Fourier series involving certain products of generalized class of polynomials, Aleph-function and the multivariable Aleph-function

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ABSTRACT

The aim of the present document is to establish some finite integrals and Fourier serie expansion for the products of class of polynomials, Aleph-function and multivariable Aleph-function. The results established in this paper are of general nature and hence encompass several particular cases.

Keywords: Multivariable Aleph-function, Aleph-function, Fourier serie, general class of polynomials.

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1. Introduction and preliminaries.

The Aleph- function, introduced by Südland [10] et al., however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integral:

\begin{equation}
N(z) = N_{P, Q, c, r}^{M, N} \left( \left. \begin{array}{c}
(a_j, A_j)_{1, n}, [c_i(a_{ji}, A_{ji})]_{n+1, p_i, r} \\
(b_j, B_j)_{1, m}, [c_i(b_{ji}, B_{ji})]_{m+1, q_i, r}
\end{array} \right| z \right)
\end{equation}

\begin{equation}
= \frac{1}{2\pi \omega} \int_{L} \Omega_{P, Q, c, r}^{M, N}(s) z^{-s} ds
\end{equation}

for all \( z \) different to 0 and

\begin{equation}
\Omega_{P, Q, c, r}^{M, N}(s) = \frac{\prod_{j=1}^{M} \Gamma(b_j + B_j s) \prod_{j=1}^{N} \Gamma(1 - a_j - A_j s)}{\sum_{i=1}^{r} c_i \prod_{j=N+1}^{P_i} \Gamma(a_{ji} + A_{ji} s) \prod_{j=M+1}^{Q_i} \Gamma(1 - b_{ji} - B_{ji} s)}
\end{equation}

With:

\(| arg z | < \frac{1}{2} \pi \Omega \) \quad \text{Where} \quad \Omega = \sum_{j=1}^{M} \beta_j + \sum_{j=1}^{N} \alpha_j - c_i( \sum_{j=M+1}^{Q_i} \beta_j + \sum_{j=N+1}^{P_i} \alpha_j ) > 0 \quad \text{with} \quad i = 1, \cdots, r

For convergence conditions and other details of Aleph-function, see Südland et al [10].

Serie representation of Aleph-function is given by Chaurasia et al [1].

\begin{equation}
N_{P, Q, c, r}^{M, N}(z) = \sum_{G=1}^{M} \sum_{g=0}^{\infty} \left( - \right)^{g} \Omega_{P, Q, c, r}^{M, N}(s) z^{-s}
\end{equation}

With \( s = \eta G, g = \frac{b_G + g}{B_G}, P_i < Q_i, | z | < 1 \) and \( \Omega_{P, Q, c, r}^{M, N}(s) \) is given in (1.2)

The Aleph-function of several variables generalize the multivariable h-function defined by H.M. Srivastava and R. Panda [9], itself is an a generalisation of G and H-functions of multiple variables. The multiple Mellin-Barnes integral occurring in this paper will be referred to as the multivariables Aleph-function throughout our present study and will be defined and represented as follows.

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We have: 
\[ N(z_1, \ldots, z_r) = \prod_{i=1}^{p_i} n_i^{a_{ij}} n_i^{P_i(i)} n_i^{\tau_{i(i)}} n_i^{R(i)} \ldots \prod_{r=1}^{P_i(r)} n_i^{\tau_{i(r)}} n_i^{R(r)} \left( \begin{array}{c} z_1 \\ \vdots \\ z_r \end{array} \right) \]

\[ = \prod_{i=1}^{p_i} \left[ \tau_i(a_{ij}) \gamma_j^{(i)} \beta_j^{(i)} \cdots \tau_i(b_{ij}) \beta_j^{(i)} \right] \]

\[ = \prod_{i=1}^{p_i} \left[ \tau_i(c_{ij}) \gamma_j^{(i)} \beta_j^{(i)} \cdots \tau_i(d_{ij}) \beta_j^{(i)} \right] \]

\[ = \frac{1}{(2\pi \omega)^r} \int_{L_1} \cdots \int_{L_r} \psi(s_1, \ldots, s_r) \prod_{k=1}^{r} \theta_k(s_k) z_k^{s_k} ds_1 \cdots ds_r \]  \hspace{1cm} \text{(1.5)}

with \( \omega = \sqrt{-1} \)

\[ \psi(s_1, \ldots, s_r) = \prod_{i=1}^{r} \frac{\Gamma(1 - a_j + \sum_{k=1}^{r} b_{ij} s_k)}{\prod_{j=n+1}^{n} \Gamma(1 - a_j + \sum_{k=1}^{r} b_{ij} s_k) \prod_{j=1}^{n} \Gamma(1 - c_j + \sum_{k=1}^{r} b_{ij} s_k)} \]  \hspace{1cm} \text{(1.6)}

and \( \theta_k(s_k) = \prod_{i=1}^{r} \frac{\Gamma(d_k^{(i)} + s_k)}{\prod_{j=m+k+1}^{m+1} \Gamma(1 - d_k^{(i)} + s_k) \prod_{j=n+k+1}^{n+1} \Gamma(1 - c_j + \sum_{k=1}^{r} b_{ij} s_k)} \]  \hspace{1cm} \text{(1.7)}

where \( j = 1 \) to \( r \) and \( k = 1 \) to \( r \)

Suppose, as usual, that the parameters:

\[ a_j, j = 1, \ldots, p; b_j, j = 1, \ldots, q; \]

\[ c_j^{(k)}, j = 1, \ldots, n_k; c_j^{(i)}; j = n_k + 1, \ldots, p_i; \]

\[ d_j^{(k)}, j = 1, \ldots, m_k; d_j^{(i)}; j = m_k + 1, \ldots, q_i; \]

with \( k = 1, \ldots, r, i = 1, \ldots, R, i^{(k)} = 1, \ldots, R^{(k)} \)

are complex numbers, and the \( \alpha', \beta', \gamma' \) and \( \delta' \) are assumed to be positive real numbers for standardization purpose such that

\[ U_i^{(k)} = \sum_{j=1}^{n} \alpha_j^{(k)} + \tau_i \sum_{j=n+1}^{p_i} \alpha_j^{(i)} + \tau_i \sum_{j=1}^{n_k} \gamma_j^{(i)} + \tau_i \sum_{j=n_k+1}^{m_k} \beta_j^{(i)} - \tau_i \sum_{j=1}^{q_i} \delta_j^{(i)} \]

\[ -\tau_i \sum_{j=m_k+1}^{m_k} \delta_j^{(i)} \leq 0 \]  \hspace{1cm} \text{(1.8)}

The real numbers \( \tau_i \) are positives for \( i = 1 \) to \( R \), \( \tau_i^{(i)} \) are positives for \( i^{(k)} = 1 \) to \( R^{(k)} \)

The contour \( L_k \) is in the \( s_k-p \) lane and run from \( \sigma - i \infty \) to \( \sigma + i \infty \) where \( \sigma \) is a real number with loop, if necessary, ensure that the poles of \( \Gamma(c_j^{(k)} - d_j^{(k)} s_k) \) with \( j = 1 \) to \( n_k \) are separated from those of \( \Gamma(1 - a_j + \sum_{i=1}^{r} b_{ij} s_k) \) with \( j = 1 \) to \( n \) and \( \Gamma(1 - c_j^{(k)} + \gamma_j^{(k)} s_k) \) with \( j = 1 \) to \( n_k \) to the left of the contour \( L_k \). The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by
extension of the corresponding conditions for multivariable H-function given by as:

\[ |\arg z_k| < \frac{1}{2} A_i^{(k)} \pi, \quad \text{where} \]

\[ A_i^{(k)} = \sum_{j=1}^{n} \alpha_{ij}^{(k)} - \tau_i \sum_{j=n+1}^{p_i} \alpha_{ij}^{(k)} - \tau_i \sum_{j=1}^{q_i} \beta_{ij}^{(k)} + \sum_{j=1}^{n_k} \gamma_j \tau_{i(k)} \sum_{j=n_k+1}^{p_i(k)} \gamma_j^{(k)} \]

\[ + \sum_{j=1}^{m_k} \delta_j^{(k)} - \tau_{i(k)} \sum_{j=m_k+1}^{q_i(k)} \delta_j^{(k)} > 0, \quad \text{with} \quad k = 1 \ldots , r, \quad i = 1, \ldots , R, \quad i^{(k)} = 1, \ldots , R^{(k)} \quad (1.9) \]

The complex numbers \( z_i \) are not zero. Throughout this document, we assume the existence and absolute convergence conditions of the multivariable Aleph-function.

We may establish the asymptotic expansion in the following convenient form:

\[ \mathcal{N}(z_1, \ldots , z_r) = 0( |z_1|^{\alpha_1} \ldots |z_r|^{\alpha_r}); \quad \max( |z_1| \ldots |z_r|) \to 0 \]

\[ \mathcal{N}(z_1, \ldots , z_r) = 0( |z_1|^{\beta_1} \ldots |z_r|^{\beta_r}); \quad \min( |z_1| \ldots |z_r|) \to \infty \]

where, with \( k = 1, \ldots , r : \alpha_k = \min[\Re(\beta_j^{(k)}/\gamma_j^{(k)})], j = 1, \ldots , m_k \) and

\[ \beta_k = \max[\Re((c_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \ldots , n_k \]

We will use these following notations in this paper:

\[ U = p_i, q_i, \tau_i; \quad R ; \quad V = m_1, n_1; \ldots ; m_r, n_r \quad (1.10) \]

\[ W = p_i^{(1)}, q_i^{(1)}, \tau_i^{(1)}; \quad R^{(1)} ; \ldots ; p_i^{(r)}, q_i^{(r)}, \tau_i^{(r)}; \quad R^{(r)} \quad (1.11) \]

\[ A = \{ (\alpha_j, \alpha_j^{(1)}, \ldots , \alpha_j^{(r)}) \}_{1,n} \{ \tau_i (\alpha_j, \alpha_j^{(1)}, \ldots , \alpha_j^{(r)})_{n+1,p_i} \} \quad (1.12) \]

\[ B = \{ \tau_i (\beta_j, \beta_j^{(1)}, \ldots , \beta_j^{(r)})_{m+1,q_i} \} \quad (1.13) \]

\[ C = \{ (c_j, \gamma_j^{(1)}, \ldots , \gamma_j^{(r)})_{1,n} \}, \tau_i (c_j, \gamma_j^{(1)}, \ldots , \gamma_j^{(r)})_{n+1,p_i}, \ldots , \tau_i (c_j, \gamma_j^{(1)}, \ldots , \gamma_j^{(r)})_{n+r-1,p_i} \} \quad (1.14) \]

\[ D = \{ (d_j, \delta_j^{(1)}, \ldots , \delta_j^{(r)})_{1,m} \}, \tau_i (d_j, \delta_j^{(1)}, \ldots , \delta_j^{(r)})_{m+1,q_i}, \ldots , \tau_i (d_j, \delta_j^{(1)}, \ldots , \delta_j^{(r)})_{m+r-1,q_i} \} \quad (1.15) \]

The multivariable Aleph-function write:

\[ \mathcal{N}(z_1, \ldots , z_r) = \mathcal{N}^{(0,n,V)}_{U;W}(z_1, \ldots , z_r; A; C; \ldots ; B; D) \quad (1.16) \]

The generalized polynomials of multivariables defined by Srivastava [7], is given in the following manner:

\[ S^{(n_1, \ldots , n_u)}_{(\mathfrak{m}_1, \ldots , \mathfrak{m}_u)} [y_1, \ldots , y_u] = \sum_{K_1=0}^{[N_1/\mathfrak{m}_1]} \cdots \sum_{K_u=0}^{[N_u/\mathfrak{m}_u]} \frac{(-N_1)_{\mathfrak{m}_1}K_1}{K_1!} \cdots \frac{(-N_u)_{\mathfrak{m}_u}K_u}{K_u!} A[N_1, K_1; \ldots ; N_u, K_u] y_1^{K_1} \cdots y_u^{K_u} \quad (1.17) \]

Where \( \mathfrak{m}_1, \ldots , \mathfrak{m}_u \) are arbitrary positive integers and the coefficients \( A[N_1, K_1; \ldots ; N_u, K_u] \) are arbitrary constants, real or complex.

Srivastava and Garg [8] introduced and defined a general class of multivariable polynomials as follows

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2. Formulas

We have the following integrals, see ([3], p.16(15), [2], p.480(3.891))

\[\begin{align*}
\text{a) } & \int_0^{\pi/2} (\cos y)^t (\cos xy)^{y} \, dy = \frac{\pi \Gamma(t+1)}{2^{t+1} \Gamma \left(1 + \frac{t+\alpha}{2}\right)} \quad \text{where } Re(t) > -1 \\
\text{b) } & \int_0^{\pi} \sin(2h+1)y(siny)^{y} \, dy = \frac{\sqrt{\pi} \Gamma \left(\frac{1-t}{2} + h\right) \Gamma \left(1 + \frac{t}{2}\right)}{\Gamma \left(h + \frac{t+3}{2}\right) \Gamma \left(\frac{1-t}{2}\right)} \quad \text{where } Re(t) > -1 \\
\text{c) } & \int_0^{\pi} e^{(2m+1)y \sin(2n+1)y} \, dy = \frac{i\pi}{2} \delta_{m,n} \quad \text{where } \delta_{m,n} = 1 \text{ if } m = n, 0 \text{ else}
\end{align*}\]  

(2.3)

3. Main Integrals

In the document, we note:

\[a = \frac{(-N_1)\cdots K_1}{K_1!} \cdots \frac{(-N_u)\cdots K_u}{K_u!} \cdot A[N_1, K_1; \cdots; N_u, K_u]\]

\[b = (-E)_{F_1, L_1; \cdots; F_u, L_u} B(E; L_1, \cdots, L_u) \quad \text{if } U_{12} = p_i + 1, q_i + 2, \tau_i; R, U_{22} = p_i + 2, q_i + 2, \tau_i; R\]

Integral 1

\[\int_0^{\pi/2} \cos(u \theta) (\cos \theta)^t S_{E_1, \cdots, E_u} \left[x_1 (\cos \theta)^{l_1}, \cdots, x_u (\cos \theta)^{l_u}\right] S_{M_1, \cdots, M_u} \left[y_1 (\cos \theta)^{k_1}, \cdots, y_u (\cos \theta)^{k_u}\right] \, d\theta = \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_u=0}^{[N_u/M_u]} \sum_{L_1=0}^{[U_{12}/2]} \cdots \sum_{L_u=0}^{[U_{22}/2]} \frac{(-\Omega)^{M,N} A_{G,g}}{B_{G,g}!} \left(\begin{array}{c}
\sum_{i=1}^{u} L_i l_i - \sum_{i=1}^{u} K_i k_i; h_1, \cdots, h_r \ni A : C \\
\sum_{i=1}^{u} L_i l_i - \sum_{i=1}^{u} K_i k_i; h_1, \cdots, h_r \ni B : D
\end{array}\right)\]

(3.1)

Provided

a) \( Re(\alpha) > 0, h_i > 0, i = 1, \cdots, r \ni h > 0 \)

b) \( Re[\ell + h \min_{1 \leq j \leq M} \frac{b_j}{B_j} + \sum_{i=1}^{r} h_i \min_{1 \leq j \leq M} \delta_{j}^{(i)}] > -1 \)

c) \( |argz| < \frac{1}{2} \pi \Omega \) Where \( \Omega = \sum_{j=1}^{M} \beta_j + \sum_{j=1}^{N} \alpha_j - c_i \sum_{j=M+1}^{P_i} \beta_j + \sum_{j=N+1}^{P_i} \alpha_j > 0 \)
d) \(|argz_k| < \frac{1}{2} A_i^{(k)} \pi\), where \(A_i^{(k)}\) is given in (1.9)

**Integral 2**

\[
\int_{0}^{\pi/2} \sin(2h+1)y(siny)^{t} \mathcal{N}(z(siny)^{2k}) S_{E}^{F_1, \cdots, F_v}[x_1(siny)^{2l_1}, \cdots, x_v(siny)^{2l_v}]
\]

\[
S_{N_1, \cdots, N_u}^{2k_1, \cdots, 2k_v} \{y_1(siny)^{2k_1}, \cdots, y_u(siny)^{2k_v}\} \mathcal{N}(z_1(siny)^{2h_1}, \cdots, z_u(siny)^{2h_v}) d\theta
\]

\[
= \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \sum_{K_u=0}^{[N_u/M_u]} \sum_{L_1, \cdots, L_v=0}^{F_1, \cdots, F_v, L_v \in E} \frac{ab(-)^{g} \Omega_{P_1, Q_1, \cdots, P_v, Q_v}}{B_G g!} x^{\eta_{G,g} K_1} y^{\eta_{G,g} K_u} x_{1}^{L_1} \cdots x_{v}^{L_v}
\]

\[
= \left( \frac{z_1}{z_r} \right)^{-(t-2)/-k\eta_{G,g} - \sum_{i=1}^{u} K_i L_i} \cdots \left( \frac{z_r}{z_r} \right)^{-(h-t-1)/-k\eta_{G,g} - \sum_{i=1}^{v} K_i L_i}
\]

\[
(1-t)/2 - k\eta_{G,g} - \sum_{i=1}^{u} L_i L_i \cdots (h-1)/2 - k\eta_{G,g} - \sum_{i=1}^{v} L_i L_i, A : C
\]

\[
(-t+1)/2 - k\eta_{G,g} - \sum_{i=1}^{u} L_i L_i \cdots (-h+1)/2 - k\eta_{G,g} - \sum_{i=1}^{v} L_i L_i, B : D)
\]

Provided

a) \(Re(\alpha) > 0, h_i > 0, i = 1, \cdots, r; \ h > 0\)

b) \(Re(2k) \min_{1 \leq j \leq M} \frac{b_j}{B_j} + 2 \sum_{i=1}^{r} \min_{1 \leq j \leq m_i} \frac{d_j^{(i)}}{\delta_j^{(i)}} > -1\)

c) \(|argz| < \frac{1}{2} \pi\Omega\) where \(\Omega = \sum_{j=1}^{M} \beta_j + \sum_{j=1}^{N} \alpha_j - \sum_{j=M+1}^{Q_v} \beta_{ji} + \sum_{j=N+1}^{P} \alpha_{ji} > 0\)

d) \(|argz_k| < \frac{1}{2} A_i^{(k)} \pi\), where \(A_i^{(k)}\) is given in (1.9)

**Proof of (3.1)**

To establish the finite integral (3.1), express the generalized classes of polynomials \(S_{N_1, \cdots, N_u}^{M_1, \cdots, M_v}\) and \(S_{E}^{F_1, \cdots, F_v}\) occurring on the L.H.S in the series form given by (1.17) and (1.18) respectively, the Aleph-function in series form given by (1.3) and the multivariable Aleph-function involving there in terms of Mellin-Barnes contour integral by (1.5). We interchange the order of summation and integration (which is permissible under the conditions stated). Now evaluating the \(\theta\)-integral by using the formula (2.1), after simplifications and on reinterpreting the Mellin-Barnes contour integral, we get the desired result.

The proof of the integral (3.2) can be developed by proceeding on similar method with the help of (2.2).

4. Fourier series

**First Fourier serie 1**

\[
S_{E}^{F_1, \cdots, F_v}[x_1(cos\theta)^{l_1}, \cdots, x_v(cos\theta)^{l_v}] S_{N_1, \cdots, N_u}^{2k_1, \cdots, 2k_v} \{y_1(cos\theta)^{k_1}, \cdots, y_u(cos\theta)^{k_v}\}(cos\theta)^{h_1} \mathcal{N}(z(cos\theta)^{h_v})
\]

\[
\mathcal{N}(z_1(cos\theta)^{h_1}, \cdots, z_r(cos\theta)^{h_v})
\]
which holds true under the same conditions from (3.1)

Second Fourier series

\[
(\sin \theta)^k N(z_1(\sin \theta)^{2h_k}, \ldots, z_u(\sin \theta)^{2h_u})
\]

\[
S^{2N_1, \ldots, 2N_u}[y_1(\sin \theta)^{2k_1}, \ldots, y_u(\sin \theta)^{2k_u}] N(z_1(\sin \theta)^{2h_1}, \ldots, z_u(\sin \theta)^{2h_u})
\]

Proof of (4.1)
To establish (4.1), let

\[
f(\theta) = \cos(u\theta)(\cos \theta)^k \frac{N_{z_1(\cos \theta)}^{k_1}, \ldots, N_{z_u(\cos \theta)}^{k_u}}{S^{E_k, \ldots, E_u}[x_1(\cos \theta)^{h_1}, \ldots, x_u(\cos \theta)^{h_u}]}
\]
The equation (4.3) is valid since $f(\theta)$ is continuous and of bounded variation in the open interval $(0, \pi)$, multiplying both the sides of (4.3) by $\cos(n\theta)$ and integrate with respect to $y$ from 0 to $\pi$ and use the orthogonal property of cosinus function and the integral (2.1), with substitution we get

$$A_0 = \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{K_1=0}^{N_M} \cdots \sum_{K_u=0}^{N_u} \frac{F_l L_1 + \cdots + F_v L_v \leq E}{L_1, \ldots, L_v = 0} \frac{\pi (-)^g \Omega_{P, Q, c, r}^{M, N} (\eta G, g)}{2^{l+1} + \eta G, g B GG!} x^{\eta G, g} y_1^{L_1} \cdots y_v^{L_v} \pi 2^{-(t+1) + \eta G, g} y_1^{K_1} \cdots y_u^{K_u}$$

Putting the value of $A_0$ in (4.3), we get the formula (4.1). To establish (4.2), let

$$f(\theta) = (\sin(\theta))^t N(z(\sin(\theta))^{2k}) S_{E^1, \ldots, E^v}^{F_1, \ldots, F_v} [x_1(\sin(\theta))^{2l_1}, \ldots, x_v(\sin(\theta))^{2l_v}]$$

$$S_{N_1, \ldots, N_u}^{M_1, \ldots, M_u} [y_1(\sin(\theta))^{2k_1}, \ldots, y_u(\sin(\theta))^{2k_u}] N(z(\sin(\theta))^{2h_1}, \ldots, z_u(\sin(\theta))^{2h_u})$$

$$= \sum_0^\infty B_n e^{(2n+1)iy} 0 < y < \infty$$

The equation (4.5) is valid since $f(\theta)$ is continuous and of bounded variation in the open interval $(0, \pi)$, multiplying both the sides of (4.5) by $\sin((2k + 1)\theta)$ and integrate with respect to $y$ from 0 to $\pi$ and use the integral (2.3), with substitution, we get

$$B_n = \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{K_1=0}^{N_M} \cdots \sum_{K_u=0}^{N_u} \frac{F_l L_1 + \cdots + F_v L_v \leq E}{L_1, \ldots, L_v = 0} \frac{(-)^g \Omega_{P, Q, c, r}^{M, N} (\eta G, g)}{B GG!} x^{\eta G, g} y_1^{K_1} \cdots y_u^{K_u}$$

$$x_1^{L_1} \cdots x_v^{L_v} \frac{2}{i\sqrt{\pi}} N_{U_{22}; W}^{0, n+2; V} \begin{pmatrix} z_1 \\ \vdots \\ z_r \\ \cdots \\ \cdots \\ \cdots \\ (t/2 - \eta G, g - \sum_{i=1}^{u} K_i k_i - \sum_{i=1}^{u} L_i l_i; h_1, \cdots, h_r), A : C \\ \cdots \\ \cdots \\ (-t/2 - \eta G, g - \sum_{i=1}^{u} L_i l_i; h_1, \cdots, h_r), B : D \end{pmatrix} e^{(2n+1)i\theta}$$

(4.6)

5. Multivariable I-function

In these section, we get two formulas concerning Fourier series and multivariable I-function defined by Sharma et al [4]

Let $\tau_1 = \tau_{(1)} = \cdots = \tau_{(r)} = 1$

First Fourier serie

$$S_{E^1, \ldots, E^v}^{F_1, \ldots, F_v} [x_1(\cos(\theta))^l_1, \ldots, x_v(\cos(\theta))^l_v] S_{N_1, \ldots, N_u}^{M_1, \ldots, M_u} [y_1(\cos(\theta))^k_1, \ldots, y_u(\cos(\theta))^k_u] N(z(\cos(\theta))^h)$$

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which holds true under the same conditions from (3.1)

**Second Fourier serie**

\[
(siny)^{r} \sum_{E} \mathcal{N}(z(x\sin\theta)^{2k}) F_{E}^{f_{1}, \ldots, f_{v}} \left[ x_{1}(\sin\theta)^{2l_{1}}, \ldots, x_{v}(\sin\theta)^{2l_{v}} \right]
\]

\[
\sum_{E} \mathcal{N}(z_{1}, \ldots, z_{v}) F_{n_{1}, \ldots, n_{v}} \left[ y_{1}(\sin\theta)^{2k_{1}}, \ldots, y_{v}(\sin\theta)^{2k_{v}} \right] I(z_{1}(\sin\theta)^{2h_{1}}, \ldots, z_{v}(\sin\theta)^{2h_{v}})
\]

which holds true under the same conditions from (3.2)

6. Multivariable H-function

If \( \tau_{\nu} = \tau_{\nu(1)} = \cdots = \tau_{\nu(r)} = 1 \) and \( \tau = \tau^{(1)} = \cdots = \tau^{(r)} = 1 \), then the multivariable Aleph-function degenerates to the multivariable H-function defined by Srivastava et al [9]. And we have the following results.
First Fourier serie

\[ S_{E}^{F_1, \ldots, F_\nu} \left[ x_1(\cos \theta)^{l_1}, \ldots, x_\nu(\cos \theta)^{l_\nu} \right] S_{N_1, \ldots, N_\nu}^{y_1, \ldots, y_\nu} \left[ y_1(\cos \theta)^{k_1}, \ldots, y_\nu(\cos \theta)^{k_\nu} \right] \mathcal{N}(z(\cos \theta)^{h_\nu}) \]

\[ H(z(\cos \theta)^{h_1}, \ldots, z_\nu(\cos \theta)^{h_\nu}) \]

\[ = \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{K_1=0}^{\left[ N_1/M_1 \right]} \sum_{L_1=0}^{\left[ N_1/M_1 \right]} \ldots \sum_{K_\nu=0}^{\left[ N_\nu/M_\nu \right]} \sum_{L_\nu=0}^{\left[ N_\nu/M_\nu \right]} (-)^g \Omega_{P_i, Q_i, c_i, r_i}^{M, N}(\eta_{G,g}) \frac{x^{\eta_{G,g}}}{B_{G,g}!} \frac{x_{L_1, \ldots, L_\nu} y_{K_1, \ldots, K_\nu}}{2^{l_1 + \cdots + l_\nu} x_{L_1, \ldots, L_\nu} y_{K_1, \ldots, K_\nu}} \]

Second Fourier serie

\[ (\sin \gamma)^{\nu} \mathcal{N}(z(\sin \theta)^{2k_\nu}) S_{E}^{F_1, \ldots, F_\nu} \left[ x_1(\sin \theta)^{2l_1}, \ldots, x_\nu(\sin \theta)^{2l_\nu} \right] \]

\[ S_{N_1, \ldots, N_\nu}^{y_1, \ldots, y_\nu} \left[ y_1(\sin \theta)^{2k_1}, \ldots, y_\nu(\sin \theta)^{2k_\nu} \right] H(z(\sin \theta)^{2h_1}, \ldots, z_\nu(\sin \theta)^{2h_\nu}) \]

\[ = \sum_{G=1}^{M} \sum_{g=-\infty}^{\infty} \sum_{K_1=0}^{\left[ N_1/M_1 \right]} \sum_{L_1=0}^{\left[ N_1/M_1 \right]} \ldots \sum_{K_\nu=0}^{\left[ N_\nu/M_\nu \right]} \sum_{L_\nu=0}^{\left[ N_\nu/M_\nu \right]} (-)^g \Omega_{P_i, Q_i, c_i, r_i}^{M, N}(\eta_{G,g}) \frac{x^{\eta_{G,g}} y_{K_1, \ldots, K_\nu}}{B_{G,g}!} 2^{l_1 + \cdots + l_\nu} \]

\[ H_{p+1,q+2:W}^{0,n+1:V} \left[ \begin{array}{c}
2^{-h_1} z_1 \\
\vdots \\
2^{-h_\nu} z_\nu 
\end{array} \right] \cos \theta (6.1) \]

\[ \left( -t \eta_{G,g} - \sum_{i=1}^{\nu} L_i l_i - \sum_{i=1}^{\nu} K_i k_i ; h_1, \ldots, h_\nu, A'' : C'' \right) + \]

\[ \left( \begin{array}{c}
(-t \pm u - h \eta_{G,g} - \sum_{i=1}^{\nu} L_i l_i - \sum_{i=1}^{\nu} K_i k_i )/2 ; h_1/2, \ldots, h_\nu/2, B'' : D'' \n\end{array} \right) \]

which holds true under the same conditions from (3.1)
7. Aleph-function of two variables

In these section, we get the two formulas of Fourier series concerning the Aleph-function of two variables defined by K. Sharma [6].

First Fourier serie

\[
S_{E}^{F_1, \ldots, F_{s}} [x_1 (\cos \theta)^{l_1}, \ldots, x_v (\cos \theta)^{l_v}] S_{N_1, \ldots, N_{u}}^{\Omega_{1}, \ldots, \Omega_{u}} [y_1 (\cos \theta)^{k_1}, \ldots, y_u (\cos \theta)^{k_u}] (\cos \theta)^{l} \mathbb{N}(z(\cos \theta)^{h})
\]

which holds true under the same conditions from (3.2)

Second Fourier serie

\[
S_{E}^{F_1, \ldots, F_{s}} [x_1 (\sin \theta)^{2l_1}, \ldots, x_v (\sin \theta)^{2l_v}] S_{N_1, \ldots, N_{u}}^{\Omega_{1}, \ldots, \Omega_{u}} [y_1 (\sin \theta)^{2k_1}, \ldots, y_u (\sin \theta)^{2k_u}] \mathbb{N}(z_1 (\sin \theta)^{2h_1}, z_2 (\sin \theta)^{2h_2})
\]

which holds true under the same conditions from (3.1)
(1-t)/2 -k_\eta G, g - \sum_{i=1}^{v} L_i l_i - \sum_{i=1}^{u} K_i k_i; h_1, h_2), A_2 : C_2
\right) e^{2n+1}i\theta (7.2)

which holds true under the same conditions from (3.2)

8. I-function of two variables

In these section, we get two results of double series concerning the I-function of two variables defined by Sharma and Mishra [5]. Let \( \tau = \tau' = \tau'' = 1 \)

First Fourier serie

\[ S_E^{F_1, \ldots, F_v} \left[ x_1 (\cos \theta)^{l_1}, \ldots, x_v (\cos \theta)^{l_v} \right] S_{N_1, \ldots, N_v}^{M_1, \ldots, M_v} \left[ y_1 (\cos \theta)^{k_1}, \ldots, y_v (\cos \theta)^{k_v} \right] N(z(\cos \theta)^h) \]

\[ I(z_1 (\cos \theta)^{h_1}, z_2 (\cos \theta)^{h_2}) \]

\[ = \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \sum_{K_2=0}^{[N_2/M_2]} \sum_{L_1=0}^{[N_1/M_1]} \sum_{L_2=0}^{[N_2/M_2]} ab \frac{(-\eta G, g)}{B G!} \frac{x^{\eta G, g}}{2^{-1+\eta G, g}} x_{l_1} \ldots x_{l_2} y_{k_1} \ldots y_{k_2} \]

\[ \begin{pmatrix} 2^{-h_1} & z_1 \\ \vdots \\ 2^{-h_2} & z_2 \end{pmatrix} \begin{pmatrix} (-t -bh_\eta G, g - \sum_{i=1}^{u} L_i l_i - \sum_{i=1}^{u} K_i k_i; h_1, h_2), A_2 : C_2 \\ \vdots \\ \frac{(-t \pm u - h_\eta G, g - \sum_{i=1}^{u} L_i l_i - \sum_{i=1}^{u} K_i k_i/2; h_1/2, h_2/2, B_2 : D_2} \end{pmatrix} \]

which holds true under the same conditions from (3.1)

Second Fourier serie

\[ (\sin \theta)^k N(z(\sin \theta)^{2k}) S_E^{F_1, \ldots, F_v} \left[ x_1 (\sin \theta)^{2l_1}, \ldots, x_v (\sin \theta)^{2l_v} \right] \]

\[ S_{N_1, \ldots, N_v}^{M_1, \ldots, M_v} \left[ y_1 (\sin \theta)^{2k_1}, \ldots, y_v (\sin \theta)^{2k_v} \right] N(z_1 (\sin \theta)^{2h_1}, z_2 (\sin \theta)^{2h_2}) \]

\[ = \sum_{G=1}^{M} \sum_{g=0}^{\infty} \sum_{K_1=0}^{[N_1/M_1]} \sum_{K_2=0}^{[N_2/M_2]} \sum_{L_1=0}^{[N_1/M_1]} \sum_{L_2=0}^{[N_2/M_2]} \sum_{\eta G, g} \frac{(-\eta G, g)}{B G!} \frac{x^{\eta G, g} y_{k_1} \ldots y_{k_v} \sin \theta}{2^{1+i \eta G, g}} \]

\[ \begin{pmatrix} 2^{-h_1} & z_1 \\ \vdots \\ 2^{-h_2} & z_2 \end{pmatrix} \begin{pmatrix} (-t -bh_\eta G, g - \sum_{i=1}^{u} L_i l_i - \sum_{i=1}^{u} K_i k_i; h_1, h_2), A_2 : C_2 \\ \vdots \\ \frac{(-t \pm u - h_\eta G, g - \sum_{i=1}^{u} L_i l_i - \sum_{i=1}^{u} K_i k_i/2; h_1/2, h_2/2, B_2 : D_2} \cos \theta \end{pmatrix} \]

\( (8.1) \)
Due to the nature of the multivariable Aleph-function and the general classes of polynomials \( S_{N_1, \ldots, N_t}^{M_1, \ldots, M_t} \) and \( S_{E_1, \ldots, E_v}^{F_1, \ldots, F_v} \), we can get general product of Laguerre, Legendre, Jacobi and other polynomials, the special functions of one and several variables.

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