Difference Speed sequence graph

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Abstract: A (p,q) graph  G(V,E) is said to be a difference speed sequence graceful graph if there exists a bijection f: V(G) → { 0, 1, 2, ...q } such that the induced mapping f: E(G) →{∆(x)/ i=1, 2, 3, ...n} defined by f(uv) = |f(u) – f(v)| is a bijection. Here ∆m(x) = (∆ mxk) = |xk – xk+m| and (x) is the Fibonacci sequence. The function f is called a difference speed sequence graceful graph.

Keywords: graceful labeling, speed sequence labeling, speed sequence graphs

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1. Introduction

Throughout this paper, by a graph we mean a finite, undirected graph G(V, E) with 'p ' vertices and 'q' edges. A detailed survey of graph labeling can be found in the dynamic survey of labeling by J.A. Gallian. In this paper we introduce a new labeling called speed sequence labeling. We use the following definitions in the following sections.

Definition: 1.1. A (p,q) graph  G(V,E) is said to be a difference speed sequence graceful graph if there exists a bijection f: V(G) → { 0, 1, 2, ...q } such that the induced mapping f: E(G) →{∆(x)/ i=1, 2, 3, ...n} defined by f(uv) = |f(u) – f(v)| is a bijection. Here ∆m(x) = (∆ mxk) = |xk – xk+m| and (x) is the Fibonacci sequence. The function f is called a difference speed sequence graceful graph.

Definition: 1.2. A complete bipartite graph  K₁,n is called a star and it has n + 1 vertices and n edges.

Definition: 1.3. The bistar graph B_m,n is the graph obtained from a copy of star K₁,m and a copy of start K₁,n by joining the vertices of maximum degree by an edge.

Definition: 1.4. A star S_n is the complete bipartite graph K₁,n is a tree with one internal node and n leaves.

Definition: 1.5. The graph obtained by joining a pendant edge at each vertex of a path P_n is called a comb and is denoted by P_n ⊙ K₁ or P_n *

Definition: 1.6. The corona G₁ ⊙ G₂ is defined as the graph obtained by taking one copy of G₁ to every point in the i-th copy of G₂.

Definition: 1.7. The graph (P_m, S_n) is obtained from m copies of the star graph S_n and the path P_m: {u₁, u₂, ..., u_m} by joining u_i with the centre of the j-th copy of S_n by means of an edge 1≤ j≤ m.

Definition: 1.8. A subdivision of a graph G is a graph that can be obtained from G by a sequence of edge subdivisions.

Definition: 1.9. An (n,t) kite graph consists of a cycle of length n with a t edge path attached to one vertex.

2. Main Results:

In this paper, we prove that path P_n, K₁,n, bistar B_m,n, the graph by subdividing the edges of the star K₁,n, the generalized crown C₃ ◯ K₁,n, the comb P_n ⊙ K₁, the graph (P_m, S_n), (3,t) kite graph are the difference speed sequence graphs.

Theorem: 2.1

The Path P_n is a difference speed sequence graceful for all n ≥ 2.

Proof:

Let u₁, u₂, ..., u_n be the vertices of the path P_n.
Define \( f : V(G) \rightarrow \{ 0, 1, 2, \ldots, q^2 \} \) and \( E(G) \rightarrow \{ \Delta(x)/ i=1, 2, 3, \ldots n \} \) and \( x \) is the fibonacci sequence. Then \( f \) induces a bijection \( f : E(G) \rightarrow \{ \Delta(x)/ i=1, 2, 3, \ldots n \} \)

**Example:**

\[
\begin{array}{c}
\Delta_2(x) \\
\text{Figure 1}
\end{array}
\]

\[
\begin{array}{c}
\Delta_3(x) \\
\text{Figure 2}
\end{array}
\]

**Theorem: 2.2**

Every comb graph \( P_n \oplus k_1 \) is a difference speed sequence graceful graph.

**Proof:**

Let \( u_1, u_2, \ldots, u_n \) be the vertices of the path \( P_n \).

Let \( v_1, v_2, \ldots, v_n \) be the vertices adjacent to each vertex of \( P_n \).

Define \( f : V(P_n \oplus k_1) \rightarrow \{ \Delta(x)/ i=1, 2, 3, \ldots n \} \)

Then \( f \) induces a bijection \( f : E(P_n \oplus k_1) \rightarrow \{ \Delta(x)/ i=1, 2, 3, \ldots n \} \)

**Example:**

\[
\begin{array}{c}
\Delta_2(x) \\
\text{Figure 3}
\end{array}
\]

\[
\begin{array}{c}
\Delta_3(x) \\
\text{Figure 4}
\end{array}
\]

**Theorem :2.3**

The star \( K_{1 , n} \) is a difference speed sequence graph.

**Proof:**

Let \( V (K_{1 , n}) = \{ u_i / 1 \leq i \leq n + 1 \} \)

Let \( E (K_{1 , n}) = \{ \Delta(x)/ i=1, 2, 3, \ldots n \} \)

Define an injection \( f : V (K_{1 , n}) \rightarrow \{ 0, 1, 2, \ldots, 2q^2 \} \) by \( f(u_i) = \Delta(x) \) if \( 1 \leq i \leq n \)

Then \( f \) induces a bijection \( f : E (K_{1 , n}) \rightarrow \{ \Delta(x)/ i=1, 2, 3, \ldots n \} \)

**Example:**

\[
\begin{array}{c}
\Delta_2(x) \\
\text{Figure 5}
\end{array}
\]
Theorem: 2.4

The graph obtained by the subdivision of the edges of the star $K_{1,n}$ is a difference speed sequence graph.

Proof:

Let $G$ be the graph obtained by the subdivision of the edges of the star $K_{1,n}$.

Let $V(G) = \{ v, u_i, w_i / 1 \leq i \leq n \}$ and $E(G) = \{ vu_i, u_iw_i / 1 \leq i \leq n \}$

Define an injection $f: V(G) \rightarrow \{ 0, 1, 2, \ldots, 3q \}$ and $E(G) \rightarrow \{ \Delta_i(x) / i=1, 2, 3, \ldots, n \}$

and $f(v) = 0$.

Then $f$ induces a bijection $f: E(G) \rightarrow \{ \Delta_i(x) / i=1, 2, 3, \ldots, n \}$

Therefore the subdivision of the edges of the star $K_{1,n}$ is a difference speed sequence graph.

Example:

Theorem: 2.5

Every bistar $B_{m,n}$ is a difference speed sequence graceful.

Proof:

Let $V(B_{m,n}) = \{ 0, 1, 2, \ldots, 2(m+n+1) \}$ and $E(B_{m,n}) \rightarrow \{ \Delta_i(x) / i=1, 2, 3, \ldots, n \}$

Then $f$ induces a bijection $f: E(B_{m,n}) \rightarrow \{ \Delta_i(x) / i=1, 2, 3, \ldots, n \}$

Case(i) : $m > n$

Define an injection $f: V(B_{m,n}) \rightarrow \{ 0, 1, 2, \ldots, 2(m+n+1) \}$ by

$f(u_i) = |m+n-6i|$ for $i=1$

$f(u_i) = |m+n-2i+1|$ for $i=2$

$f(u_i) = |m+n-5i|$ for $i=3$

$f(u_i) = |m+n-4i|$ for $i=4$

and $f(u) = 0$, $f(v) = m+n+1$

$f(v_i) = |m+n-4i(m+n)|$ for $i=1$

$f(v_i) = |m+n-(5i/2)(m+n)+1|$ for $i=2$

$f(v_i) = |m+n-mn|$ for $i=3$

Case(ii) : $m < n$
Define an injection \( f: V(B_{m,n}) \rightarrow \{0, 1, 2, \ldots, 2(m+n+1)^2\} \) by

\[
f(u_i) = |m+n-4i(m+n)| \text{ for } i=1
\]
\[
f(u_i) = |m+n-(5i/2)(m+n) + 1| \text{ for } i=2
\]
\[
f(u_i) = |m+n-mn| \text{ for } i=3
\]
and \( f(u) = m+n+1 ; f(v) = 0 \)

\[
f(v_i) = | m+n-6i| \text{ for } i=1
\]
\[
f(v_i) = |m+n-(2i+1)| \text{ for } i=2
\]
\[
f(v_i) = |m+n- 5i| \text{ for } i=3
\]
\[
f(v_i) = |m+n-4i| \text{ for } i=4
\]

Then \( f \) induces a bijection \( E(G) \rightarrow \{\Delta_i(x)/ i=1, 2, 3, \ldots n\} \)

Therefore the bistar \( B_{m,n} \) is a difference speed sequence graph.

**Example:**

![Image of a graph](image9)

\( \Delta_2(x) \)

Figure 9

Let \( \{v_1, v_2, v_3, u_1, u_2, u_3, u_4, u_5, u_6\} \) be the vertices of \( C_3 \odot k_{1,2} \)

Here \( \{v_1, v_2, v_3\} \) are the vertices of the cycle \( C_3 \) and \( \{u_1, u_2, u_3, u_4, u_5, u_6\} \) are the vertices of the copies of \( k_{1,2} \) adjacent to \( v_i \) for \( i=1, 2, 3 \)

The size of the graph is \( q = 3n+3 \)

Define an injection \( f: V(C_3 \odot k_{1,2}) \rightarrow \{0, 1, 2, \ldots, (3n+3)^2\} \) by \( f(v_1) = 0, f(v_2) = 1, f(v_3) = 3, \)

and \( E(G) \rightarrow \{\Delta_i(x)/ i=1, 2, 3, \ldots n\} \)

Then \( f \) induces a bijection \( E(G) \rightarrow \{\Delta_i(x)/ i=1, 2, 3, \ldots n\} \)

Therefore the crown graph \( C_3 \odot k_{1,2} \) is a difference speed sequence graph.

**Example:**

![Image of a graph](image10)

\( \Delta_2(x) \)

Figure 10

**Theorem 2.6**

The crown graph \( C_3 \odot k_{1,2} \) is a difference speed sequence graceful

**Proof:**
Theorem: 2.7

The kite graph \( (3,t) \) is difference speed graceful for \( t \geq 1 \)

Proof:

Let \( \{v_1, v_2, v_3\} \) be the vertices of a cycle \( C_3 \) and \( \{u_1, u_2, \ldots, u_t\} \) be the \( t \) vertices of the tail with \( v_1 \) adjacent to \( u_1 \).

The size of \( G \) is \( q = 3 + t \)

Define a bijection \( f: V(G) \to \{0, 1, 2, \ldots, (3+2t)^2\} \) for all \( t = 1 \ldots n \) and

\( E(G) \to \{\Delta_i(x)/ i=1, 2, 3, \ldots n\} \)

Therefore \( f \) induces a bijection.

Hence \( (3,t) \) is a kite difference speed sequence graceful graph.

Example:
Example:

![Diagram for Example 1](image1)

\[ \Delta_2(x) \]

Figure 14

![Diagram for Example 2](image2)

\[ \Delta_3(x) \]

Figure 15

Conclusion:

Here we have introduced a new labeling called difference speed sequence labeling. We have also proved it for various types of graphs.

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