On the Monophonic Number of Line Graph

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Abstract.

For a connected graph G = (V, E) of order at least 2, a subset S of V is said to be a monophonic set of G if each vertex V of G lies on an x-y monophonic path for some elements x and y in S. The minimum cardinality of a monophonic set of G is the monophonic number of G. In this paper, we obtain the monophonic number of line graph.

Keywords: Monophonic set, monophonic number, monophonic distance, line graph.

1. Introduction

By a graph G = (V, E) we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For any graph G = (V, E), the Line graph L(G) whose vertices correspond to the edges of G and two vertices in L(G) are adjacent if and only if the corresponding edges in G are adjacent.

The vertex set and edge set of L(G) is V(G) \{V(G)} and E(V(G)) respectively. A chord of a path P is an edge joining two non-adjacent vertices of P. A path P is called monophonic if it is a chordless path. A vertex v is an extreme vertex if the subgraph induced by its neighbours is complete. For any two vertices u and v in a connected graph G, the monophonic distance d_u(v) from u to v is defined as the length of a longest u-v monophonic path in G. A set S of vertices of a graph G is a monophonic set of G if each vertex v of G lies on an x - y monophonic path in G for some x, y \in S. The minimum cardinality of a monophonic set of G is the monophonic number of G and is denoted by m(G).

The monophonic eccentricity e_v(G) of a vertex v in G is e_v(G) = max\{d_u(v) : u \in V(G)\}. The monophonic radius, rad_u(G) of G is rad_u(G) = min\{e_v(G) : v \in V(G)\} and the monophonic diameter, diam_u(G) of G is diam_u(G) = max\{e_v(G) : v \in V(G)\}. A vertex u in G is a monophonic eccentric vertex of a vertex v in G if e_v(u) = d_u(v, u). The monophonic closure J_u[S] is the set formed by the union of all monophonic closed intervals J_u[v, v] with u, v \in S. If e = [u, v] is an edge of a graph G with d(u) = 1 and d(v) > 1, then we call ‘e’ a pendant edge.

In this paper, we find the monophonic number line graph of a graph G. A set S' \subseteq V' is called a monophonic set of L(G) if J_u[S'] = V' for some vertices in S’ and the minimal cardinality of S’ is the monophonic number of L(G) and is denoted by m(L(G)) = p’.

2. Preliminary Results

Theorem 2.1: [2] Each extreme vertex of a connected graph G belongs to every monophonic set of G. Moreover, if the set S of all extreme vertices of G is a monophonic set, then S is the unique minimum monophonic set of G.

Corollary 2.2: [2] If T is a tree with k end vertices, then m(T) = k.

Theorem 2.3: [2] For any connected graph G, 2 \leq m(G) \leq p.

Corollary 2.4: [2] For the path P_p, m(P_p) = 2.

Corollary 2.5: [2] For the path C_p, m(C_p) = 2.

Theorem 2.6: [3] For integers m, n \geq 2, m(K_m \times K_n) = 2.

3. Monophonic number of line graph

Theorem 3.1. Each vertices in L(G) corresponds to the pendant edges e_i (1 \leq i \leq r) in G must belongs to S’ of L(G).

Proof. Let e = v_v_i be the pendant edge in G and Let S and S’ be the monophonic set of G and L(G) respectively. By the definition of pendant edge, consider d(v_i) = 1 and d(v) \geq 2, this shows that v_i is pendant vertex in G for all i, which is an extreme vertex of G. By Theorem 2.1 all v_i \in S. Since d(v) \geq 2, let v_1, v_2, ..., v_r be the vertices adjacent with v then the edge v_1, v_2, ..., v_i has a common vertex v in G, then vertices in L(G) corresponds to the pendant edges forms a complete graph K, in L(G). Then by Theorem 2.1 the result follows.

Figure 1.1G
Example 3.2 For the graph G given in Figure 2, a monophonic set $S = \{v_1, v_4, v_6\}$ and the line graph $L(G)$ in Figure 3, $S' = \{E_5, E_6, E_7\}$

**Figure 2. G**

**Figure 3. L(G)**

Theorem 3.2: For any tree $T$ with $k$ pendant edges, $m(L(T)) = k$.

Proof: Let $S$ be the set of all end vertices of $T$. By Corollary 2.2 $|S| = m(T) = k$. Each vertex in $S$ has a pendant edge in $T$. By Theorem 3.1 the corresponding pendant edges forms a unique extreme vertex set $S'$ in $L(T)$, which is minimal, then by Theorem 2.1 $|S'| = m(L(T)) = k$.

Corollary 3.3: For the path $P_p (p \geq 2)$, $m(L(P_p)) = 2$.

Corollary 3.4: For the cycle $C_p (p \geq 2)$, $m(L(C_p)) = 2$.

Proof: Since the line graph of $C_p$ is again $C_p$, then by Corollary 2.5 $m(L(C_p)) = 2$.

Theorem 3.5: Let $G$ be a connected graph of order $p$ and $q$, $m(L(G)) = p'$ if and only if $G$ is star graph.

Proof: Let $G$ be a graph with $m(L(G)) = p'$. Since $m(L(G)) = p'$ then each vertex in $L(G)$ must be a extreme vertex. Then the edges in $G$ corresponds to the vertices in $V'$ are pendant edges. So that it must be star graph. Conversely let $G$ be a star graph, then all the edges in $G$ are pendant edge in $G$ then by Theorem 3.1, that correspond edges forms a unique minimal monophonic set $S'$ of $L(G)$, so that $m(L(G)) = p'$

4. Bounds on monophonic number of a line graph

Theorem 4.1. For any connected graph $G$, $2 \leq m(L(G)) \leq p'$.

Remark 4.2. The bound in the above theorem are sharp. For the star graph $K_{1,p+1} (p \geq 2)$, the set of two end edges of path $P_p (p \geq 2)$ is its unique minimal monophonic set of $L(P_p)$, so that $m(L(P_p)) = 2$.

Theorem 4.3. For any integer $k$ such that $2 \leq k \leq p'$, there is a connected graph of order $p$ such that $m(L(G)) = k$.

Proof. For $k = p-1$, the theorem follows from theorem 3.5 by taking $G = K_{1,p-1}$. For $2 \leq m(L(G)) \leq p-2$, the theorem follows from theorem 3.2 by taking $G = T \neq K_{1,p-1}$.

Theorem 4.4. For any non-complete connected graph $G$ of order $p$, $2 \leq m(L(G)) \leq m(G)$. The other inequality is trivial.

Corollary 4.5. For the complete bipartite graph $K_{m,n}$, then $m(L(K_{m,n})) = 2$.

Proof. By result 7.1.8 in [4], $L(K_{m,n}) = K_m \times K_n$, then $m(L(K_{m,n})) = m(K_m \times K_n)$ and result follows by Theorem 2.6.

Remark 4.6. The bounds in Theorem 4.4 are sharp. For the star graph $K_{1,p+1}$, $m(K_{1,p+1}) = m(L(K_{1,p+1}))$. For the non-trivial path $P_p$, $m(P_p) = m(L(P_p)) = 2$. For the complete bipartite graph $K_{m,n}$, $m(L(K_{m,n})) < m(K_{m,n})$.

Conclusion

In this paper we define the monophonic number of line graph of $G$. We also determine Lower and upper bound of the monophonic number line graph $L(G)$. Also we establish a relation between monophonic number of graph and line graph

REFERENCES