) and \( \lambda_{H} \) be any two intuitionistic anti-fuzzy HX subrings of a HX ring \( \mathfrak{R} \) then their union, \( \gamma_{G} \cup \lambda_{H} \) is also an intuitionistic anti-fuzzy HX subring of a HX ring \( \mathfrak{R} \).
Proof

Let \( G = \{ \langle x, \alpha(x), \beta(x) \rangle \mid x \in \mathbb{R} \} \) and \( H = \{ \langle x, \mu(x), \eta(x) \rangle \mid x \in \mathbb{R} \} \) be any two intuitionistic fuzzy sets defined on a ring \( R \).

Then, \( \gamma_G = \{ \langle A, \gamma_G(A), \gamma_H(A) \rangle \mid A \in \mathcal{F} \} \) and \( \lambda_H = \{ \langle A, \lambda_G(A), \lambda_H(A) \rangle \mid A \in \mathcal{F} \} \) be any two intuitionistic anti-fuzzy subrings of a HX ring \( R \).

Then,

\[
\gamma_G \circ \lambda_H = \{ \langle A, (\gamma_G \circ \lambda_H)(A) \rangle \mid A \in \mathcal{F} \}
\]

where

\[
(\gamma_G \circ \lambda_H)(A) = \min \{ \gamma_G(A), \lambda_H(A) \}
\]

Hence, \( \gamma_G \circ \lambda_H \) is an intuitionistic anti-fuzzy HX subring of a HX ring \( R \).

3.1.7 Definition

Let \( G = \{ \langle x, \alpha(x), \beta(x) \rangle \mid x \in \mathbb{R} \} \) and \( H = \{ \langle x, \mu(x), \eta(x) \rangle \mid x \in \mathbb{R} \} \) be any two intuitionistic fuzzy sets defined on a ring \( R \).

Let \( \gamma_G = \{ \langle A, \gamma_G(A), \gamma_H(A) \rangle \mid A \in \mathcal{F} \} \) and \( \lambda_H = \{ \langle A, \lambda_G(A), \lambda_H(A) \rangle \mid A \in \mathcal{F} \} \) be any two intuitionistic anti-fuzzy subrings of a HX ring \( R \).

The cartesian anti-product of \( \gamma_G \) and \( \lambda_H \) is defined as

\[
(\gamma_G \times \lambda_H) = \{ \langle A, (\gamma_G \times \lambda_H)(A, B) \rangle \mid (A, B) \in \mathcal{F}_1 \times \mathcal{F}_2 \}
\]

where

\[
(\gamma_G \times \lambda_H)(A, B) = \min \{ \gamma_G(A), \lambda_H(B) \}
\]

for all \((A, B) \in \mathcal{F}_1 \times \mathcal{F}_2\).

3.1.8 Theorem

Let \( G \) and \( H \) be any two intuitionistic fuzzy sets of \( R \). Then, \( \mathcal{F}_1 \subset 2^{R_1} \) \( - \) \{\( \emptyset \)\} and \( \mathcal{F}_2 \subset 2^{R_2} \) \(-\) \{\( \emptyset \)\} be any two HX rings. If \( g^H \) and \( h^H \) are any two intuitionistic anti-fuzzy HX subrings of \( R \) and \( \mathcal{F}_1 \) respectively, then \( g^H \times h^H \) is also an intuitionistic anti-fuzzy HX subring of a HX ring \( R \).

Proof

It is clear.

3.1.9 Theorem

Let \( H \) be an intuitionistic fuzzy set defined on \( R \). Let \( \lambda^H \) be an intuitionistic anti-fuzzy HX subring of \( R \) if and only if \( (\lambda^H)^\wedge \) is an intuitionistic anti-fuzzy HX subring of \( R \).

Proof

Let \( \lambda^H \) be an intuitionistic fuzzy HX subring of \( R \). Let \( A, B \in \mathcal{F} \).

i. \( \lambda^H(A-B) \geq \min \{ \lambda^H(A), \lambda^H(B) \} \)

ii. \( \lambda^H(A) \geq \min \{ \lambda^H(A), \lambda^H(B) \} \)

iii. \( \lambda^H(A) \leq \min \{ \lambda^H(A), \lambda^H(B) \} \)

Hence, \( \gamma_G \circ \lambda_H \) is an intuitionistic anti-fuzzy HX subring of a HX ring \( R \).

3.1.10 Definition

Let \( H \) be an intuitionistic fuzzy set of \( R \). Let \( \mathcal{F}_1 \subset 2^{R_1} \) \(-\) \{\( \emptyset \)\} be a HX ring. If \( \lambda_H \) be an intuitionistic fuzzy set of \( R \). We define the following “necessity” and possibility” operations:

\[
\square \lambda_H = \{ \langle A, \lambda_H(A), 1- \lambda_H(A) \rangle \mid A \in \mathcal{F} \}
\]

\[
\Diamond \lambda_H = \{ \langle A, 1- \lambda_H(A), \lambda_H(A) \rangle \mid A \in \mathcal{F} \}
\]

3.1.11 Theorem

Let \( H \) be an intuitionistic fuzzy set on \( R \). Let \( \lambda_H \) be an intuitionistic anti-fuzzy HX subring of \( R \).
3.2 Homomorphism and anti homomorphism

In this section, we introduce the concept of an anti-image, anti-pre-image of an intuitionistic anti-fuzzy HX subring of a HX ring and discuss its properties under homomorphism and anti homomorphism.

3.2.1 Definition

Let R_1 and R_2 be any two rings. Let \( \mathfrak{R}_1 \subset 2^{R_1} - \{\phi\} \) and \( \mathfrak{R}_2 \subset 2^{R_2} - \{\phi\} \) be any two HX rings defined on \( R_1 \) and \( R_2 \) respectively. Let H and G be any two intuitionistic fuzzy subsets in \( R_1 \) and \( R_2 \) respectively. Let \( \lambda_1 \) and \( \gamma_0 \) be any two intuitionistic anti-fuzzy HX subrings of \( \mathfrak{R}_1 \) and \( \mathfrak{R}_2 \) respectively induced by H and G. Let f: \( \mathfrak{R}_1 \rightarrow \mathfrak{R}_2 \) be a mapping then the image of \( \lambda_1 \) denoted as f(\( \lambda_1 \)) is an intuitionistic fuzzy subset of \( \mathfrak{R}_2 \) and is defined as for each \( U \in \mathfrak{R}_2 \),

\[
(f(\lambda_1))(U) = \begin{cases} 
\sup\{\lambda_2(X) : X \in f^{-1}(U)\} & \text{if } f^{-1}(U) \neq \phi \\
1, & \text{otherwise}
\end{cases}
\]

Also the pre-image of \( \gamma_0 \) denoted as \( f^{-1}(\gamma_0) \) under f is an intuitionistic fuzzy subset of \( \mathfrak{R}_1 \) defined as for each \( X \in \mathfrak{R}_1 \),

\[
(f^{-1}(\gamma_0))(X) = \inf\{\lambda_1(Y) : Y \in f(X)\} \text{ and } (f^{-1}(\gamma_0))(X) = \gamma_0(f(X)).
\]
\[(f(\lambda_G))(f(X)) \geq \min\{f(\lambda_G)(f(X)), f(\lambda_H)(f(Y))\}\]

iv. \[(f(\lambda_H))(f(X)) f(Y)) \geq \min\{f(\lambda_H)(f(X)), f(\lambda_H)(f(Y))\}\]

Hence, \(f(\lambda_H)\) is an intuitionistic anti-fuzzy HX subring of \(R_2\).

3.2.3 Theorem

Let \(R_1\) and \(R_2\) be any two HX rings on \(R_1\) and \(R_2\) respectively. Let \(f : R_1 \to R_2\) be a homomorphism on HX rings. Then, \(G\) be an intuitionistic anti-fuzzy HX subring of \(R_2\), then \(f^{-1}(\gamma_0)\) is an intuitionistic anti-fuzzy HX subring of \(R_1\).

Proof

Let \(G = \{ (\gamma, \alpha(y), \beta(y)) / y \in R_2 \} \) be an intuitionistic anti-fuzzy subsets defined on a ring \(R_2\).

Then, \(\gamma_0 = \{ (X, \gamma_0(X), \gamma_0(Y)) / Y \in R_2 \} \) be an intuitionistic anti-fuzzy HX subring of a HX ring \(R_2\).

Then, \(f^{-1}(\gamma_0) = \{ (X, f^{-1}(\gamma_0)(X), f^{-1}(\gamma_0)(Y)) / Y \in R_1 \} \).

For any \(X, Y \in R_1, f(X), f(Y) \in R_2\),

i. \((f^{-1}(\gamma_0))(X-Y) = \gamma_0(f(X)-f(Y)) \)

\[= \gamma_0(f(X))-\gamma_0(f(Y)) \leq \max\{\gamma_0(f(X)), \gamma_0(f(Y))\} \]

\[= \max\{(f^{-1}(\gamma_0)) (X), (f^{-1}(\gamma_0)) (Y)\}\]

\((f^{-1}(\gamma_0))(X-Y) \leq \max\{(f^{-1}(\gamma_0)(X)),(f^{-1}(\gamma_0))(Y)\} \)

ii. \((f^{-1}(\gamma_0))(XY) = \gamma_0(f(X)f(Y)) \)

\[= \gamma_0(f(X)f(Y)) \leq \max\{\gamma_0(f(X)), \gamma_0(f(Y))\} \]

\[= \max\{(f^{-1}(\gamma_0))(X), f^{-1}(\gamma_0)(Y)\}\]

\((f^{-1}(\gamma_0))(XY) \leq \max\{(f^{-1}(\gamma_0))(X), (f^{-1}(\gamma_0))(Y)\}\)

iii. \((f^{-1}(\gamma_0))(X-Y) = \gamma_0(f(X)-f(Y)) \)

\[= \gamma_0(f(X)-f(Y)) \geq \min\{\gamma_0(f(X)), \gamma_0(f(Y))\} \]

\[= \min\{(f^{-1}(\gamma_0))(X), f^{-1}(\gamma_0)(Y)\}\]

\((f^{-1}(\gamma_0))(X-Y) \geq \min\{(f^{-1}(\gamma_0))(X), (f^{-1}(\gamma_0))(Y)\}\)

Hence, \(f^{-1}(\gamma_0)\) is an intuitionistic anti-fuzzy HX subring of \(R_1\).

3.2.4 Theorem

Let \(R_1\) and \(R_2\) be any two HX rings on the rings \(R_1\) and \(R_2\) respectively. Let \(f : R_1 \to R_2\) be an homomorphism onto HX rings. Let \(H\) be an intuitionistic anti-fuzzy subset of \(R_1\). Let \(\lambda_H\) be an intuitionistic anti-fuzzy HX subring of \(R_1\) then

\(f(\lambda_H)\) is an intuitionistic anti-fuzzy HX subring of \(R_2\), if \(\lambda_H\) has an infimum property and \(\lambda_H\) is \(f\)-invariant.

Proof

Let \(H = \{ (x, \mu(x), \nu(x)) / x \in R_1 \} \) be an intuitionistic anti-fuzzy subsets defined on a ring \(R_1\).

Then, \(\lambda_H = \{ (X, \lambda_H(X), \lambda_H(Y)) / X \in R_1 \} \) be an intuitionistic anti-fuzzy HX subring of a HX ring \(R_1\).

Then, \(f(\lambda_H) = \{ (X, f(\lambda_H)(f(X)), f(\lambda_H)(f(Y))) / X \in R_1 \}\).

There exist \(X, Y \in R_1\) such that \(f(X), f(Y) \in R_2\),

i. \((f(\lambda_H))(f(X)-f(Y)) = (f(\lambda_H))(f(X)-f(Y)) \)

\[= \lambda_H(X-Y) \leq \max\{\lambda_H(X), \lambda_H(Y)\} \]

\[= \max\{(f(\lambda_H))(f(X)), (f(\lambda_H))(f(Y))\}\]

ii. \((f(\lambda_H))(f(X)f(Y)) = (f(\lambda_H))(f(X)f(Y)) \)

\[= \lambda_H(XY) \leq \max\{\lambda_H(X), \lambda_H(Y)\} \]

\[= \max\{(f(\lambda_H))(f(X)), (f(\lambda_H))(f(Y))\}\]

\((f(\lambda_H))(f(X)f(Y)) \leq \max\{(f(\lambda_H))(f(X)), (f(\lambda_H))(f(Y))\}\)

iii. \((f(\lambda_H))(f(X)-f(Y)) = (f(\lambda_H))(f(X)-f(Y)) \)

\[= \lambda_H(X-Y) \geq \min\{\lambda_H(X), \lambda_H(Y)\} \]

\[= \min\{(f(\lambda_H))(f(X)), (f(\lambda_H))(f(Y))\}\]

\((f(\lambda_H))(f(X)-f(Y)) \geq \min\{(f(\lambda_H))(f(X)), (f(\lambda_H))(f(Y))\}\)

Hence, \(f(\lambda_H)\) is an intuitionistic anti-fuzzy HX subring of \(R_2\).

3.2.5 Theorem

Let \(R_1\) and \(R_2\) be any two HX rings on \(R_1\) and \(R_2\) respectively. Let \(f : R_1 \to R_2\) be an anti homomorphism on HX rings. Let \(G\) be an intuitionistic anti-fuzzy subset of \(R_2\). Let \(\gamma_0\) be an intuitionistic anti-fuzzy HX subring of \(R_2\), then \(f^{-1}(\gamma_0)\) is an intuitionistic anti-fuzzy HX subring of \(R_1\).

Proof

Let \(G = \{ (\gamma, \alpha(y), \beta(y)) / y \in R_2 \} \) be an intuitionistic anti-fuzzy subsets defined on a ring \(R_2\).

Then, \(\gamma_0 = \{ (X, \gamma_0(X), \gamma_0(Y)) / Y \in R_2 \} \) be an intuitionistic anti-fuzzy HX subring of a HX ring \(R_2\).

Hence, \(f^{-1}(\gamma_0)\) is an intuitionistic anti-fuzzy HX subring of \(R_1\).
Then, \( f^{-1}(\gamma_G) = \{ <X, f^{-1}(\gamma_G)(X), f^{-1}(\gamma_H)(X)> | X \in \mathfrak{R}_1 \} \). For any \( X, Y \in \mathfrak{R}_1, f(X), f(Y) \in \mathfrak{R}_2, \)

i. \( (f^{-1}(\gamma_G))(X, Y) = \gamma_G(f(X, Y)) = \gamma_G(f(X) - f(Y)) \leq \max \{ \gamma_G(f(X)), \gamma_G(f(Y)) \} = \max \{ (f^{-1}(\gamma_G)(X)), (f^{-1}(\gamma_G)(Y)) \} \)

\( (f^{-1}(\gamma_G))(X, Y) \leq \max \{ (f^{-1}(\gamma_G)(X)), (f^{-1}(\gamma_G)(Y)) \}. \)

ii. \( (f^{-1}(\gamma_G))(XY) = \gamma_G(f(XY)) = \gamma_G(f(Y) f(X)) \leq \max \{ \gamma_G(f(Y)), \gamma_G(f(X)) \} = \max \{ (f^{-1}(\gamma_G)(X)), (f^{-1}(\gamma_G)(Y)) \} \)

\( (f^{-1}(\gamma_G))(XY) \leq \max \{ (f^{-1}(\gamma_G)(X)), (f^{-1}(\gamma_G)(Y)) \}. \)

iii. \( (f^{-1}(\gamma_H))(X, Y) = \gamma_H(f(X, Y)) = \gamma_H(f(X) - f(Y)) \geq \min \{ \gamma_H(f(X)), \gamma_H(f(Y)) \} = \min \{ (f^{-1}(\gamma_H)(X)), (f^{-1}(\gamma_H)(Y)) \} \)

\( (f^{-1}(\gamma_H))(X, Y) \geq \min \{ (f^{-1}(\gamma_H)(X)), (f^{-1}(\gamma_H)(Y)) \}. \)

iv. \( (f^{-1}(\gamma_H))(XY) = \gamma_H(f(XY)) = \gamma_H(f(Y) f(X)) \geq \min \{ \gamma_H(f(Y)), \gamma_H(f(X)) \} = \min \{ (f^{-1}(\gamma_H)(X)), (f^{-1}(\gamma_H)(Y)) \} \)

\( (f^{-1}(\gamma_H))(XY) \geq \min \{ (f^{-1}(\gamma_H)(X)), (f^{-1}(\gamma_H)(Y)) \}. \)

Hence, \( f^{-1}(\gamma_G) \) is an intuitionistic anti-fuzzy HX subring of \( \mathfrak{R}_1. \)

4. Conclusion

In this paper we introduce the concept of intuitionistic anti-fuzzy HX ring and discuss the basic results on intuitionistic anti-fuzzy HX ring.

Also we introduce the image and pre-image of an intuitionistic anti-fuzzy HX subring and discuss some of its properties.

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