Super Root Square Mean Labeling of Some Graphs

R. Chitra devi¹, S. Saravana Kumar²

¹Research Scholar, Department of Mathematics, Sri S.R.N.M college, Sattur-628 203, TamilNadu, India.
²Department of Mathematics, Sri S.R.N.M College, Sattur-628 203, TamilNadu, India.

Abstract: Let G be a (p, q) graph and f: V(G)→{1,2,...,p+q} be an injective function. For a vertex labeling f, the induced edge labeling \( f^*(e)=\sqrt{\frac{f(u)^2+f(v)^2}{2}} \) or \( \sqrt{\frac{f(u)^2+f(v)^2}{2}} \), then f is called a super root square mean labeling if \( f(V)\cup\{f^*(e):e\in E(G)\}={1,2,3,...,p+q} \). A graph which admits super root square mean labeling is called super root square mean graph.

Definition 1.2: A Walk in which \( u_1u_2u_3...u_n \) are distinct is called a path. A path on n vertices is denoted by \( P_n \).

Definition 1.3: A closed path is called a cycle. A cycle on n vertices is denoted by \( C_n \).

Definition 1.4: The corona \( G_1\overrightarrow{\bigcirc} G_2 \) of two graph \( G_1 \) and \( G_2 \) is defined as the graph \( G \) obtained by taking one copy of \( G_1 \), (which has \( P_1 \) vertices) and \( P_1 \) copies of \( G_2 \) and then joining the \( i \)th vertex of \( G_1 \) to every vertex in the \( i \)th copy of \( G_2 \).

Definition 1.5: The middle graph \( M(G) \) of a path \( G \) is the graph whose vertex set is \( V(G)\cup E(G) \) and in which two vertices are adjacent if and only if either they are adjacent edges of \( G \) or one is a vertex of \( G \) and the other is an edge incident with it.

I. INTRODUCTION

All graphs in this paper are finite, simple and undirected graph \( G=(V,E) \) with \( p \) vertices and \( q \) edges. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices(edges) then the labeling is called a vertex labeling(an edges labeling). Several types of graph labeling and a detailed survey is available in [1]. For all other standard terminology and notations we follow Harary[2].

Mean labeling was introduced by S. Somasundaram and R. Ponraj[3],[4]. Root square mean labeling was introduced by S.S. Sandhya, S. Somasundaram and S. Anusa in[5]. In this paper, we introduce super root square mean labeling of graphs and investigate super root square mean labeling of \( nP_m, nK_1, P_i\overrightarrow{\bigcirc} K_2, \) middle graph of path \( P_n, \) dragon \( C_n\overrightarrow{\bigcirc} P_m. \) The following definitions are useful for the present study.

Definition 1.1: [6] Let \( f: V(G)\rightarrow\{1,2,...,p+q\} \) be an injective function. For a vertex labeling \( f, \) the induced edge labeling \( f^*(e) \) is defined by \( f^*(e)=\sqrt{\frac{f(u)^2+f(v)^2}{2}} \) or \( \sqrt{\frac{f(u)^2+f(v)^2}{2}} \), then \( f \) is called a super root square mean labeling if \( f(V)\cup\{f^*(e):e\in E(G)\}={1,2,3,...,p+q} \). A graph which admits super root square mean labeling is called super root square mean graph.

II. Main Results

Theorem 2.1: \( nP_m \) is a super root square mean graph.

Proof: Let the vertices of \( nP_m \) be \( \{v_{ij}: 1\leq i \leq n, 1\leq j \leq m\} \) and the edge of \( nP_m \) be \( \{e_i: (v_{ij}, v_{ij+1}): 1\leq i \leq n, 1\leq j \leq m-1\} \).

Define a function \( f: V(nP_m)\rightarrow\{1,2,...,p+q\} \) by \( f(v_{ij})=(2m-1)(i-1)+2j-1, 1\leq i \leq n, 1\leq j \leq m. \)

Then the induced edge labels of \( nP_m \) is \( f^*(v_{ij}, v_{ij+1})=(2m-1)(i-1)+2j, 1\leq i \leq n, 1\leq j \leq m-1. \)

Thus the vertices and edges together get distinct labels.

Hence \( nP_m \) is a super root square mean graph.

Example 2.2: super root square mean labeling of \( 4P_3 \) is shown in figure 2.1
Theorem 2.3: \(nK_3\) is a super root square mean graph.

**Proof:** Let \(G_1, G_2, \ldots, G_n\) be \(n\) copies of \(K_3\).

Let \(v_1, v_2, v_3\) be the vertices of \(G_1\),
\(v_4, v_5, v_6\) be the vertices of \(G_2\),
\(v_{3i-2}, v_{3i-1}, v_{3i}\) be the vertices of \(G_n\).

Here \(|V(nK_3)|=3n\), \(|E(nK_3)|=3n\).

Define a function \(f : V(nK_3) \rightarrow \{1, 2, \ldots, p+q\}\) by
\[
\begin{align*}
    f(v_{3i-2}) &= 6i-5, \quad 1 \leq i \leq n \\
    f(v_{3i-1}) &= 6i-3, \quad 1 \leq i \leq n \\
    f(v_{3i}) &= 6i, \quad 1 \leq i \leq n \\
\end{align*}
\]

Then the induced edge labels of \(nK_3\) is
\[
\begin{align*}
    f^*(v_{3i-2}v_{3i-1}) &= 6i-4, \quad 1 \leq i \leq n \\
    f^*(v_{3i-1}v_{3i}) &= 6i-2, \quad 1 \leq i \leq n \\
    f^*(v_{3i}v_{3i+1}) &= 6i, \quad 1 \leq i \leq n-1 \\
\end{align*}
\]

Thus the vertices and edges together get distinct labels.

Hence \(nK_3\) is a super root square mean graph.

**Example 2.4:** super root square mean labeling of \(4K_3\) is shown in figure 2.2

Theorem 2.5: \(P_n \circ \overline{K_2}\) is a super root square mean graph.

**Proof:** Let \(P_n\) be the path \(u_1, u_2, \ldots, u_n\) and \(v_i, w_i\) be the vertices of \(\overline{K_2}\) which are attached to the vertex \(u_i\) of \(P_n\).

Let \(G = P_n \circ \overline{K_2}\).

Here \(|V(G)|=3n\), \(|E(G)|=3n-1\).

Define a function \(f : V(G) \rightarrow \{1, 2, \ldots, p+q\}\) by
\[
\begin{align*}
    f(u_i) &= 6i-3, \quad 1 \leq i \leq n \\
    f(v_i) &= 6i-5, \quad 1 \leq i \leq n \\
    f(w_i) &= 6i-1, \quad 1 \leq i \leq n \\
\end{align*}
\]

Then the induced edge labels of \(G\) as follows,
\[
\begin{align*}
    f^*(u_iv_i) &= 6i-4, \quad 1 \leq i \leq n \\
    f^*(v_iw_i) &= 6i-2, \quad 1 \leq i \leq n \\
    f^*(u_iu_{i+1}) &= 6i, \quad 1 \leq i \leq n-1 \\
\end{align*}
\]

Then the vertices and edges together get distinct labels.

Hence \(P_n \circ \overline{K_2}\) is a super root square mean graph.

**Example 2.6:** super root square mean labeling of \(P_n \circ \overline{K_2}\) is shown in figure 2.3

Theorem 2.7: The middle graph \(M(P_n)\) of a path \(P_n\) is a super root square mean graph.

**Proof:** Let \(M(P_n)= (V, E)\), where \(V(M(P_n)) = \{u_i : 1 \leq i \leq n, v_j : 1 \leq i \leq n-1\}\)
\(E(M(P_n)) = \{v_ju_i, u_iu_{i+1}, v_{j+1}v_j : 1 \leq i \leq n-1, 1 \leq j \leq n-2\}\).

Here \(|V(M(P_n))|=2n-1\) and \(|E(M(P_n))|=3n-4\).

Define a function \(f : V(M(P_n)) \rightarrow \{1, 2, \ldots, p+q\}\) by
\[
\begin{align*}
    f(u_i) &= 1, \quad 1 \leq i \leq n \\
    f(v_i) &= 5i-5, \quad 2 \leq i \leq n \\
    f(w_i) &= 5i-3, \quad 1 \leq i \leq n-1 \\
\end{align*}
\]

Then the induced edge labels of \(M(P_n)\) as follows,
\[
\begin{align*}
    f^*(v_ju_i) &= 5i-4, \quad 1 \leq i \leq n-1 \\
    f^*(v_jv_{i+1}) &= 5i-2, \quad 1 \leq i \leq n-1 \\
    f^*(u_iu_{i+1}) &= 5i+1, \quad 1 \leq i \leq n-2 \\
\end{align*}
\]

Then the vertices and edges together get distinct labels.

Hence \(M(P_n)\) is a super root square mean graph.

**Example 2.8:** super root square mean labeling of the middle graph of a path \(P_5\) is shown in figure 2.4
Theorem 2.9: Dragon $C_n \square P_m$ is a super root square mean graph.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of the cycle $C_n$ and $v_1, v_2, \ldots, v_m$ be the vertices of the path $P_m$.

Here $u_n = v_1$.

Let $G = C_n \square P_m$, where $V(G) = \{u_i : 1 \leq i \leq n, v_j : 1 \leq j \leq m\}$, $E(G) = \{e_i = u_iu_{i+1}, 1 \leq i \leq n - 1, e_n = u_1u_n, e_j = v_jv_{j+1}, 1 \leq j \leq m - 1\}$.

Define a function $f : V(C_n \square P_m) \rightarrow \{1, 2, \ldots, p+q\}$ by

$$f(u_i) = \begin{cases} 2i - 1; & 1 \leq i \leq \frac{k}{2} \text{ if } k = \left\lfloor \frac{4n^2+1}{2} \right\rfloor \text{ is even} \\ 2i; & \frac{k+2}{2} \leq i \leq n \text{ if } k = \left\lfloor \frac{4n^2+1}{2} \right\rfloor \text{ is odd} \\ 
\end{cases}$$

$$f(v_j) = 2n+2j-2, \quad 1 \leq j \leq m$$

Then the induced edge labeling of $C_n \square P_m$ is as follows

$$f^*(u_iu_{i+1}) = \begin{cases} k; & i = n \\ 2i; & 1 \leq i \leq \frac{k-1}{2} \text{ if } k = \left\lfloor \frac{4n^2+1}{2} \right\rfloor \\ 2i + 1; & \frac{k+1}{2} \leq i \leq n - 1 \text{ if } k = \left\lfloor \frac{4n^2+1}{2} \right\rfloor \text{ is odd} \\ \end{cases}$$

is odd

$$f^*(u_iu_{i+1}) = \begin{cases} k; & i = n \\ 2i; & 1 \leq i \leq \frac{k}{2} - 1 \text{ if } k = \left\lfloor \frac{4n^2+1}{2} \right\rfloor \\ 2i + 1; & \frac{k}{2} \leq i \leq n - 1 \text{ if } k = \left\lfloor \frac{4n^2+1}{2} \right\rfloor \text{ is even} \\ \end{cases}$$

Thus the vertices and edges together get distinct labels.

Hence dragon $C_n \square P_m$ is a super root square mean graph.

Example 2.10: super root square mean labeling of $C_5 \square P_6$ is shown in figure 2.5

References:


