1 -Near Mean Cordial Labeling of $D_2(P_n)$, $P_n(+)N_m$ (when n is even), Jelly Fish $J(m,n)$ graphs

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Abstract — Let $G=(V,E)$ be a simple graph. A surjective function $f : V \rightarrow \{0,1,2\}$ is said to be 1-Near Mean Cordial labeling if for each edge uv, the induced map

$$f(\{u, v\}) = 0 \text{ if } \frac{f(u) + f(v)}{2} \text{ is an integer}$$

Satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with 0 label and $e_f(1)$ is the number of edges with 1 label. $G$ is said to be 1-Near Mean Cordial Graph if it has a 1-Near Mean Cordial Labeling. In this paper we proved that $D_2(P_n), P_n(+)N_m$ (When n is even), Jelly Fish $J(m,n)$ graphs are 1-Near Mean Cordial Graphs.

Keywords — 1-Near Mean Cordial Labeling, 1-Near Mean Cordial Graph

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1. Introduction

Let us consider the graphs to be finite, undirected and simple. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. The cardinality of $V(G)$ and $E(G)$ are respectively called order and size of $G$. Labeling of graphs has enormous application in many practical problems involved in circuit designing, communication network, astronomy etc. [1]. The concept of mean cordial labeling was introduced by Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram in the year 2012 in [5]. Some results on Mean Cordial Labeling was discussed in [3,4,5,6]. Let $f$ be a function $V(G) \rightarrow \{0,1,2\}$. For each edge uv of $G$ assign the label $\frac{f(u) + f(v)}{2}$. $f$ is called a mean cordial labeling of $G$ if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with $x (x = 0,1,2)$ respectively. A graph with a mean cordial labeling is called mean cordial graph. K.Palani, J.Rejila Jeya Surya [2] introduced a new concept called 1-Near Mean Cordial labeling and investigated the 1-Near Mean Cordial Labeling behavior of Paths, Combs, Fans and Crowns. Terms defined here are used as in F. Harary [7].

2. PRELIMINARIES

K.Palani, J.Rejila Jeya Surya [2] define the concept of 1-Near Mean Cordial labeling as follows

Let $G=(V,E)$ be a simple graph. A surjective function $f : V \rightarrow \{0,1,2\}$ is said to be 1-Near Mean Cordial Labeling if for each edge uv, the induced map

$$f^+(\{u,v\}) = 0 \text{ if } \frac{f(u) + f(v)}{2} \text{ is an integer}$$

Satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is the number of edges with zero label and $e_f(1)$ is the number of edges with one label. $G$ is said to be a 1-Near Mean Cordial Graph if it has a 1-Near Mean cordial labeling. In this paper we proved that $D_2(P_n), P_n(+)N_m$ (When n is even), Jelly Fish $J(m,n)$ graphs are 1-mean near cordial graphs

Definition : 2.1 : $D_2(P_n)$ is the graph of two copies of path graph $P_n$ consisting of vertices $u_i$ and $v_i$ for $1 \leq i \leq n$. Hence $D_2(P_n)$ consists of $V(G) = 2n$ and $E(G) = 4(n-1)$. 

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Definition 2.2: $P_n(+)N_m$ is the graph with order of vertices $m+n$ and size of edges $2m+n-1$.

Definition 2.3: Jelly Fish $J(m,n)$ is a graph with order of vertices $m+n+4$ and size of edges is $m+n+5$.

3. Main Results

Theorem 3.1: Graph $D_2(P_n)$ is 1-Near Mean Cordial graph

Proof: Let $G=(V,E)$ be a simple graph and let $G$ be $D_2(P_n)$.

Let $V(G)=\{u_i,v_i:1\leq i \leq n\}$,

$E(G)=\{(u_i,u_{i+1}):1\leq i \leq n-1\} \cup \{(v_i,v_{i+1}):1\leq i \leq n-1\} \cup \{(u_i,v_{i+1}):1\leq i \leq n-1\}$

Define $f : V \rightarrow \{0,1,2\}$ by

$$f(u_i)=0 \text{ if } i \equiv 1 \mod 2$$

$$=2 \text{ if } i \equiv 0 \mod 2$$

$$f(v_i)=1 \text{ if } i \equiv 1 \mod 2$$

$$=2 \text{ if } i \equiv 0 \mod 2$$

The induced edge labeling are

$$f^*(u_i,v_{i+1})=0 \text{ for } i \text{ is odd} \quad \{1 \leq i \leq n-1\}$$

$$=1 \text{ for } i \text{ is even}$$

$$f^*(v_i,u_{i+1})=1 \text{ for } i \text{ is odd} \quad \{1 \leq i \leq n-1\}$$

$$=0 \text{ for } i \text{ is even}$$

Hence, the graph satisfies the condition $|e_f(0) - e_f(1)| \leq 1$

Therefore, the graph $D_2(P_n)$ is a 1-Near Mean Cordial Graph.
\[ f^*(v_iu_i) = 1 \text{ when } i \text{ is odd} \]
\[ = 0 \text{ when } i \text{ is even} \]

For \( f^*(u_iu_{i+1}) \) is a sequence of 0’s and 1’s

For \( P_4(+) \) \( N_m \); \( e_f(0) \) is one more than \( e_f(1) \)

Other wise for all \( n \) even \( P_n(+) \) \( N_m \) either \( e_f(0) \) or \( e_f(0) \) is one more than \( e_f(1) \).

Hence it satisfies the condition \( |e_f(0) - e_f(1)| \leq 1 \)

Therefore the graph \( P_n(+) \) \( N_m \) is a 1-Near Mean Cordial graph when \( n \) is even and for all \( n, m \geq 1 \)

**Theorem 3.3:** Jelly Fish \( J(m,n) \) graph is 1-Near Mean Cordial Graph

**Proof:** Let \( G = (V,E) \) is a simple graph and let \( G = J(m,n) \) be a Jelly Fish graph. Let the vertices of \( G \) be defined as \( V(G) = V_1 \cup V_2 \) where \( V_1 = \{x,u,v\} \) and \( V_2 = \{u,v;1 \leq i \leq m, 1 \leq j \leq n\} \) and the edges \( E = E_1 \cup E_2 \) where \( E_1 = \{xu,uy,vv,xy,vx\} \) and \( E_2 = \{uu,vv;1 \leq i \leq m, 1 \leq j \leq n\} \).

For labeling the vertices let us assign

\[ f(x) = 0; f(u) = 1; f(y) = 1; f(v) = 1 \]

\[ f(u_i) = 1 \text{ if } i \equiv 1 \text{ mod } 2 \]
\[ = 2 \text{ if } i \equiv 0 \text{ mod } 2 \]

\[ f(v_j) = 1 \text{ if } j \equiv 1 \text{ mod } 2 \]
\[ = 2 \text{ if } j \equiv 0 \text{ mod } 2 \]

Then the induced labeling for edges

\[ f^*(uu_i) = 0 \text{ if } i \equiv 1 \text{ mod } 2 \]
\[ = 1 \text{ if } i \equiv 0 \text{ mod } 2 \]

\[ f^*(vv_j) = 0 \text{ if } j \equiv 1 \text{ mod } 2 \]
\[ = 1 \text{ if } j \equiv 0 \text{ mod } 2 \]

Also we have \( f(xu) = 1; f(uy) = 0; f(yv) = 0; f(xv) = 1; f(xy) = 1 \)

We can have the following cases

Case.1 When both \( m, n \) are equal and odd
We find that \( e_f(0) \) is one more than \( e_f(1) \)

Case.2 When both \( m, n \) are odd , \( m > n \)
We find that \( e_f(0) \) is one more than \( e_f(1) \)

Case.3 When both \( m, n \) are odd, \( m < n \)
We find that \( e_f(0) \) is one more than \( e_f(1) \)

Case.4 When both \( m, n \) are equal and even
We find that \( e_f(1) \) is one more than \( e_f(0) \)
Case.5 When both m,n are even, \( m \succ n \)
We find that \( e_f(1) \) is one more than \( e_f(0) \)

Case.6 When both m,n are even, \( m \prec n \)
We find that \( e_f(1) \) is one more than \( e_f(0) \)

Case.7 When m is even , n is odd
We find that \( e_f(0) = e_f(1) \)

Case.8 When m is odd, n is even
We find that \( e_f(0) = e_f(1) \)

Clearly in all the cases mentioned above it satisfies the condition \( |e_f(0) - e_f(1)| \leq 1 \) where \( e_f(0) \) is the number of edges with label 0 and \( e_f(1) \) is the number of edges with label 1.

Hence the Jelly Fish \( J(m,n) \) graph is 1- Near Mean Cordial graph.

REFERENCES


[7]. F.Harary, Graph Theory, Addison-Wesley Publishing Company Inc, USA, 1969