Volume mensuration relation of two Cuboids
(Relation All Mathematics)

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Abstract:
In this research paper, Cube, Cuboid and two cuboids volume relation is explained with the help of formula. We can understand the difference between their volumes, also with this formula. This volume relation is considered in two parts as i) when height is same and ii) when height is un-equal.

We are trying to give a new concept “Relation All Mathematics” to the world. I am sure that this concept will be helpful in Agricultural, Engineering, Mathematical world etc.

Keywords:
Volume, Sidemeasurement, Relation, Cube, Cuboid

I. Introduction

Relation All Mathematics is a new field and the various relations shown in this research, “Volume mensuration relation of two cuboids” is a 3rd research paper of Relation All Mathematics. And in future, the research related to this concept, that must be part of “Relation Mathematics” subject. Here, we have studied and shown new variables, letters, concepts, relations, and theorems. Inside the research paper explained relation between two cuboids explained in two parts. i.e. i) When height is same and ii) when height is un-equal.

Sidemeasurement is an explained new concept which is very important related to Relation Mathematics subject.

In this “Relation All Mathematics” we have proved the relation between cube – cuboid and two cuboids with the help of formula. This “Relation All Mathematics” research work is near by 300 pages. This research is done considering the Agricultural sector mainly, but I am sure that it will also be helpful in other sector also.

II. Basic concept of Cube and Cuboids

2.1. Sidemeasurement(B):
- If sides of any geometrical figure are in right angle with each other, then those sides or considering one of the parallel and equal sides after adding them, the addition is the sidemeasurement. Sidemeasurement indicated with letter ‘B’

Sidemeasurement of right angled triangle - B(ΔPQR) = b+h
In ΔPQR, sides PQ and QR are right angle, performed to each other.

Sidemeasurement of rectangle-B(□PQRS)= l₁+b₁
In □PQRS, opposite sides PQ and RS are similar to each other and m<Q = 90°. Here side PQ and QR are right angle performed to each other.

Sidemeasurement of cuboid–E_B(□PQRS) = l₁+b₁+h₁
In E(□PQRS), opposite sides are parallel to each other and QM are right angle performed to each other. Sidemeasurement of cuboid written as $E_B(□PQRS)$

### 2.2) Important points of square-rectangle relation :-

I) For explanation of square and rectangle relation following variables are used
i) Area – $A$
ii) Perimeter – $P$
iii) Sidemeasurement – $B$

II) For explanation of square and rectangle relation following letters are used
i) Area of square $ABCD$ – $A(□ABCD)$
ii) Perimeter of square $ABCD$ – $P(□ABCD)$
iii) Sidemeasurement of square $ABCD$ – $B(□ABCD)$
iv) Area of rectangle $PQRS$ – $A(□PQRS)$
v) Perimeter of rectangle $PQRS$ – $P(□PQRS)$
vi) Sidemeasurement of rectangle $PQRS$ – $B(□PQRS)$

### 2.3) Important points of cube-cuboid relation:-

I) For explanation of Cube-Cuboid relation following variables are used
i) Cuboid - $E$
ii) Cube - $G$
iii) Volume - $V$
iv) Vertical surface area - $U$
v) Total surface area - $A$
vi) Sidemeasurement - $B$

II) Concept of explanation of Cube-Cuboid

![Figure I: Concept of cube and cuboid](image)

Explanation of cube and cuboid is given with the reference of its upper side.
In Fig.I, cuboid explained with the reference of rectangle i.e. $E(□PQRS)$ and cube explained with reference of square. i.e. $G(□ABCD)$.

III) For explanation of Cube-Cuboid relation following letters are used
i) Volume of cube $ABCD$ – $G_V(□ABCD)$
ii) Volume of cuboid $PQRS$ – $E_V(□PQRS)$
iii) Vertical surface area of the cube $ABCD$ – $G_U(□ABCD)$
iv) Total surface area of the cube $ABCD$ – $G_A(□ABCD)$
v) Vertical surface area of the cuboid $PQRS$ – $E_U(□PQRS)$
vi) Total surface area of the cuboid $PQRS$ – $E_A(□PQRS)$
vii) Sidemeasurement of cube $ABCD$ – $G_B(□ABCD)$
ix) Sidemeasurement of cuboid $PQRS$ – $E_B(□PQRS)$
2.4) Un-equal height Volume Relation formula of cube and cuboid(Z):

In cube and cuboid when perimeter of square and rectangle is same but height of both are unequal then difference between volume of both are maintained with the help of ‘Un-equal height Volume Relation formula of cube and cuboid(Z)’ and both sides volume relation of cube and cuboid become equal.

Un-equal height Volume Relation formula of cube and cuboid indicated with letter ‘Z’

\[ Z = [l_1 \cdot b_1 \cdot (h - h_1)] \]  

\[ \text{here } h = \frac{(l_1 + b_1)}{2} \text{ and } l_1 \cdot b_1 = L^2 \]

2.5) Important Reference theorem of previous paper which used in this paper:-

Theorem :- Basic theorem of area relation of square and rectangle

Perimeter of square and rectangle is same then area of square is more than area of rectangle, at that time area of square is equal to sum of the, area of rectangle and Relation area formula of square-rectangle(K).

Proof formula :- \[ A(\square ABCD) = A(\square PQRS) + \left[ \frac{(l_1 + b_1)}{2} - b_1 \right]^2 \]

[Note :- The proof of this formula given in previous paper and that available in reference]

Theorem :- Basic theorem of perimeter relation of square-rectangle

Area of square and rectangle is same then perimeter of rectangle is more than perimeter of square, at that time perimeter of rectangle is equal to product of the, perimeter of square and Relation perimeter formula of square-rectangle(V).

Proof formula :- \[ P(\square PQRS) = P(\square ABCD) \times \frac{1}{2} \left[ \frac{(n^2+1)}{n} \right] \]

[Note :- The proof of this formula given in previous paper and that available in reference]
III. Relation between cube and cuboids.

Relation –I: Volume relation of cube and cuboid when height is same

Known information: Side of cube \( G(□ABCD) \) is ‘l’. and length, width and height of cuboid \( E(□PQRS) \) is \( l_1, b_1 \) and \( h_1 \) respectively.

\[ G_B(□ABCD) = E_B(□PQRS) \quad \ldots l_1 > l. \]

Figure –IV : Equal height Volume relation of cube and cuboid

To prove: \( G_V(□ABCD) = E_V(□PQRS) + h \cdot \left[ \frac{(l_1+b_1)}{2} - b_1 \right]^2 \)

Proof: In \( G(□ABCD) \) and \( E(□PQRS) \),

\[ A(□ABCD) = A(□PQRS) + \left[ \frac{(l_1+b_1)}{2} - b_1 \right]^2 \ldots K=\left[ \frac{(l_1+b_1)}{2} - b_1 \right]^2 \]

\( \ldots \) (Basic theorem of area relation of square and rectangle)

Multiply to both side with height ‘h’.

\[ A(□ABCD) \times h = [A(□PQRS) \times h] + h \cdot \left[ \frac{(l_1+b_1)}{2} - b_1 \right]^2 \]

Here, \( h \cdot \left[ \frac{(l_1+b_1)}{2} - b_1 \right]^2 \) is a ‘Volume Relation formula of cube and cuboid’ and it explains with letter ‘T’

\[ G_V(□ABCD) = E_V(□PQRS) + h \cdot \left[ \frac{(l_1+b_1)}{2} - b_1 \right]^2 \]

Hence, we are prove that Volume relation of cube and cuboid when height is same.

This Relation cleared that following points-

1) Sidemeasurement of cube and cuboid are equal.

2) But volume of cube is more than volume of cuboid and that relation explained with the help of formula.
Example :-

<table>
<thead>
<tr>
<th>WATER TANK</th>
<th>□ABCD (CUBE)</th>
<th>□PQRS (CUBOID)</th>
<th>+ T</th>
<th>REMARK</th>
</tr>
</thead>
<tbody>
<tr>
<td>GIVEN</td>
<td>LENGTH</td>
<td>10</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>WIDTH</td>
<td>10</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HEIGHT</td>
<td>10</td>
<td>10</td>
<td>EQUAL</td>
</tr>
<tr>
<td>EXPLANATION</td>
<td>VERTICAL SURFACE AREA</td>
<td>400</td>
<td>400</td>
<td>EQUAL COST</td>
</tr>
<tr>
<td></td>
<td>TOTAL SURFACE AREA</td>
<td>600</td>
<td>568</td>
<td>DIFFERENCE 32 NEGLIGIBLE</td>
</tr>
<tr>
<td></td>
<td>LHS</td>
<td>1000</td>
<td>840</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RHS</td>
<td>1000</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>RESULT IN WATER CAPACITY</td>
<td>WATER CAPACITY ON TANK</td>
<td>27,300 ltr (27.3 ltr/sqft)</td>
<td>22,932 ltr (27.3 ltr/sqft)</td>
<td>Water store Difference 4368 ltr</td>
</tr>
<tr>
<td></td>
<td>ANSWER</td>
<td>1000</td>
<td>1000</td>
<td>LHS=RHS</td>
</tr>
</tbody>
</table>

Relation –II: Volume relation of Cube and Cuboid when height is un-equal.

**Known information:** Side of cube G(□ABCD) is ‘l’. and length, width and height of cuboid E(PQRS) is l1, b1 and h1 respectively.

B(□ABCD)=B(□PQRS) … l1 > l.

But, side of cube (h)≠ height of cuboid (h1)

**Figure –V** : Un-equal height Volume relation of Cube and Cuboid

To prove : \( G_V(□ABCD) = E_V(□PQRS) + h \left[ \frac{(l_1 + b_1)}{2} - b_1 \right]^2 + A(□PQRS) \times (h-h_1) \)

**Proof:** In G(□ABCD) and E(□PQRS),

\( G_V(□ABCD) = E_V(□PQRS) + h \left[ \frac{(l_1 + b_1)}{2} - b_1 \right]^2 \) ...(i) ...( Volume relation of cube and cuboid when height is same)

But,
\[ G_V(\Box ABCD) \neq E_V(\Box PQRS) + h \left( \frac{(l_1 + b_1)}{2} - b_1 \right)^2 \]

Here height of cube and cuboid is unequal, so add value of ‘Z’ in RHS. So equation become,

\[ G_V(\Box ABCD) = E_V(\Box PQRS) + h \left( \frac{(l_1 + b_1)}{2} - b_1 \right)^2 + Z \]

\[ G_V(\Box ABCD) = E_V(\Box PQRS) + h \left( \frac{(l_1 + b_1)}{2} - b_1 \right)^2 + [l_1b_1 \times (h-h_1)] \quad \ldots[Z = l_1b_1 \times (h-h_1)] \]

\[ G_V(\Box ABCD) = E_V(\Box PQRS) + h \left( \frac{(l_2 + b_1)}{2} - b_1 \right)^2 + [A(\Box PQRS) \times (h-h_1)] \]

Hence, we are prove that Volume relation of Cube and Cuboid when height is un-equal.

This relation cleared that following points,

1) In cube and cuboid, side measurement of square and rectangle are same but height is unequal.

2) Be remember related to cube, length and width of cube are equal but height is not necessary to equal with its side.

3) Height of cube—cuboid is unequal at that time volume relation between them explained here with the help of formula.

Relation—III: Volume relation of two cuboids when height is same

**Known information:** The length, width and height of cuboid E(PQRS) is \(l_1, b_1\) and \(h_1\) and cuboid E(LMNO) is \(l_2, b_2\) and \(h_2\) respectively.

\[ G_B(\Box ABCD) = E_B(\Box PQRS) = E_B(\Box LMNO) \quad \ldots l_2 > l_1 > h \quad \& \quad h = h_1 = h_2 \]

**Figure VI:** Equal height Volume relation of two cuboids

To prove: \(E_V(\Box PQRS) = E_V(\Box LMNO) + h \times (b_1 - b_2) \times (l_1 + b_1 - (b_1 + b_2))\)

**Proof:** In \(G(\Box ABCD)\) and \(E(\Box PQRS)\),

\[ G_V(\Box ABCD) = E_V(\Box PQRS) + h \left( \frac{(l_1 + b_1)}{2} - b_1 \right)^2 \ldots (i) \]

\[ \ldots (\text{Volume relation of cube and cuboid when height is same}) \]

In \(G(\Box ABCD)\) and \(E(\Box LMNO)\),
G_V(□ABCD) = E_V(□LMNO) + h\left[\frac{(l_2+b_2)}{2} - b_2\right]^2 \ldots (ii)

\ldots (Volume relation of cube and cuboid when height is same)

E_V(□PQRS) + h\left[\frac{(l_1+b_1)}{2} - b_1\right]^2 = E_V(□LMNO) + h\left[\frac{(l_2+b_2)}{2} - b_2\right]^2

E_V(□PQRS) = E_V(□LMNO) + h\left[\frac{(l_2+b_2)}{2} - b_2\right]^2 - h\left[\frac{(l_1+b_1)}{2} - b_1\right]^2 \ldots \text{From equation no. (i)}

= E_V(□LMNO) + h \times \left[\left(\frac{(l_2+b_2)}{2} - b_2\right)^2 - \left(\frac{(l_1+b_1)}{2} - b_1\right)^2\right]

= E_V(□LMNO) + h \times \left(\frac{(l_2+b_2)}{2} - \frac{(l_1+b_1)}{2} + b_1\right) \times \left(\frac{(l_2+b_2)}{2} - b_2 + \frac{(l_1+b_1)}{2} - b_1\right)

\ldots (a^2 - b^2) = (a+b) (a-b)

E_V(□PQRS) = E_V(□LMNO) + h \times (b_1 - b_2) \times [(l_1+b_1) - (b_1+b_2)]

Hence, we are prove that, Volume relation of two cuboids when height is same

This relation cleared that following points,

1) Side measurement of two cuboids are equal

2) But among the both cuboid, minimum length of cuboid compared with another cuboid that’s volume is more than volume of another cuboid, and that relation here explained with the help of formula.

Relation -IV: Volume relation of two cuboids when height is un-equal.

**Known information:** The length, width and height of cuboid E(PQRS) is $l_1, b_1$ and $h_1$ and cuboid E(LMNO) is $l_2, b_2$ and $h_2$ respectively.

$B(□ABCD) = B(□PQRS) = B(□LMNO) \ldots l_2 > l_1 \geq l \& h \neq h_1 \neq h_2$

But, side of cube (h) ≠ height of cuboid (h1)

![Diagram of cuboids](http://www.ijmmtjournal.org)

**Figure – VII:** Un-equal Volume relation of two cuboids

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To prove:
\[ E_V(\square PQRS) = E_V(\square LMNO) + \frac{l_1b_1}{2}(h - h_1) \]

Proof:

In \(G(\square ABCD)\) and \(E(\square PQRS)\),
\[ G_V(\square ABCD) = E_V(\square PQRS) + h \left[ \frac{(l_1 + b_1)}{2} - b_1 \right]^2 + [l_1b_1(h-h_1)] \]  
...(i)

...(Volume relation of Cube and Cuboid when height is un-equal.)

In \(G(\square ABCD)\) and \(E(\square LMNO)\),
\[ G_V(\square ABCD) = E_V(\square LMNO) + h \left[ \frac{(l_2 + b_2)}{2} - b_2 \right]^2 + [l_2b_2(h-h_2)] \]  
...(ii)

...(Volume relation of Cube and Cuboid when height is un-equal.)

\[ E_V(\square PQRS) + \frac{l_1b_1}{2}(h - h_1) = E_V(\square LMNO) + h \left[ \frac{(l_2 + b_2)}{2} - b_2 \right]^2 + [l_2b_2(h-h_2)] \]

...From equation no.(i) and (ii)

\[ E_V(\square PQRS) = E_V(\square LMNO) + h \left[ \frac{(l_2 + b_2)}{2} - b_2 \right]^2 + \frac{l_1b_1}{2}(h - h_1) \]  

Hence, we are prove that Volume relation of two cuboids when height is un-equal.

This Relation cleared that following points-
1)Two cuboid inside sidemeasurement of two rectangle are same but height is un-equal.
2)As like project, inside height are depends upon their situation.
3)In this relation two cuboide relation explained with the help of formula when height is unequal.

Note: In above relation \(l_1+b_1=l_2+b_2\) and \(h\) is defined as \(h = \frac{l_1+b_1}{2}\)
References

[1]. Surrounding agricultural life.
[2]. Answers.yahoo.com (www.answers.yahoo.com)